B.E.

Sixth Semester Examination, December-2008

Heat Transfer (ME-306-E)

Note : Answer any five questions. All questions carry equal marks.

Q. 1. (a) Define the term 'Critical Insulation thickness.' What is its significance ? Derive an expression for the critical insulation radius of a spherical system.

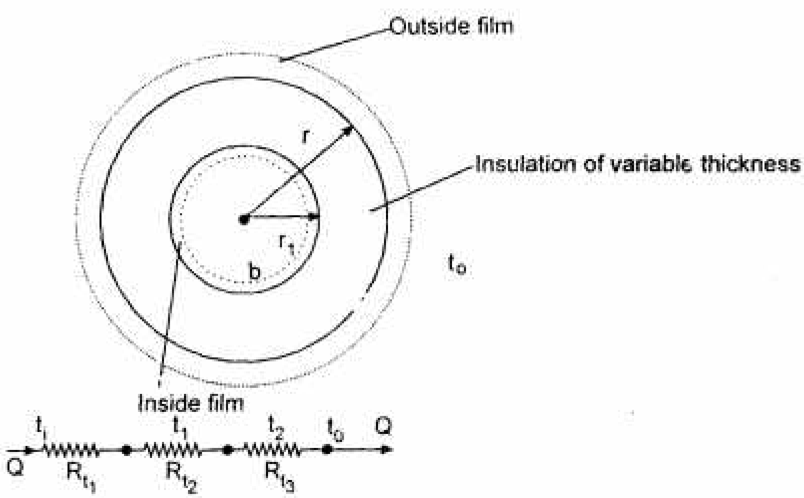
Ans. Critical Thickness of Insulation : It is the thickness of insulation to avoid the heat conduction through it.

Common belief is that addition of insulating material on a surface decreases heat transfer rate. But it is true at certain limit of thickness. After that heat transfer rate may increase. So, determination of critical thickness is necessary.

To establish a relation, consider a thin walled metallic cylinder of length l, radius r_i and transporting a fluid at temperatures t_i which is higher than ambient temperature t_o. Surrounding this cylinder is an annular section of insulation of thickness $(r - r_i)$ and conductivity K. The heat transmission can be expressed as

$$Q = \frac{(t_i - t_o)}{1/2\pi r_i l h_i + 1/2\pi k l \times \log_e (r/r_i) + 1/2\pi r l h_o}$$

where h_i and h_o are the film coefficient at inner and outer surface.



$$R_t = \frac{1}{2\pi r_i l h_i} + \frac{1}{2\pi k l}\log_e \frac{r}{r_i} + \frac{1}{2\pi r l h_o}$$

Differentiating w.r.t. r

$$\frac{dR_t}{dr} = \frac{d}{dr}\left[\frac{1}{2\pi r_i l h_i} + \frac{1}{2\pi k l}\log_e \frac{r}{r_i} + \frac{1}{2\pi r l h_o}\right] = 0$$

$$\Rightarrow \qquad \frac{1}{2\pi k l}\frac{1}{r} - \frac{1}{2\pi r^2 l h_o} = 0$$

$\therefore$ It gives $\frac{k}{h_o}$.

Differentiating again w.r.t. r

$$\frac{d^2 R_t}{dr^2} = \frac{-1}{2\pi k l}\frac{1}{r^2} + \frac{1}{\pi r^3 l h_o} \qquad \text{at } r = \frac{k}{h_o}$$

$$= \frac{h_o^2}{2\pi k^3 l} \text{ which is +ve.}$$

$\therefore \quad r = \frac{k}{h_o}$ is the condition for minimum resistance and $\therefore$ maximum heat flow rate

$$\therefore \qquad \gamma_{\text{critical}} = \frac{k}{h_o}$$

Q. 1. (b) Derive from 1st principles, the steady-state temperature distribution in a plane slab of thickness '2L', in which there is uniform internal heat generation of qw/m^3 throughout. The end faces of the wall are exposed to a fluid at temperature T_{f1} T_{f2} with h_1, h_2 being the corresponding convective heat transfer coefficients. Locate the occurrence of maximum/minimum temperature in the wall.

