UNIT - 5

ADVANCED TOPICS IN BENDING OF BEAM.

1. Fatigue strength :- (Apr/May 2010)

The maximum stress in a material can endure for a given number of stress cycle without break. It is also known as endurance strength.

2. Endurance ratio : (Apr/May 2010)

The ratio of the endurance limit for completely reversed flexural stress to the tensile strength.

3. Winkler - Bach formula for a curved beam (Apr/May 2010)

$$\sigma = \frac{M}{RA}\left[1 + \frac{R^2}{h^2}\left(\frac{y}{R+y}\right)\right]$$

σ - Bending stress.
R - Radius of curvature.
A - Area of cross section
M - Bending Moment.
h - Constant for c/s / link radius.

4. Distinguish between symmetrical & Unsymmetrical sections of beam. (Dec 2010)

If the section is symmetrical, the principal axes are along the axes of symmetry.

If the section is unsymmetrical, the plane of loading does not lie in a plane that contains the principal centroidal axis of c/s.

5. Causes of fatigue in beams? (Dec 2010)

machine parts are subjected to fluctuating loads which cause variation of stress in the member. Even if the fluctuation stresses are smaller than the ultimate tensile strength of the materials under static load, failure may occur if the load is repeated a sufficient number of times.

Structural members like aircraft, ships and

6. Reasons for unsymmetrical bending. (Apr 2011, June 2012)

* The section is symmetrical but the load line is inclined to the both the principal axis.
* The section itself is unsymmetrical and the load line is along the centroidal axis.

7. Expression for ~~position of neutral axis in case of curved bar~~ principal centroidal moments of Inertia

$$I_{uu} = \left(\frac{I_{yy} + I_{xx}}{2}\right) + \sqrt{\left(\frac{I_{yy} - I_{xx}}{2}\right)^2 + (I_{xy})^2}$$

$$I_{vv} = \left[\frac{I_{yy} + I_{xx}}{2}\right] - \sqrt{\left(\frac{I_{yy} - I_{xx}}{2}\right)^2 + (I_{xy})^2}$$

$$I_{xx} + I_{yy} = I_{uu} + I_{vv}.$$

8. Expression for position of neutral axis in case of curved bars. (Apr 2011)

$$y = \frac{-Rh^2}{R^2 + h^2}$$

where, h - link Radius
R - Radius of curvature.
y - Position of neutral axis.

9. Stress Concentration :- (Dec 2011)

Sectional discontinuities are called stress raisers, and the distribution of stresses in the neighbourhood of regions are higher than in other regions. They are called regions of stress Concentration.

10. Assumptions made in Winkler-Bach Theory for determination of stress in curved beam. (June 2012, Dec 2012)

* Plane sections remain plane during bending. and bend
* The materials obeys Hooke's law
* Radial strain is negligible.
* The fibres are free to expand (or) contract without any constraining effect from the adjacent fi

11. Shear ~~stress~~ centre (Dec 2012)

The shear centre (for any transverse section) is the point of intersection of the bending axis and plane of ~~stress~~ transverse section.

Part – B

1. A curved bar of rectangular section, initially unstressed is subjected to bending moment of 2000 N-m tends to straighten the bar. The section is 5 cm wide and 6 cm deep in the plane of bending and the mean radius of curvature is 10 cm. Find the position of neutral axis and the stress at the inner and outer face. (Dec 2011, Apr 2010)

Solution: $D = 6\text{ cm} = 0.06\text{ m}$; $w = 5\text{ cm} = 0.05\text{ m}$;

$R = 10\text{ cm} = 0.1\text{ m}$; $M = 2000\text{ N-m} = -2\text{ kN m}$. (-ve BM tends to straighten the bar)

$$A = 0.06 \times 0.05 = 0.003\text{ m}^2.$$

$$h^2 = \frac{R^3}{D}\log_e\left(\frac{2R+D}{2R-D}\right) - R^2$$

$$= \frac{(0.1)^3}{0.6}\log_e\left[\frac{(2\times 0.1)+(0.06)}{(2\times 0.1)-(0.06)}\right] - (0.1)^2$$

$$h^2 = 3.173\times 10^{-4}\text{ m}^2.$$

(i) Position of neutral axis ; $y = \dfrac{-Rh^2}{R^2+h^2}$

$$= \frac{-(0.1)\times 3.173\times 10^{-4}}{(0.1)^2+(3.173\times 10^{-4})}$$

$$y = -3.08\text{ mm}.$$

(ii) Bending stress :

