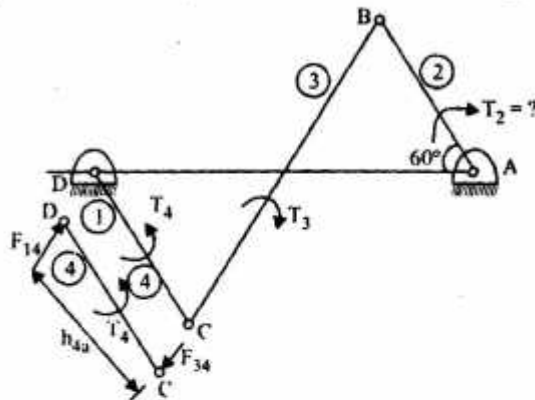


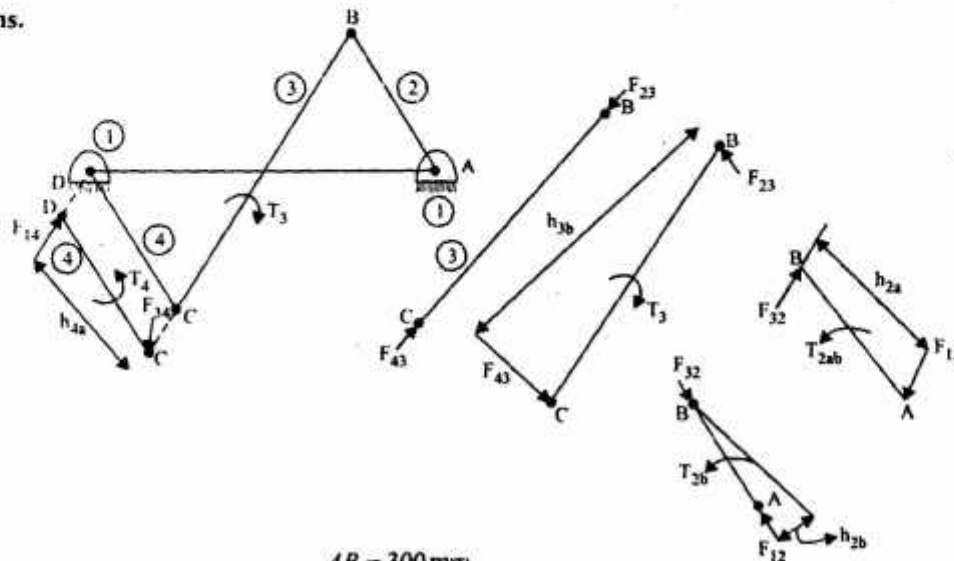
**B.E.**  
**Sixth Semester Examination, 2010**  
**Dynamics of Machines (ME-302-E)**

**Note :** Attempt any five questions.

**Q. 1.** In a four link mechanism shown in figure, torque  $T_3$  and  $T_4$  have magnitude of 30 N-m and 20 N-m respectively. The link lengths are  $AD = 800$  mm,  $AB = 300$  mm,  $BC = 700$  mm and  $CD = 400$  mm. For the static equilibrium of mechanism, determine the required input Torque  $T_2$ .



**Ans.**



$AB = 300$  mm  
 $AD = 800$  mm  
 $BC = 700$  mm

$CD = 400 \text{ mm}$

$$T_1 = 30 \text{ N-m}$$

$$T_4 = 20 \text{ N-m}$$

### Applying superposition principal

Link (4) is balance by two equal and opposite parallel forces  $C$  and  $D$ .

Link (3) having  $F_{43}$  or  $F_{34}$  &  $F_{14}$  parallel to  $BC$ .

$$F_{34} = F_{14} = \frac{T_4}{h_{4x}} = \frac{20}{0.38} = 52.1 \text{ N}$$

$$F_{34} = F_{43} = F_{23} = F_{32} = F_{12} = 521 \text{ N}$$

$$T_{2ab} = F_{32} \times h_{2a} = 521 \times 0.274 = 14.26 \text{ N-m (counter-clockwise)}$$