Ans. Conduction Through a Plane Wall : Consider one dimensional heat conduction through a homogeneous, isotopic wall of thickness δ with constant thermal conductivity k & constant cross-sectional area A. The wall is insulated on its lateral faces & constant but different temperature t_1 & t_2 are maintained at its boundary surfaces. Obviously temperature varies only in the direction normal to the wall and the temperature potential causes heat temperature transfer in the +ve direction.

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = \frac{1}{\alpha}\frac{\partial t}{\partial \tau}$$

with stipulations of

$$\frac{\partial t}{\partial \tau} = 0 \text{ (steady state)}$$

$$\frac{\partial^2 y}{\partial y^2} = \frac{\partial^2 t}{\partial z^2} = 0 \text{ (one-dimensional)}$$

$$\frac{q_g}{k} = 0 \text{ (no generation of heat)}$$

The conduction equation transforms to

$$\frac{\partial^2 t}{\partial x^2} = 0 \qquad \text{or} \qquad \frac{d^2 t}{dx^2} = 0$$

The second order differential equation can be twice integrated with respect to x to give,

$$\frac{dt}{dx} = c_1 \qquad \& \qquad t = c_1 x + c_2$$

The constants of integration are evaluated with regard to the boundary conditions relevant to the flow situation. Here the boundary conditions are known temperatures. That is

$$t = t_1 \quad \text{at} \quad x = 0 \quad \& \quad t = t_2 \quad \text{at} \quad x = \delta$$

when these boundary conditions are applied to the equation for temperature distribution,

$$t_1 = 0 + c_2 \qquad \& \qquad t_2 = c_1\,\delta + c_2$$

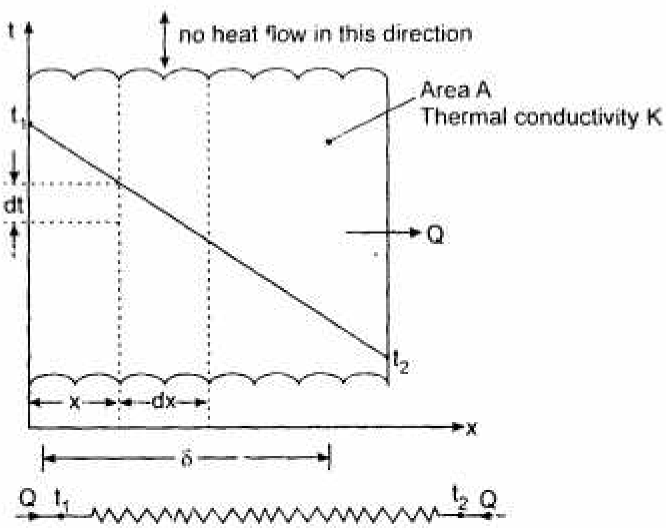
From these identities, the integration constants are obtained as

$$c_2 = t_1 \qquad \& \qquad c_1 = \frac{t_2 - t_1}{\delta}$$

Accordingly the expression for temperature profile become

$$t = t_1 + \left(\frac{t_2 - t_1}{\delta}\right)x \qquad \text{...(i)}$$

The temperature distribution is thus linear across the wall. Since equation (i) does not involve thermal conductivity, a conclusion may be drawn that temperature distribution is independent of the material; whether it is steel, wood or asbestos.



Stready state conduction through a plane wall

Computation for heat flow can be made by substituting the value of temperature gradient into the Fourier equation

$$Q = -kA\frac{dt}{dx}$$

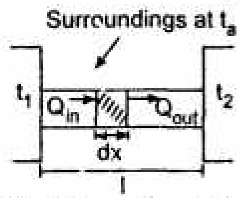
$$\frac{dt}{dx} = \frac{d}{dx}\left[t_1 + \frac{t_2 - t_1}{\delta}x\right] = \frac{t_2 - t_1}{\delta}$$

$$\therefore \qquad Q = -kA\frac{t_2 - t_1}{\delta} = \frac{KA\,(t_1 - t_2)}{\delta}$$

Q. 2. A bar of square Cross-section connects two metallic structures. One structure is maintained at a temperature of 200°C and the other at 50°C. The bar is 20mm × 20mm in Cross-section and 100 mm long and has a thermal conductivity of K = 60 W/mK. The ambient air is at 20°C and the heat

transfer coefficient between the bar and the ambient air is 10W/m^2 K. Derive an expression for the temperature distribution along the bar and calculate the steady-state heat flow rate from the bar to the surroundings.