At inside face $\sigma = \dfrac{M}{AR}\left[1 - \dfrac{R^2}{h^2}\left(\dfrac{y}{R-y}\right)\right]$

$$y = D/2 = \frac{0.06}{2} = 0.03.$$

$$\sigma = -\frac{2\times 10^3\times 10^{-6}}{0.003\times 0.1}\left[1 - \frac{(0.1)^2}{3.173\times 10^{-4}}\left(\frac{0.03}{0.1-0.03}\right)\right]$$

$$\sigma = 83.373 \text{ MN/m}^2 \text{ (tensile)}$$

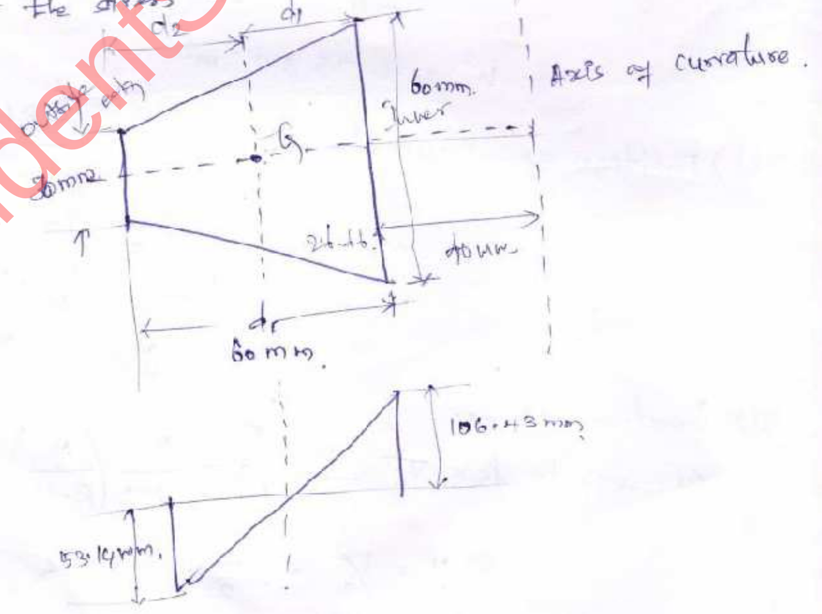
Bending stress at outside face :

$$\sigma = \frac{M}{AR}\left[1 + \frac{R^2}{h^2}\left(\frac{y}{R+y}\right)\right]$$

$$= \frac{-2\times10^3 \times 10^{-6}}{0.003 \times 0.1}\left[1 + \frac{(0.1)^2}{3.173\times10^{-4}}\left(\frac{0.03}{0.1+0.03}\right)\right]$$

$$\sigma = -55.15 \text{ MN/m}^2 \text{ (comp)}$$

2. A central horizontal section of hook is a symmetric ~~other~~ trapezium 60mm deep, the inner width being 60mm and outer being 30mm. Estimate the extreme intensities of stress when the hook carries a load of 30kN. The load line passing 40mm from the inside edge of the section and the centre of curvature being in the load line. Also plot the stress distribution across the section. (Apr 2011)



$B = 60$ mm $\quad b = 30$ mm

$d = 60$ mm $\quad P = 30$ kN.

$$A = \left(\frac{30+60}{2}\right)\times 60 = 2700\ \text{mm}^2.$$

$$d_1 = \frac{d}{3}\left[\frac{B+2b}{B+b}\right]$$

$$= \frac{60}{3}\left[\frac{60+(2\times30)}{60+30}\right]$$

$d_1 = 26.667$ mm

$d_2 = d - d_1$

$= 60 - 26.667 = 33.333$ mm.