For  $T_x$  torque,

$$F_{43} = F_{34} \text{ along CD}$$

$$F_{43} = F_{23} = \frac{T_3}{h_{3b}} = \frac{30}{0.67} = 44.8 \text{ N}$$

$$F_{23} = F_{32} = F_{12} = 44.8 \text{ N}$$

$$T_{2h} = F_{32} \times h_{2h} = 44.8 \times 0.042 = 1.88 \text{ N-(ccw)}$$

$$T_2 = T_{2a} + T_{2b} = 14.26 + 188 = 1614 \text{ N(ccw) Ans.}$$

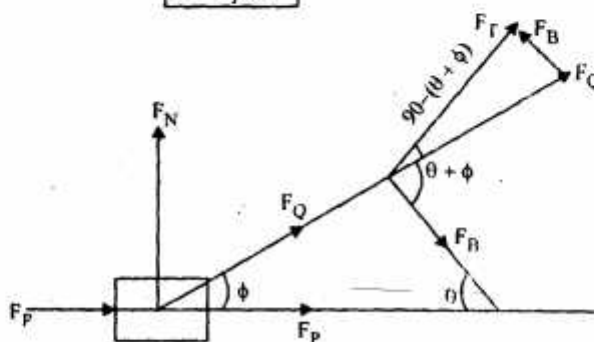
**Q. 2. (a) What is crank effort? Derive an expression for turning moment on crankshaft.**

**Ans. Crank Effort :** The product of the crank-pin effort ( $F_T$ ) and the crank pin radius ( $r$ ) is known as crank effort or turning moment or torque on the crank shaft.

$$T = F_T \times r$$

### Expression for turning moment on crank shaft

$$T = F_T \times r$$



$$T = \frac{F_p \sin(\theta + \phi)}{\cos \phi} \times r$$

$$= \frac{F_p (\sin \theta \cos \phi + \cos \theta \sin \phi)}{\cos \phi} \times r$$

$$= F_p (\sin \theta + \cos \theta \tan \phi) r \quad \dots (i)$$

We know that  $n = \frac{l}{r}$ ,  $l \sin \phi = r \sin \theta$

$$\sin \phi = \frac{\sin \theta}{n}$$

$$\tan \phi = \frac{\sin \phi}{\cos \phi} = \frac{\sin \theta}{\sqrt{n^2 - \sin^2 \theta}}$$

From equation (i)

$$T = F_p \times r \left[ \sin \theta + \frac{\cos \theta \sin \theta}{\sqrt{n^2 - \sin^2 \theta}} \right]$$

$$= F_p \times r \left[ \sin \theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2 \theta}} \right]$$

If

$$n^2 \gg \sin^2 \theta$$

Then

$$T = F_p \times r \left[ \sin \theta + \frac{\sin 2\theta}{2n} \right]$$

**Q. 2. (b) Explain the different types of engine shaking force.**

**Ans.** Shaking forces on the reciprocating parts of an engine are from figure (i).

**(i) Piston Effort :** It is the net force acting on the piston or cross-head pin, along the line of piston movement. It is denoted by  $F_p$ .

$$F_p = F_L \mp F_I - F_R$$

If frictional force considered

$F_R = 0$  frictional forces are absent

$$F_I = \text{Inertia force} = m_R \omega^2 r \left( \cos \theta + \frac{\cos 2\theta}{n} \right)$$

$F_L = \text{Net load on piston} = \text{Pressure of gas } (p) \times \text{Area}$

**(ii) Force Acting Along Connecting Rod :** It is denoted by  $F_Q$

$$F_Q = \frac{F_p}{\cos \theta} \quad \text{from figure (i)}$$

**(iii) Normal Reaction on the Guide Bars or Thrust on the Side of Cylinder Wall :**

$$F_N = F_Q \sin \phi$$

**(iv) Crank Pin-Effort :** Force acting along the connecting rod can be resolved in two components, one is  $\perp$  to the crank is known as crank pin effort. It is denoted by ( $F_T$ )

$$F_T = F_Q [\sin(\theta + \phi)]$$

**(v) Thrust on Crank Shaft Bearing :** The second component of connecting rod is along the crank, is known as thrust on the crank shaft bearing. It is denoted by

