BT4/M05

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Mathematics–III

Paper : MATH–201E

Time : Three Hours] [Maximum Marks : 100

Note :— Attempt any **FIVE** questions, selecting at least **ONE** question from each unit.

UNIT–I

1\. (a) If $f(x) = 0, \quad -\pi < x < 0,$

$\quad = \sin x, \quad 0 < x < \pi,$

prove that

$$f(x) = \frac{1}{\pi} + \frac{\sin x}{2} - \frac{2}{\pi}\sum_{m=1}^{\infty} \frac{\cos 2mx}{4m^2 - 1}$$

Hence show that

$$\frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} - \ldots\ldots\infty = \frac{1}{4}(\pi - 2).$$ 10

(b) If $f(x) = |\cos x|$, expand $f(x)$ as a Fourier series in the interval $(-\pi, \pi)$. 10

2\. (a) Find the Fourier cosine transform of $f(x) = \frac{1}{1 + x^2}$. 10

(b) Solve $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$, $u(x, 0)$ [illegible]

bounded where $x > 0, t > 0$. 10

UNIT–II

3\. (a) If $\tan(\theta + i\phi) = e^{i\alpha}$, show that $\theta = \left(n + \frac{1}{2}\right)\frac{\pi}{2}$ and

$$\phi = \frac{1}{2}\log\tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)$$ 10

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... analytic at the origin even though C-R equations are satisfied at that point. 10

4. (a) Find the analytic function $f(z) = u + iv$, if

$$u - v = \frac{x - y}{x^2 + 4xy + y^2}.$$ 10

(b) Find the condition that the transformation $w = \frac{az + b}{cz + d}$ transforms the circle $|w| = 1$ into a straight line in the z-plane. 10

UNIT–III

5. (a) A and B throw alternatively with a pair of dice. The one who throws 9 first wins. Show that chances of their winning are 9 : 8. 10

(b) In a bolt factory, machines A, B and C manufacture 25%, 35% and 40% of the total. Of their output 5%, 4% and 2% are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A, B or C ? 10

6. (a) If the probability that a new-born child is a male 0·6, find the probability that in a family of 5 children there are exactly 3 boys. 10

(b) Show that for a normal distribution mean deviation about mean is $\frac{4}{5}$ of its standard deviation. 10

UNIT–IV

7. (a) Solve graphically the following LPP :

Minimize $Z = 20x_1 + 10x_2$

subject to $x_1 + 2x_2 \le 40$, $3x_1 + x_2 \ge 30$,

$4x_1 + 3x_2 \ge 60$, $x_1, x_2 \ge 0$. 10

(b) Using Simplex method, solve the LPP

Maximize $Z = 2x_1 + 3x_2$

subject to $x_1 + x_2 \leq 400$, $2x_1 + x_2 \leq 600$,

$x_1, x_2 \geq 0$.

8. (a) Find an optimal solution to the following LPP by comp basic solutions and then finding one that maximizes the o function :

Maximize $Z = 2x_1 + 3x_2 + 4x_3 + 7x_4$,

subject to $2x_1 + 3x_2 - x_3 + 4x_4 = 8$,

$x_1 - 2x_2 + 6x_3 - 7x_4 = -3$,

$x_1, x_2, x_3, x_4 \geq 0$.

(b) Using Dual Simplex method, solve the following LPP :

Minimize $Z = x_1 + 2x_2 + 3x_3$

subject to $2x_1 - x_2 + x_3 \geq 4$, $x_1 + x_2 + 2x_3 \leq 8$,

$x_2 - x_3 \geq 2$, $x_1, x_2, x_3 \geq 0$.