Ans. Consider heat flow along the bar between two temperatures at t_1 and t_2. The system will be subjected to convective heat flow from the bar to the surroundings.



Let a infinitesimal element of bar with thickness δ_m at distance x from temperature t_1

$$Q_{in} = -k\,A_c\,\frac{dt}{dx}$$

$$Q_{out} = -k\,A_c\left[t + \frac{dt}{dx}\,\delta x\right]$$

$$Q_{conv} = h\,P\,\delta x\,(t - t_a)$$

where A_c is the cross-sectional area, P is the perimenter of bar, h is convective transfer coefficient, k is thermal conductivity and t_a is the ambient temperature.

$$\because \quad Q_{in} = Q_{out} + Q_{conv.}$$

$$\Rightarrow \quad -k\,A_c\,\frac{dt}{dx} = -kA_c\,\frac{d}{dx}\left[t + \frac{dt}{dx}\,\delta x\right] + h\,P\,\delta_x\,(t - t_a)$$

Upon simplification,

$$\frac{d^2 t}{dx^2} - \frac{hp}{kA_c}(t - t_a) = 0$$

Since, ambient temperature is assumed constant, $(t - t_a)$ is replaced by θ.

$$\therefore \quad \frac{d^2 t}{dx^2} \text{ becomes } \frac{d^2\theta}{dx^2}$$

$$\therefore \quad \frac{d^2\theta}{dx^2} - \frac{hp}{kA_c}\,\theta = 0$$

Solving the differential equation

$$\theta = c_1 e^{mx} + c_2\,e^{-mx}$$

where $m = \sqrt{\dfrac{hP}{kA_c}}$

Applying boundary conditions

$$\theta = \theta_1 \quad \text{at} \quad x = 0$$

$$\theta = \theta_1 \quad \text{at} \quad x = l.$$

Applying these conditions, equation becomes

$$\theta_1 = c_1 + c_2 \qquad \text{...(i)}$$

$$\theta_2 = c_1 e^{ml} + c_2 e^{-ml} \quad \text{...(ii)}$$

Solving, we get

$$c_1 = \frac{\theta_2 - \theta_1 e^{-ml}}{e^{ml} - e^{-ml}} \text{ and } c_2 = \frac{\theta_1 e^{ml} - \theta_2}{e^{ml} - e^{-ml}}$$

Substituting c_1 and c_2 in the equation

$$\theta = \frac{\theta_2 - \theta_1 e^{-ml}}{e^{ml} - e^{-ml}} e^{mx} + \frac{\theta_1 e^{ml} - \theta_2}{e^{ml} - e^{-ml}} e^{-mx}.$$

$$= \frac{\theta_1 \sin hm(l-x)}{\sin hml} + \frac{\theta_2 \sin hmx}{\sin hml}$$

$$\theta = \frac{\theta_1 \sin hm(l-x) + \theta_2 \sin hmx}{\sin hml}$$ is the expression for temperature distribution.

Also, rate of heat flow

$$Q = \int_0^l h P\, dx (t - t_a) = \int_0^l h P\, dx\, \theta$$

$$= hP \int_0^l \frac{\theta_1 \sin hm(l-x) + \theta_2 \sin hmx}{\sin hml} dx$$

$$= \frac{hP}{m \sin hml} (\theta_1 + \theta_2) \{\cos h(ml) - 1\}$$

Substituting $m = \sqrt{Ph/A_c}$

$$Q = \sqrt{PhkA_c} (\theta_1 + \theta_2) \left[\frac{\cos h(ml) - 1}{\sin hml}\right]$$

Q. 3. (a) Obtain the correlation between Nusselt number, Grashoff number and Prandtl number for a vertical heated plate losing heat to the surrounding air in Natural Convection using. Dimensional analysis.

Ans. Let Nu = Nusselt No., Gr = Grashoff No., Pr = Prandtl No.

