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VH 3977

Unit - 4

STATE OF STRESS IN 3 DIMENSION.

1. What are the types of failure?
   Two types: 1. Brittle failure
   2. Ductile failure.

2. Brittle failure:
   Failure of a material represents direct seperation of particles from each other, accompanied by considerable deformation is known as brittle failure.

3. Ductile failure:
   Slippling of a material accompanied, by considerable plastic deformation is known as ~~Brittle~~ ductile failure.

4. Define tensor:
   State of stress at a point is defined by three components on each of the three mutually perpendicular axis in mathematical terminology is called tensor.

5. List out the theories of failure.
   - Maximum principal stress theory (Rankine's theory)
   - Maximum principal strain theory (St. Venant's theory)
   - Maximum shear stress theory (Trescas & guest theory)
   - Maximum shear strain energy theory (Von-Mises-Hencky theory)
   - Maximum strain energy theory (Haigh's theory)

6) Maximum principal stress theory (Rankine's theory)

The failure of a material will occur when maximum principal tensile stress (σ_1) in the complex system reaches the value of the maximum stress (σ_t^*) at elastic limit in simple tension or minimum principal stress (i.e maximum principal compressive stress reaches the value of the maximum stress at the elastic limit in simple compression.

$$\sigma_1 = \sigma_t^*.$$

7). Maximum Principal strain theory (st. Venant's theory)

The failure of a material will occur when the maximum principal strain (e_1) reaches the strain due to the yield stress in simple tension (σ_t^*/E)

$$e_1 = \frac{\sigma_t^*}{E}$$

In 3D, $e_1 = [\sigma_1 - \mu(\sigma_2 + \sigma_3)] = \frac{\sigma_t^*}{E}$

2D, $e_1 = [\sigma_1 - \mu\sigma_2] = \frac{\sigma_t^*}{E}$

8) Maximum shear stress theory (Tresca's & Guest theory)

The failure of a material will occur when the maximum shear stress in the body will reach the value of maximum shear stress in simple tension at elastic limit.

In 3D $(\sigma_1 - \sigma_3) = \sigma_t^*.$

2D $\sigma_1 = \sigma_t^*.$

9) Maximum shear strain energy theory (Von-Mises-Hencky theory)

The failure of a material will occur when the total shear strain energy per unit volume in a stressed material reaches a value equal to shear strain energy per unit volume at the elastic limit in simple tensile test.

In 3D shear strain energy due to distortion

$$U = \frac{1}{12C}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right]$$

In 3D shear strain energy due to simple tensio.

$$U = \frac{\sigma_t^{*2}}{6C}$$

$$U = \frac{1}{6C}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right] = \frac{\sigma_t^{*2}}{6C}$$

In 2D shear strain energy due to simple tension

$$U = \frac{1}{6C}\left[(\sigma_1-\sigma_2)^2 + \sigma_2^2 + \sigma_1^2\right] = \sigma_t^{*2}/6C$$

10) Maximum strain energy theory: (Haigh's theory)
The failure of a material will occur when the total strain energy per unit volume in the stressed material reaches the strain energy per unit volume of the material at the elastic limit in a simple tensile tent.

In 3D, shear energy due to deformation

$$U = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3 \ldots\right.$$

In 3D shear strain energy due to simple tension

$$U = \sigma_t^{*2} / 2E$$

$$U = \left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right] = \frac{\sigma_t^{*}}{2}$$

In 2D,

$$U = \sigma_1^2 + \sigma_2^2 - 2\mu(\sigma_1 \sigma_2) = \sigma_t^{*2}$$

11) Limitations of Maximum Principal stress theory (Rankine's theory).

This theory disregards the effects of other principal stresses and effect of shearing stresses on other planes through the planes through the element.

Material in tension test piece along 45° to the axis of the test piece, where normal stress is neither maximum nor minimum, but the shear stress is maximum.

Failure is not brittle but cleavage failure.

12) Limitation of shear stress theory (Tresca's & Guest theory)

This theory does not give accurate results for the state of stress of pure shear in which the maximum amount of shear is developed, in torsion.

13) Limitation of Maximum shear strain energy theory (Von-Mises-Henky Theory)

This theory cannot be applied to materials under hydrostatic pressure.

Limitations of Maximum strain energy Theory (Haigh's theory)

This theory cannot be applied to brittle materials for which elastic limit in tension and compression is quite different.

15. Octahedral plane:

The plane which is equally inclined to the three axes of reference is called octahedral plane.

16. Octahedral stresses:

The normal and shear stress acting on the octahedral plane (The plane which is equally inclined to three axes of reference) is known as octahedral stresses.

17. Volumetric strain per unit volume.

Volumetric strain per unit volume is defined as the ratio of change in volume of a material to its unit volume.

18. What are Residual stresses:

Maximum principal stress and maximum shear stress are collectively known as residual stresses.

19). Principal plane:

The planes on which the normal stress is maximum and the shear stress is zero is called principal plane.

20. Spherical stress tensor:

The hydrostatic type of stress, s'' is known as spherical stress tensor.

$$s'' = \begin{pmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_m \end{pmatrix} \qquad \sigma_m = \frac{1}{3}(\sigma_x + \sigma_y + \sigma_z)$$

The spherical part is rather harmless, produces only uniform volume changes without any change of shape and does not cauce failure.

21. Deviatorial stress tensor.

The type of stress other than hydrostatic stress is known as deviatorial stress tensor.

$$s' = \begin{pmatrix} \sigma_x - \sigma_m & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y - \sigma_m & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z - \sigma_m \end{pmatrix}$$

The deviatorial part produces all the changes of shape in the body and finally causes failure.

22. Stress Invariants

In the context of the stress tensor, invariants are such quantities that do not change with rotation of axes or which remain unaffected under transformation, from one set of axes to another.

$$I_1 = \tau_{xx} + \tau_{yy} + \tau_{zz}$$

$$I_2 = \tau_{xx}\tau_{yy} + \tau_{yy}\tau_{zz} + \tau_{zz}\tau_{xx} - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2.$$

$$I_3 = \tau_{xx}\tau_{yy}\tau_{zz} + 2\tau_{xy}\tau_{yz}\tau_{zx} - \tau_{xx}\tau_{yz}^2 - \tau_{yy}\tau_{xz}^2 - \tau_{zz}\tau_{xy}^2.$$

The equation for principal stres.

$$\sigma_1^3 - I_1\sigma_2^2 + I_2\sigma_3' - I_3 = 0$$

24) Dialatation and distortion components.

$$E = \text{Stress tensor} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix}.$$

$$E = E'' + E'$$

$$E'' = \begin{bmatrix} e & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & e \end{bmatrix} \qquad E' = \begin{bmatrix} \varepsilon_x - e & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_y - e & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_z - e \end{bmatrix}$$

$$e = \frac{\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}}{3}.$$

The spherical component E'' produces only volume changes without any change while deviatorial component E' produces distortion or change of shape.

These components are extensively used in theories of failure and are sometimes known as dialatation and distortion components.

25) Octahedral shear stresses.

$\sigma_1, \sigma_2, \sigma_3$ for the three principal stresses.

So that $\sigma_1 > \sigma_2 > \sigma_3$

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}$$

It is very important eqn and is very often used in theories of failure, like Von-Mises-Theory of failure.