$R = 40 + d_1 = 66.667$ mm

In trapezium section;

$$h^2 = \frac{R^3}{A}\left[b \log_e\left(\frac{R+d_2}{R-d_1}\right) + \left(\frac{B-b}{d}\right)(R+d_2)\log_e\left(\frac{R+d_2}{R-d_1}\right) - (B-b)\right] - R^2$$

$$h^2 = \frac{66.667^3}{2700}\left[30\log_e\left(\frac{66.667+33.333}{66.67-26.67}\right) + \left(\frac{60-30}{60}\right)\times (66.667+33.333)\log_e\left(\frac{66.667+33.33}{66.667-26.667}\right) - 30\right] - 66.66^2$$

$h^2 = 109.741\,(27.49 + 45.81 - 30) - 4444.489$

$h^2 = 307.296$ mm^2.

BM $M = -30\times1000\times66.667 = 2000010$ N-mm

(-ve shows BM is bending to decrease the curvature)

$$\sigma_d = \frac{P}{A} = \frac{30000}{2700} = 11.111\ \text{N/mm}^2$$

Bending stress at outside edge of the section

$$(\sigma_b)_o = \frac{M}{AR}\left[1 + \frac{R^2}{h^2} \times \frac{d_2}{R+d_2}\right]$$

$$= \frac{-2000010}{(2700 \times 66.667)}\left[1 + \frac{66.667^2}{307.296}\left(\frac{33.333}{66.667+33.333}\right)\right]$$

$$(\sigma_b)_o = -64.679 \ N/mm^2.$$

Total stress at the outside edge of the section

$$\sigma_o = \sigma_d + (\sigma_b)_o$$

$$= 11.111 - 64.679$$

$$= -53.568 \ N/mm^2 \ (comp)$$

Bending stress at the inside edge of the section

$$(\sigma_b)_i = \frac{M}{AR}\left[1 - \frac{R^2}{h^2}\left(\frac{d_1}{R-d_1}\right)\right]$$

$$= \frac{-2000010}{2700 \times 66.667}\left[1 - \frac{66.667^2}{307.296} \times \frac{26.667}{66.667 - 26.667}\right]$$

$$(\sigma_b)_i = 96.025 \ N/mm^2 \ (tensile)$$

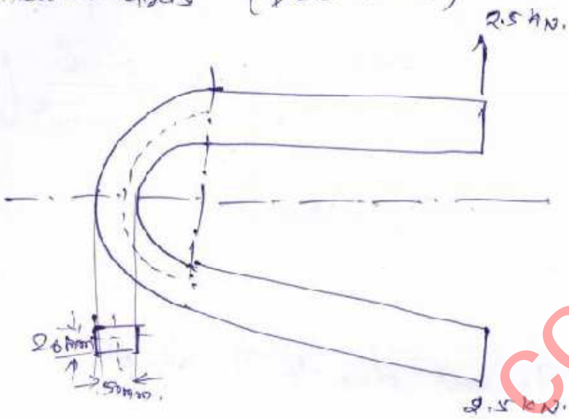
Total stress at the inside edge of the section

$$\sigma_i = \sigma_d + (\sigma_b)_i$$

$$= 11.111 + 96.025$$

$$= 107.136 \ N/mm^2 \ (tensile)$$

The fig shows a frame subjected to a load of 2.5 kN. find (i) The Resultant stresses at points 1 & 2 (ii) Position of neutral axis (Dec 2011)



(i) Resultant stresses at points 1 & 2.

Direct stress $\sigma_d = P/A$

Area of c/s 1-2 $A = 50 \times 20 \times 10^{-6} = 1 \times 10^{-3}\ m^2$.

$M = -2.5 \times 10^3 \times (110+50) \times 10^{-3}$.