Variables	Symbol	Dimension
Fluid Viscosity	μ	$ML^{-1}T^{-1}$
Fluid Density	ρ	ML^{-3}
Thermal Conductivity	K	$HL^{-1}T^{-1}\theta^{-1}$
Heat Capacity	C_p	$HM^{-1}\theta^{-1}$
Coefficient of Thermal Expansion	β	θ^{-1}
Temperature Difference	Δt	θ
Significant Length	l	L
Heat Transfer Coefficient	h	$HL^{-2}T^{-1}\theta^{-1}$

Using Buckingham's π method

Functional Relationship $f(\mu, \rho, K, C_p, \beta_g\ \Delta T, h) = 0$

There are 8 physical quantities (β_g and ΔT are counted separately) and five fundamental units.

Hence, (8 − 5) or 3π terms will be used.

$$\pi_L = \mu^a\ K^b\ (\beta_g\ \Delta T)^c\ l^d\ \rho$$

$$1 = (ML^{-1}\ T^{-1})^a\ (HL^{-1}\ T^{-1}\ \theta^{-1})^b\ (LT^{-1})^c \times (L)^d \times ML^{-3}$$

Equating exponents of fundamental dimensions

at $\quad 1 = 0; -a - b + c + d - 3 = 0; -a - b - 2c = 0; -b = 0;\ b = a$

$\therefore \quad a = -1;\ b = 0;\ c = 1/2,\ d = 3/2$

$$\therefore \quad \pi_1 = \frac{l^3\ \rho^2\ (\beta_g\ \Delta T)}{\mu^2}$$

Similarly, $\quad \pi_2 = \mu^a K^b\ (\beta_g\ \Delta T)^c\ l^d\ C_p\ ; \qquad \pi_2 = \dfrac{\mu\ C_p}{K}$

$\pi_3 = \mu^a K^b\ (\beta_g\ \Delta T)^c\ l^d\ h\ ; \qquad \pi_3 = \dfrac{hl}{K}$

$\therefore$ Relationship becomes :

$$\phi = \left[\frac{l^3\ \rho^2\ \beta_g\ \Delta T}{\mu^2}, \frac{\mu C_p}{K}, \frac{hl}{K}\right] = 0$$

$$\therefore \quad Nu = \phi(Gr,\ pr).$$

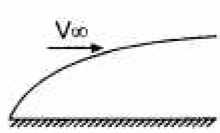
Q. 3. (b) Using the tenets of boundary layer theory derive the governing equation for boundary layer flow over a flat plate at zero angle of incidence.

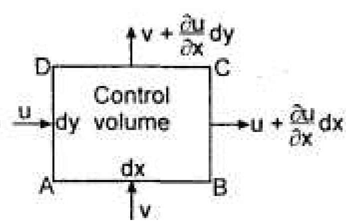
Ans. Consider an infinitesimal, two dimensional control volume ($dx \times dy \times$ unit depth) within the boundary layer region and assume that

(i) Flow is steady and fluid is in compressible.

(ii) Viscosity is constant.

(iii) Shear in y direction is negligible.





Let u represent the velocity of fluid a' the left hand face AD. The change in velocity along x axis is assumed to be $\dfrac{\partial u}{\partial x}$ and along y axis it is assumed to be $\dfrac{\partial \upsilon}{\partial y}$.

$\therefore$ Velocity at face $BC = u + \dfrac{\partial u}{\partial x} dx$

Velocity at face CD $= v + \dfrac{\partial \upsilon}{\partial y} dy$

$\therefore$ The mass entering the left face of the control volume during time interval $d\tau$ is give

$$\text{Fluid influx} = \text{density} \times (\text{velocity} \times \text{area}) \times \text{time}$$
$$= \rho\, u\, dy\, d\tau$$

During same time

$$\text{Fluid efflux} = \rho \left[u + \frac{\partial u}{\partial x} dx \right] dy\, d\tau.$$

Likewise, the mass flow entering the bottom face is $\rho\, \upsilon\, dx\, d\tau$ and mass leaving top face $\rho \left\{ \upsilon + \left(\frac{\partial \upsilon}{\partial y} \right) dy \right\} dx\, d\tau$

Mass balance of element yields :

$$\rho\, u\, dy\, d\tau + \rho\, \upsilon\, dx\, d\tau = \rho \left[u + \frac{\partial u}{\partial x} dx \right] dy\, d\tau + \rho \left[u + \frac{\partial u}{\partial x} dy \right] dx\, d\tau$$

Simplification gives

$$\boxed{\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} = 0}$$ is the required equation of boundary layer flow.