$$F_B = F_Q [\cos(\theta + \phi)]$$

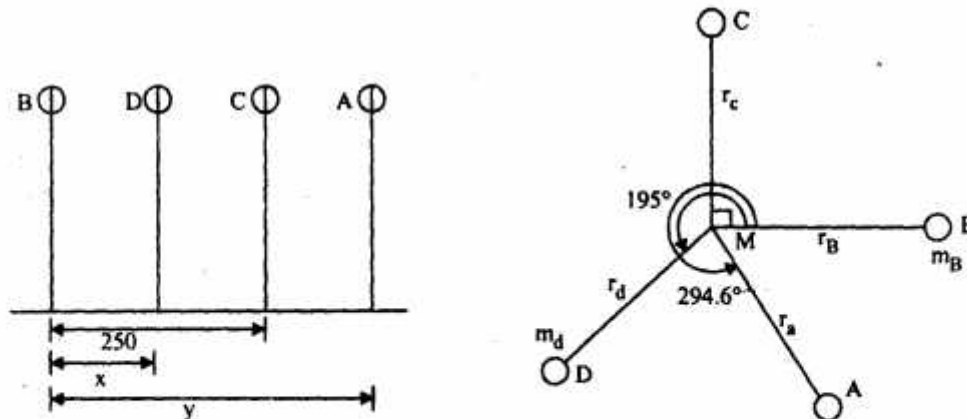
Q. 3. Four masses A, B, C and D are completely balanced. Masses C and D make angle of  $90^\circ$  and  $195^\circ$  respectively with B in same sense. The rotating mass have following properties :

$m_b = 25 \text{ kg}$	$r_a = 150 \text{ mm}$
$m_c = 40 \text{ kg}$	$r_b = 200 \text{ mm}$
$m_d = 35 \text{ kg}$	$r_c = 100 \text{ mm}$
	$r_d = 180 \text{ mm}$

Plane B and C are 250 mm apart. Determine :

- The mass A and its angular position
- The position of plane A and D.

Ans.



Let  $\omega = 1 \text{ rad/sec}$

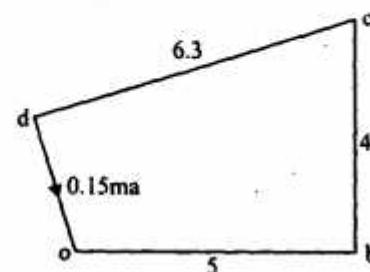
Plane	Mass (kg)	Radius (m)	Mr. (N)	Length (m)	mrl (N-m)
A	$m_a$	0.15	$0.15 m_a$	y	$0.15 m_a y$
B (ref)	25	0.20	5	0	0
C	40	0.10	4	0.25	1
D	35	0.18	6.3	x	$6.3x$

(i) Force Polygon : Scale 1cm = 1N

$$0.15 m_a = 2.6$$

$$m_a = \frac{2.6}{0.15} = \pm 7.33 \text{ kg}$$

From parallel to online draw the line from point m, and this is angular position of mass (A).



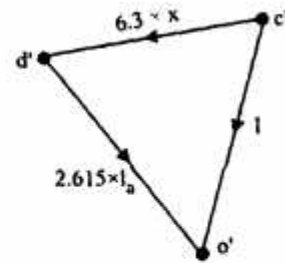
(ii) Couple Polygon :

$$c'd' = 4.25 \text{ m}$$

$$6.3 \times x = 4.25$$

$$x = 67.46 \text{ mm}$$

$$y = \frac{9.83}{2.6} = 375 \text{ mm} \quad \text{Ans.}$$



**Q. 4. (a) Explain balancing of In-line two cylinders engine.**

**Ans.** Balancing of in-line two cylinder engine can be completely balanced from balancing of primary and secondary balancing :

**(i) Balancing of Primary Forces :** The following two conditions must be satisfied in order to give the primary balancing of the reciprocating parts in two cylinder engine are :

(i) The algebraic sum of the primary forces must be equal to zero, or in other words. The primary force polygon must be close.

(ii) The algebraic sum of the couples about any point in the plane is equal to zero or, the primary couple polygon must be close.