$M = -400\ Nm$

$$\sigma_d = \frac{2.5 \times 10^3}{1 \times 10^{-3}} = 2.5\ MN/m^2 \text{(tensile)}$$

$$h^2 = \frac{R^3}{D} \log_e \left(\frac{2R+D}{2R-D}\right) - R^2$$

Here $R = 50\ mm = 0.05\ m$ $D = 50\ mm = 0.05\ mm$

$$h^2 = \frac{0.050^3}{0.05} \log_e \left[\frac{(2 \times 0.05) + 0.05}{(2 \times 0.05) - 0.05}\right] - 0.05^2$$

$$h^2 = 2.5 \times 10^{-3} \log_e \left(\frac{0.15}{0.05}\right) - 0.05^2$$

$$h^2 = 2.47 \times 10^{-4}\ m^2$$

Bending stress at point 2 due to M.

$$\sigma_{b2} = \frac{M}{AR}\left[1 - \frac{R^2}{h^2}\left(\frac{y}{R-y}\right)\right]$$

$$= \frac{-400\times10^{-6}}{1\times10^{-3}\times0.05}\left[1 - \frac{0.05}{2.47\times10^{-4}}\left(\frac{0.025}{0.05-0.025}\right)\right]$$

$$= 8\left[1-10.12\right]$$

$$\sigma_{b2} = 72.92 \ MN/m^2 . \text{ (tensile)}$$

Bending Moment due to M at pt 1,

$$\sigma_{b1} = \frac{M}{AR}\left[1 + \frac{R^2}{h^2}\left(\frac{y}{R+y}\right)\right]$$

$$= \frac{-400\times10^{-6}}{1\times10^{-3}\times0.05}\left[1 + \frac{(0.05)^2}{2.47\times10^{-4}}\left[\frac{0.025}{0.05+0.025}\right]\right]$$

$$\sigma_{b1} = -34.99 \ MN/m^2$$

$$\sigma_{b1} = 34.99 MN/m^2 \text{ (Comp)}$$

$$h^2 = \frac{d^2}{16} + \frac{1}{128}\cdot\frac{d^4}{R^2} + \ldots$$

Here $d = 10$ cm. $R = 8+5 = 13$.

$$h^2 = \frac{10^2}{16} + \frac{1}{128}\left(\frac{10^4}{13}\right).$$

$$h^2 = 12.26 \ cm^2 = 12.26\times10^{-4} \ m^2 .$$

Direct stress $\sigma_d = \dfrac{40\times10^3}{78.54\times10^{-4}} = 5.09 \ MN/m^2$ (Comp)

Bending stress at point 1 due to M.

$$\sigma_{b1} = \frac{M}{AR}\left[1 + \frac{R^2}{h^2}\left(\frac{y}{R+y}\right)\right]$$

$$= \frac{5200}{78.54\times10^{-4}\times0.13}\left[1 + \frac{(0.13)^2}{12.26\times10^{-4}}\left(\frac{0.05}{0.13+0.05}\right)\right]$$

$$\sigma_{b1} = 19.5 \ MN/m^2 \ (\text{tensile})$$

Bending stress at pt 2 due to M.

$$\sigma_{b2} = \frac{M}{AR}\left[\frac{R^2}{h^2}\times\left(\frac{y}{R-y}\right) - 1\right]$$

$$= \frac{5900}{78.54\times10^{-4}\times0.13}\left[\frac{(0.13)^2}{12.26\times10^{-4}}\left(\frac{0.05}{0.13-0.05}\right) - 1\right]$$

$$\sigma_{b2} = 38.78 \ MN/m^2 \ (\text{comp})$$

Hence, $\sigma_1 = \sigma_d + \sigma_{b1}$

$= -5.09 + 19.5$

$= 14.41 \ MN/m^2.$

$\sigma_2 = \sigma_d + \sigma_{b2} = 2.65 + 38.78$

$= 41.43 \ MN/m^2.$

Resultant stress at point 2,

$\sigma_2 = \sigma_d + \sigma_{b2}$

$= 2.5 + 72.97$

$= 75.47 \ MN/m^2. \ (\text{tensile})$

Resultant stress at point 1,

$\sigma_1 = \sigma_d + \sigma_{b1} = 2.72 \ 34.95$

$= 32.22 \ MN/m^2$

(iii) Position of Neutral axis:

$$y = -\left(\frac{Rh^2}{R^2+h^2}\right)$$

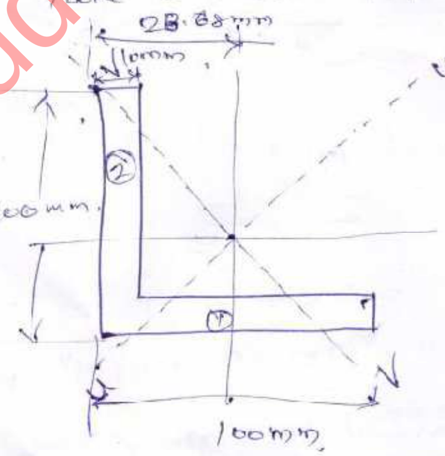
$$= \frac{-0.05 \times 2.47 \times 10^{-4}}{\left[0.05^2 + (2.47\times10^{-4})\right]}$$

$$y = 4.49 \times 10^{-3}\ m.$$

$$y = 4.5\ mm.$$

(7). A 100 mm × 100 mm × 100 mm angle section shown in figo. is used as a simply supported beam over a span of 3m. It carries a load of 500 N along the line 'yo'. where o is centroid of the section, calculate

(i) Stresses at point A, B & C of the mid section of beam. (ii) Deflection of beam at the mid section and its direction with the load line. (iii) Position of neutral axis. Take E = 200 GN/m². (APR 2011)



Let $(\bar{x}, \bar{y})$ be the Coordinates of centroid 'O'

Taking moments of area about AB,

$$\bar{x} = \frac{(100 \times 10)(50) + (90 \times 10 \times 5)}{(100 \times 10) + (90 \times 10)} = \frac{54500}{1900}$$

$= 28.68$ mm.

Taking moments of area abt BC.

$$\bar{y} = \frac{(100 \times 10 \times 5) + (90 \times 10 \times 55)}{(100 \times 10) + (90 \times 10)} = 28.68 \text{ mm.}$$

$$I_{xx} = I_{yy} = \left(\frac{100 \times 10^3}{12}\right) + (1000 \times 23.68^2) + \left(\frac{10 \times 90^3}{12}\right) + (900 \times 26.3^2)$$

$= 8333.33 + 560742.4 + 607500 + 622521$

$= 179 \times 10^4$ mm⁴.

$a_1 = 21.32$ mm $\quad b_1 = -23.68$

$a_2 = -23.68$ mm. $\quad b_2 = +16.82$.

x, y (−, +)	x, y (+, +)
x, y (−, −)	(x, y) (+, −)

$$I_{xy} = \left[I_{\bar{x}\bar{y}} + A_1 a_1 b_1\right] + \left[I_{\bar{x}\bar{y}} + A_2 a_2 b_2\right]$$

$= 0 + \left[(100 \times 10 \times 21.32 \times -23.68)\right] +$

$0 + (90 \times 10 \times -23.68 \times 16.32)$

$= -504857.6 - 347811.84.$

$= -852669$ mm⁴.

Principal Axes :-

$$\tan 2\theta = \frac{2I_{xy}}{I_{yy} - I_{xx}} = \infty \quad \left[\text{Since } I_{xx} = I_{yy}\right].$$

$$\tan 2\theta = \frac{-2I_{xy}}{} \quad \tan 90^\circ$$

$$2\theta = 90^\circ$$

$$\theta_1 = 45^\circ$$

$$\theta_2 = 90^\circ + \theta_1$$

$$\theta_2 = 135^\circ$$

Principal Moment of Inertia:

$$2\theta = 90^\circ$$

(7)

$$I_{uu} = \frac{1}{2}(I_{xx} + I_{yy}) + \frac{1}{2}(I_{xx} - I_{yy}) \cos$$

$$- I_{xy} \sin 2\theta.$$

$$= 0.5(2 \times 174 \times 10^4) + (0.5)(0) + 8526$$

$$I_{uu} = 2642669.4 \text{ mm}^4$$

$$I_{vv} = I_{xx} + I_{yy} - I_{uu}.$$

$$= 2 \times 174 \times 10^4 - 2642669.4$$

$$= 937330.6 \text{ mm}^4.$$

To find stress at point A, B, C,

$$M = \frac{W \times L}{4} = \frac{500 \times 3000}{4} = 375000 \text{N}$$

Components of M in the planes of Principal axes UU' and VV'.