Q. 4. (a) A thorium fuel rod is in the form of a Solid cylinder of 20mm diameter. It is covered with a thin aluminium cladding of 2mm thickness. The melting point of thorium is 2023 K and Aluminium loses its mechanical strength above a temperature of 700 K. The fuel rod has uniform internal heat generation of 400 MW/m^3. The system loses heat to the surrounding fluid at 90°C with h = 6kW/m^{2}K. K = 237 W/mK for Aluminium and K = 54W/mK for thorium. Determine if the system is safe.

Ans. Let r_1 and r_2 denote the inner and outer radii of the plastic insulation. Then the heat flow rate per metre length of the wire is given by

$$Q = \frac{t_1 - t_2}{\frac{1}{2\pi kl} \log(r_2 \quad r_1) + \frac{1}{2\pi r_2 lh}}$$

$$= \frac{700 - 90°}{\left[\frac{1}{2\pi \times 237 \times 1} \times \log\left(\frac{12}{10} \right) \right] + \left[\frac{1}{2\pi \times 0{\cdot}12 \times 1 \times 6000} \right]}$$

$$= \frac{610}{1{\cdot}224 \times 10^{-4} + 22{\cdot}10 \times 10^{-4}} = 26{\cdot}15 \times 10^4 \text{ w per unit length.}$$

$$\text{Internal heat generation } q_g = \frac{26{\cdot}15 \times 10^4}{\pi ({\cdot}01)^2 \times 1} = 8{\cdot}32 \times 10^8 \text{ w/m}^3$$

$\therefore$ Maximum temperature occurs at midpoint of wire and is equal to

$$t_{max} = tw + \frac{q_g}{4k} R^2$$
$$= 90° + \frac{8{\cdot}32 \times 10^7}{4 \times 54} \times ({\cdot}01)^2$$
$$= 475{\cdot}18° \text{ K which is less than 2023 K.}$$

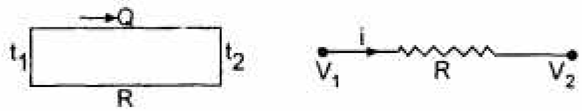
So, design is safe.

Q. 4. (b) Explain the analogy between heat conduction and electric current. What is the utility of this analogy ?

Ans. Heat conduction and electric current are analogous and is understood as :

Current (i)	analogous to	Heat Rate (Q)
Resistance (R)	analogous to	Resistance to heat flow (R)
Voltage (V)	analogous to	Temperature Difference (ΔT)

Analogy is useful as the heat conduction can be represented by the electrical circuits which can help in easy calculations and visualisation of the problems related to the heat conduction.



Q. 5. Water (Cp = 4187J/kgk) is heated at the rate of 1.4 kg/s from 40°C to 70°C by an oil (Cp = 1900J/kgk) entering at 110°C and leaving at 60°C in a counter flow heat exchanger. If the overall heat transfer coefficient. U_0 = 350W/m^{2}k. Calculate the surface area required. If the water flow rate is halved retaining the same inlet temperatures and oil flow rate, calculate the exit temperatures of both the fluids.

Ans. Heat transfer to the water, $Q = m \times Cp \times \Delta T$

$$Q = 1{\cdot}4\,\text{kg/s} \times 4{\cdot}186 \times 10^3\ (70-40)$$

$$= 1{\cdot}76 \times 10^5\ \text{J/s}$$

Log mean temperature difference,

$$\theta_m = \frac{\theta_1 - \theta_2}{\log_e(\theta_1/\theta_2)}$$

For counter flow heat exchanger

$$\theta_1 = t_{h_1} - t_{c_2} = (110° - 40°) = 70°$$

$$\theta_2 = t_{h_2} - t_{c_1} = (60° - 70°) = -10°$$

$$\theta_m = \frac{70° + 10°}{\log(70/10)} = 41{\cdot}11°K$$

Heat exchange, $Q = VA\theta_m$

Heating surface area, $$A = \frac{Q}{V\theta_m} = \frac{1{\cdot}76 \times 10^5\ \text{J/s}}{350 \times 41{\cdot}11\ \text{W/m}^2}$$

$$\boxed{A = 12{\cdot}23\ \text{m}^2}$$

Q. 6. (a) Define the terms :

(i) Black body

(ii) Gray body

(iii) Monochromatic emissive power

(iv) Diffuse Emitter

Ans. (i) Black Body : Black surfaces are effective absorbers of radiation in the wavelengths that are encountered in heat transfer. Black body is assigned to a perfect absorber of radiation.