**(ii) Balancing of Secondary Forces :** When the connecting rod is not too longer.

Then,

$$F_{\text{sec}} = m\omega^2 r \times \frac{\cos 2\theta}{n}$$

For the secondary forces

$\theta$  will become  $\rightarrow 2\theta$  &  $\omega \rightarrow 2\omega$

Then

$$r = \frac{r}{4n}$$

Then

$$F_s = m(2\omega)^2 \times \frac{r}{4n} \times \cos 2\theta$$

Following two conditions must be satisfied in order to give a complete balancing of secondary forces are :

(i) The algebraic sum of the secondary forces must be equal to zero or in other words, force polygon must be closed.

(ii) The algebraic sum of the couple about any point in the plane of secondary forces must be equal to zero or in other words couple polygon must be closed.

**Q. 4. (b) Define the terms :**

(a) Hammer blow

(b) Variation of Tractive force

(c) Swaying couple.

**Ans. (a) Hammer Blow :** The effect of an unbalanced primary force perpendicular to the line of stroke is produce variation in thrust, which result in hammering action. The maximum magnitude of the unbalanced force along the perpendicular to the line of stroke is known as hammer blow.



We know that the unbalanced force along the  $\perp$  to the line of stroke due to the balancing mass  $B$ , at the radius  $r$ , in order to balance reciprocating parts only is  $B\omega^2 r \sin \theta$ ,

This value will be maximum when,

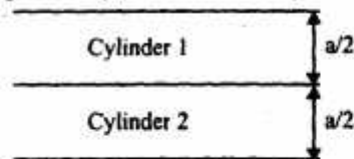
$$\theta = 90^\circ, 270^\circ$$

$$\text{Hammer blow} = B\omega^2 r$$

**(b) Variation of Tractive Force :** The resultant unbalanced force due to the two cylinders, along the line of stroke is known as tractive force.

$$\begin{aligned} \text{Variation of tractive force} &= \pm(1-C)m\omega^2 r \times \sqrt{2} \\ &= \pm\sqrt{2}(1-C)m\omega^2 r \end{aligned}$$

**(c) Swaying Couple :** The unbalanced force along the line of stroke for the two cylinders constitute a couple about the centre line of the cylinders. This couple has swaying effect about a vertical axis, and because of this couple engine sway, hence this couple is known as swaying couple.

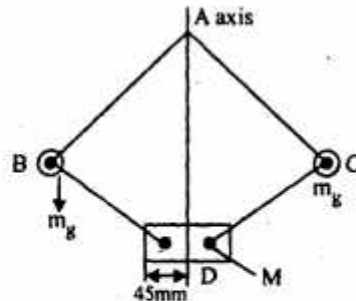


Maximum and minimum swaying couple

$$= \pm \frac{a}{\sqrt{2}} (1-C)m\omega^2 r$$

**Q. 5. In a Porter Governor, each of four arms is 400mm long. The upper arms are pivoted on the axis of the sleeve whereas the lower arms are attached to a sleeve at a distance of 45 mm from the axis. Each ball has a mass of 8 kg and load on sleeve is 60kg. What will be the equilibrium speed for the two extreme radii of 250mm and 300mm of rotation of governor balls?**

Ans.



$$\begin{aligned} M &= 60 \text{ kg} \\ m &= 8 \text{ kg} \end{aligned}$$

**Case I : When extreme radii**

$$= 250 \text{ mm}$$

$$AE = \sqrt{AB^2 - BE^2} = 312.25 \text{ mm} = h_1$$

$$h_1 = 0.312 \text{ m}$$

$$DF = \sqrt{400^2 - [250 - 45]^2} = 343.47 \text{ mm}$$

$$\tan \alpha_1 = \frac{250}{312.25}$$

$$\tan \alpha_1 = 0.8006$$

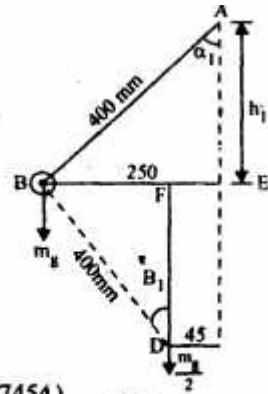
$$\tan \beta_1 = \frac{205}{343.47} = 0.5968$$