$$\text{Along } UU' = M \sin\theta = 375000 \times \sin 45^\circ$$

Bending stress due to this component at any pt $P(u,v) = \dfrac{M \sin\theta \times u}{I_{vv}}$.

Along $vv' = M \cos\theta$.

$\therefore$ Bending stress at point $P(u,v) = \dfrac{M \cos\theta \times v}{I_{uu}}$

Resultant Bending stress at pt (u,v)

$$\sigma_b = \left(\frac{M \sin\theta \times u}{I_{vv}}\right) + \left(\frac{M \cos\theta \cdot v}{I_{uu}}\right)$$

$$= 375000 \times 0.707 \left(\frac{u}{I_{vv}} + \frac{v}{I_{uu}}\right)$$

$$\sigma_b = 375000 \times 0.707 \left(\frac{u}{937830.6} + \frac{v}{2642669.4}\right)$$

~~$\sigma_b = 375000 \times 0.707$.~~

$u = x \cos\theta + y \sin\theta$.

$v = y \cos\theta - x \sin\theta$,

At pt A, $x = -\bar{x} = -28.68$.

$y = 100 - 28.68 = 71.32$ mm.

$u = -28.68 \times \cos 45^\circ + 71.32 \times \sin 45^\circ$

$= 30.22$ mm.

$v = (71.32 \times \cos 45^\circ) + 28.68 \times \sin 45^\circ)$

$= 70.71$ mm.

$$\sigma_b = 375000 \times 0.707 \left[\frac{30.22}{737330.6} + \frac{70.71}{2642669.4}\right]$$

At B, $\sigma_b = -11.45\ N/mm^2$.

At C, $\sigma_b = 1.47\ N/mm^2$.

Deflection of beam at mid section.

$$\delta = \sqrt{(\delta u)^2 + (\delta v)^2}$$

$$\delta = 1.195\ mm.$$

$$\tan\beta = \frac{I_{uu}}{I_{vv}} * \tan\theta.$$

$$\beta = 90.47^\circ.$$

Position of Neutral axis.

$$\sigma = 375000 \times 0.707 \left(\frac{u}{9373330.6} + \frac{v}{26426694}\right)$$

$$\boxed{-\frac{v}{u} = 2.819.}$$

A beam of circular section of diameter 25 mm has its centre line curved to a radius of 50 mm. Find the intensity of maximum stress in the beam, when subjected to a moment of 5 kN-mm.

Dia of section = 25 mm

Radius of curvature = $R = 50$ mm.

$M = 5 \times 10^3$ N-mm

$A = \pi/4 \times 25^2 = 156.25\pi$ mm^2.

Distance between centre line and extreme fibre,

$y = d/2 = \frac{25}{2} = 12.5$ mm.

$$h^2 = \frac{d^2}{16} + \frac{1}{8} \times \frac{d^4}{16R^2}$$

$$= \frac{25^2}{16} + \left[\frac{1}{8} \times \frac{25^4}{16 \times 50^2}\right]$$

$= 39 + 1.22$

$= 40.28$ mm.

Maximum stress, $\sigma_1 = \frac{M}{AR}\left[1 + \frac{R^2 y}{h^2(R+y)}\right]$

$$= \frac{5 \times 10^3}{156.25\pi \times 50}\left[1 + \frac{50^2 \times 12.5}{40.28(50+12.5)}\right]$$

$= 2.73$ N/mm^2 (tensile)

Maxi stress at top surface $\sigma_2 = \frac{M}{AR}\left[1 - \frac{R^2 y}{h^2(R-y)}\right]$

$= 0.098$ N/mm^2.