$$q_1 = \frac{\tan \beta_1}{\tan \alpha_1} = 0.7454$$

$$N_1^2 = \frac{m + \frac{M}{2}(1 + q_1)}{m} \times \frac{895}{h_1} = \frac{8 + \frac{60}{2}(1 + 0.7454)}{8} \times \frac{895}{0.31225}$$

$$N_1^2 = 21626.8975$$

$$N_1 = 147.06 \text{ rpm}$$



Case II : When extreme radii—300 mm

$$AG = 264.575 \text{ mm} = 0.2645m = h_2$$

$$DH = 308.1801 \text{ mm}$$

$$\tan \alpha_2 = 0.75$$

$$q_2 = 1.1036$$

$$\tan \beta_2 = 0.8274$$

$$N_2^2 = \frac{8 + \frac{60}{2}(1 + 1.1036)}{8} \times \frac{895}{0.2645}$$

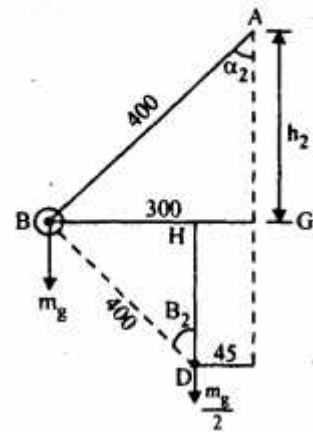
$$N_2 = 173.425 \text{ rpm}$$

Range of speed

$$= N_2 - N_1$$

$$= 173.425 - 147.06$$

$$= 26.365 \text{ rpm Ans.}$$



Q. 6. Explain with the help of diagram :

(a) Block or shoe brake

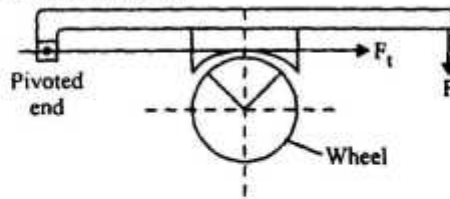
(b) Band Brake

(c) Band and Block brake

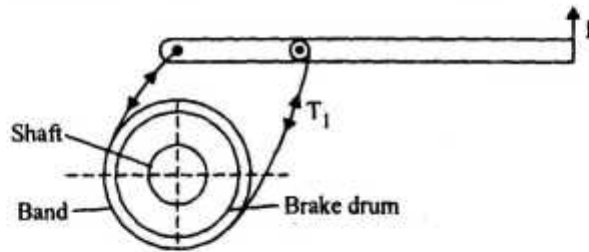
(d) Internal Expanding shoe brake

Ans. (a) **Block or Shoe Brake** : This type of brake is commonly used on railway trains and tram cars. It consists of a block or shoe which is pressed against the rim of a revolving brake wheel drum. The block is made of softer material than the rim wheel. The friction between the block and the wheel causes

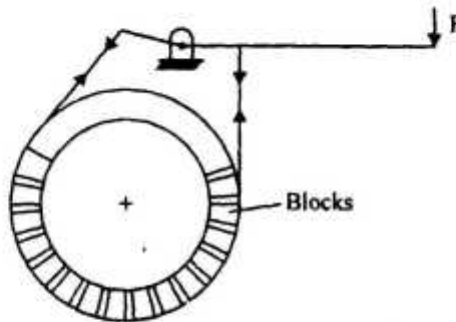
a tangential braking force to act on the wheel. Which retard the rotation of the wheel. The force applied by the lever which is pivoted on fixed fulcrum.



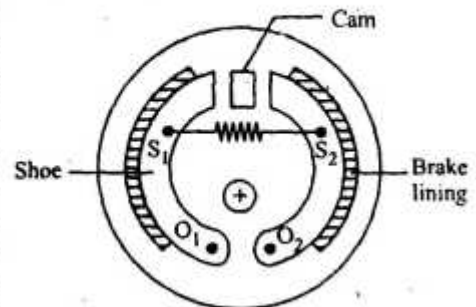
**(b) Band Brake :** A band brake consist of flexible band of rope, leather etc. with friction material, which embraces a part of the circumference of the drum. In this arrangement one end of the band is attached to a fixed pin or fulcrum of the lever while the other end is attached to the lever by the fulcrum.



**(c) Band and Block Brake :** Band and block brake consist of band lined with blocks of wood or other frictional material. The friction between the blocks and drum provides braking effect. The arrangement shown in the figure.



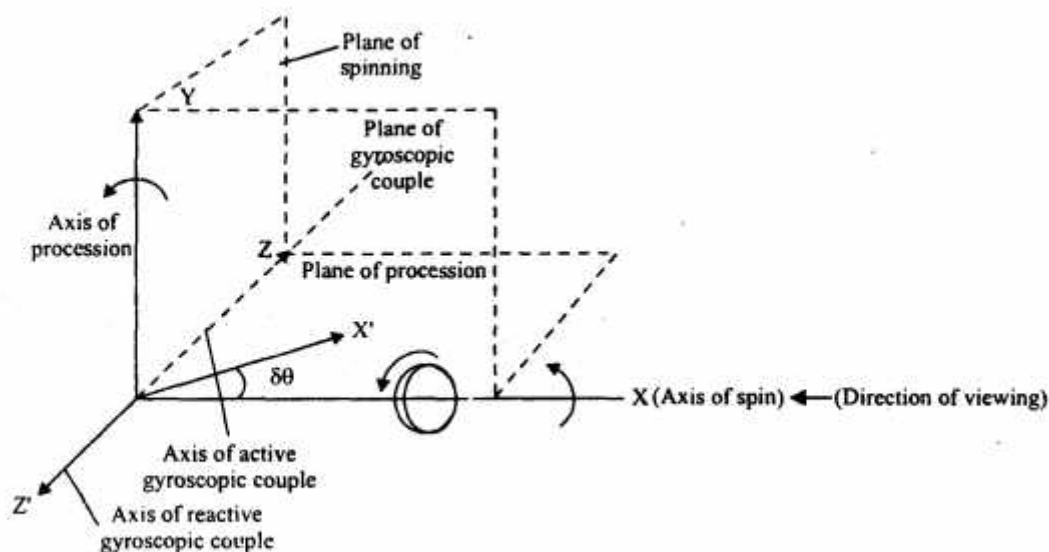
**(d) Internal Expanding Shoe Brake :** This type of brake is commonly used in motor cars and light trucks. An internal expanding brakes consist of two shoes with frictional element. The outer periphery of the shoes are lined with friction material (Ferodo) to increase the friction. Each shoe is pivoted at one end about a fixed point  $O_1$  &  $O_2$  and made to contact a cam at the other end. When the cam moves the shoes are pushed outwards against the rim. Because of this action the braking torque is produce between shoes and drum, and reduces the speed of the drum.





**Q. 7. (a) Derive the expression for gyroscopic couple.**

**Ans.**



Consider a disc spinning about the axis—OX ( $\omega$ ) seen from the front

YOZ plane  $\rightarrow$  Plane of spinning

XOZ plane  $\rightarrow$  Plane of precession

OY  $\rightarrow$  Axis of precession

Angular momentum of the disc  $= I\omega$

Where,  $I$  = MOI of the disc about OX

$\omega$  = Angular velocity of the disc

Let the disc rotate about OX, in anticlockwise direction. The axis of spin OX is also rotating anticlockwise when seen from the OY axis (from top position). Let the axis OX, turned in time  $\delta t$  sec in the plane XOZ through a small angle  $\delta\theta^\circ$  the new position OX'.

Then change in angular momentum

$$\begin{aligned} &= \text{Angular momentum of } OX' - \text{Angular momentum of } OX \\ &= \text{Angular momentum of } XX' \\ &= I\omega \times \delta\theta \end{aligned}$$

We know that rate of change of angular momentum is equal to couple

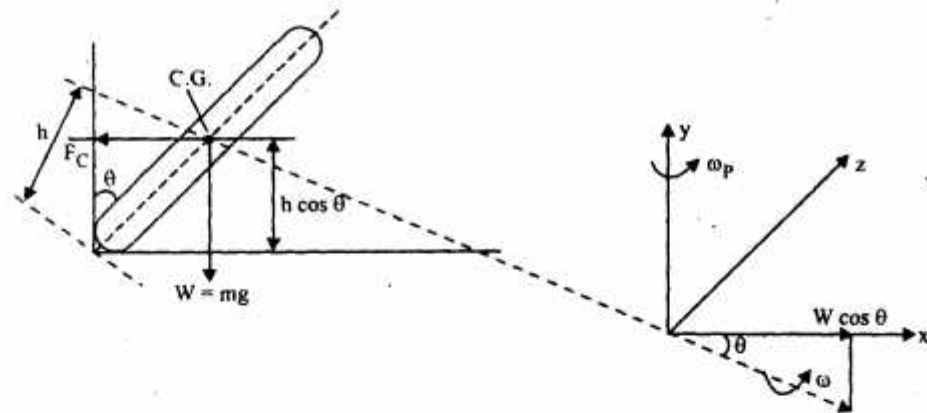
$$C = \lim_{\delta t \rightarrow 0} I\omega \times \frac{\delta\theta}{\delta t} = I \times \omega \times \frac{d\theta}{dt} = I \times \omega \times \omega_p$$

$$\boxed{C = I\omega\omega_p}$$

Where,  $\omega_p$  = Angular velocity of precession

Q. 7. (b) Derive an expression for the stability of two wheel vehicle.

Ans. Stability of Two Wheel Vehicle :



Let,  $m$  = Mass of the wheel and rider (kg)

$h$  = Height of the C.O.G. of the vehicle and rider

$r_w$  = Radius of the wheel

$R$  = Radius of the track

$I_w$  = M.O.I. of the wheel

$I_E$  = M.O.I. of the engine (rotating parts)

$\omega_w$  = Angular velocity of the wheel

$\omega_E$  = Angular velocity of the engine

$$G = \text{Gear ratio} = \frac{\omega_E}{\omega_w}$$

$$v = \text{Linear velocity of the wheel} = \omega_w \times r_w \text{ or } \omega_w = \frac{v}{r_w}$$

$\theta$  = Angle of heel

Total overturning couple = Gyroscopic couple + Centrifugal couple

$$\omega_E = G\omega_w = G \times \frac{v}{r_w}$$

Total angular momentum

$$\begin{aligned} &= 2I_w \times \omega_w \pm I_E \times \omega_E \\ &= 2I_w \times \frac{v}{r_w} \pm I_E \times G \times \frac{v}{r_w} = \frac{v}{r_w} [2I_w \pm GI_E] \end{aligned}$$

$$\omega_p = \frac{v}{R}$$

Gyroscopic couple

$$= I_w \cos \theta \times \omega_p = \frac{v^2}{Rr_w} \times (2I_w \pm GI_E) \cos \theta$$

$$\text{Centrifugal couple} = F_C \times h \cos \theta = \frac{mV^2}{R} \times h \cos \theta$$

The total overturning couple is balanced by the balancing couple (from the weight of system)

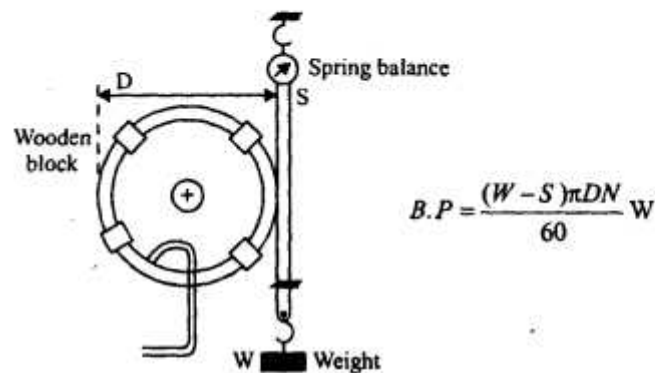
$$\Rightarrow \frac{V^2}{R} \left[ \frac{2I_w + GI_E}{r_w} + mgh \right] \cos \theta = mgh \sin \theta$$

The above expression, express the stability of two wheel.

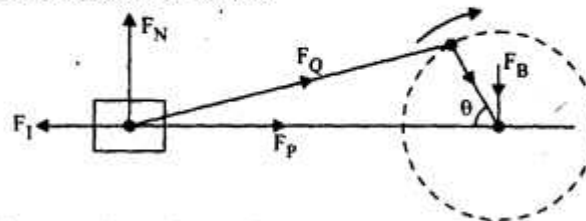
**Q. 8. Write short notes on any two :**

- (a) Rope brake absorption dynamometer
- (b) Different types of engine shaking force
- (c) Prony Brake dynamometer
- (d) Piston effort

**Ans. (a) Rope Brake Absorption Dynamometer :** It is commonly used for measuring the brake power of the engine. It consist of ropes, wound around the flywheel or rim of a pulley fixed rigidly to the shaft of an engine. The upper end of the ropes is attached to a spring balance while the lower end of the ropes is kept in position by applying a dead weight. In order to prevent the slipping of the rope over the rim wooden blocks are place. Because of this large heat is generated so a cooling arrangement is also provided.



**(b) Different Types of Engine Shaking Force :** The various forces acting on the reciprocating parts of an engine. The resultant forces acting on the body of the engine due to inertia forces only is known as shaking forces or unbalanced forces.



$F_I$  (Inertia force due to reciprocating part)

$F_P$  = Force required to accelerate the reciprocating part

$F_N$  = Force on the side of cylinder walls

$F_B$  = Force acting on the crank shaft bearing

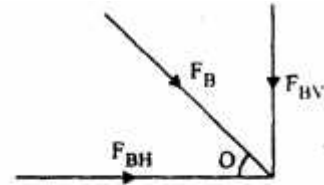
$$F_I = F_R = m\omega^2 r \left[ \cos \theta + \frac{\cos 2\theta}{n} \right]$$

From neutral equilibrium  $F_{BH} = F_I = m\omega^2 r \left[ \cos \theta + \frac{\cos 2\theta}{n} \right]$

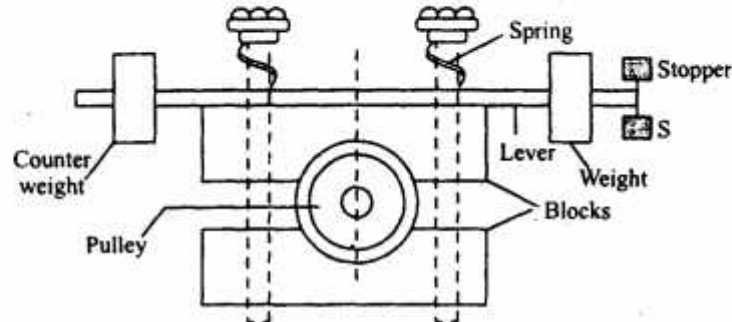
Unbalanced force  $= m\omega^2 r \cos \theta + m\omega^2 r \frac{\cos 2\theta}{n}$

Primary unbalance force  $= m\omega^2 r \cos \theta$

Secondary unbalance force  $= m\omega^2 r \cos 2\theta$



**(c) Prony Brake Dynamometer :** This is the simplest form of dynamometer. It consists of two wooden blocks placed around a pulley fixed to the shaft of an engine whose power is required to be measured. This blocks are clamped by means of two bolts and nuts. A helical spring is provided between the nut and the upper block to adjust the pressure on the pulley to control its speed. The upper block has attached with weight from a lever. A counter weight is placed at the other end of the lever which balances the brake when unloaded. Two stopper are provided as shown in figure.



When the driving torque is not uniform, this dynamometer is subjected to severe condition.

**(d) Piston Effort :** It is the net force acting on the piston or cross pin head, along the piston movement. It is denoted by  $F_P$ .

$F_P$  = Net load on the piston  $\mp$  Inertia force

(-)ve and (+ve) sign indicates the condition of retardation and acceleration respectively

$$F_P = F_L \mp F_I - F_R \quad (\text{In horizontal position})$$

Where,  $F_L$  = Net load on the piston = Pressure  $\times$  Area

$$F_I = \text{Inertia force} = m\omega^2 r \left( \cos \theta + \frac{\cos 2\theta}{n} \right)$$

$F_R$  = Frictional resistance.