(ii) Gray Body : When a surface absorbs a certain fixed percentage of impinging radiations, the surface is called gray body.

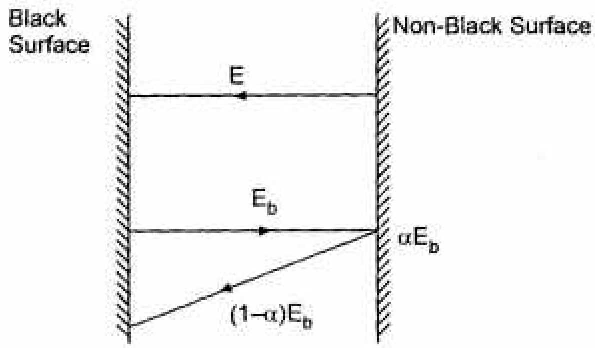
(iii) Monochromatic Emissive Power : When a surface emits a percentage of light (monochromatic) falling at it or amount of light emitted by it is called emissive power.

(iv) Diffuse Emitters : If a surface emits a diffuse light then it is called as a diffuse emitter.

Q. 6. (c) State and prove Kirchoff's law of radiation.

Ans. Kirchoff's Law : It states that the ratio of the emissive power E to absorptivity is same for all bodies and is equal to the emissive power of a black body at the same temperature.

Consider two surfaces, one absolutely black at temperatures T_b and other non-black at temperature T. The surfaces are arranged parallel to each other and so close that the radiation of one falls equally on the other. The radiant energy E emitted by the non-black surfaces impinges on the black surface and gets fully absorbed.



Under these conditions :

$$E - \alpha\, E_b = 0$$

$$\frac{E}{\alpha} = E_b$$

This relation can be extended by considering different surfaces in turn. Therefore,

$$\frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} = \frac{E_3}{\alpha_3} = \ldots\ldots = \frac{E_b}{\alpha_b} = E_b = f\,(t)$$

is the required proof of Kirchoff's law.

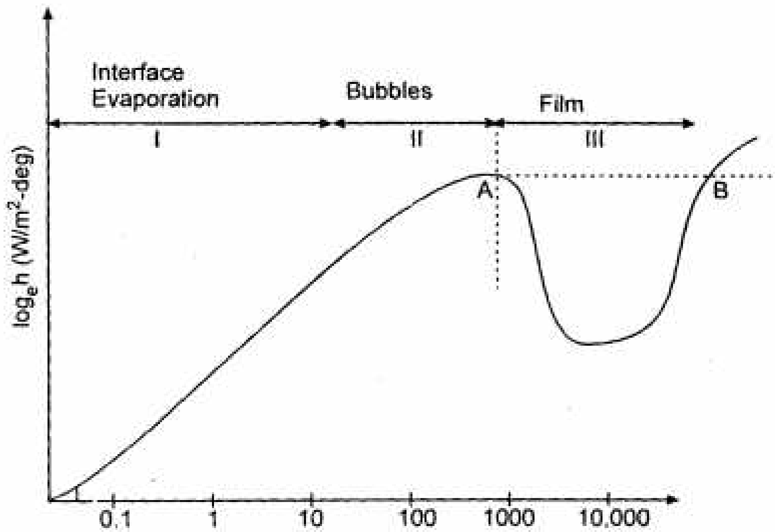
Q. 7. (a) Describe the different regimes of boiling and depict their ranges on a heat flux Vs. excess temperature curve.

Ans. Boiling Regimes : Boiling phenomenon corresponds to pool boiling or forced circulation boiling, there are three definite regimes of boiling associated with progressively increasing heat flux.

(i) Evaporation with no Bubble Formation (Interface Evaporation) : This boiling takes place in a thin layer of liquid which adjoins the heated surface. The liquid in the immediate vicinity of the wall becomes superheated. The superheated liquid rises to the liquid vapour interface where evaporation takes place.

(ii) Nucleate Boiling : When the liquid is overheated in relation to saturation temperature, vapour bubbles are formed at certain favourable spots called nucleation or active sites. The bubbles grow to certain size influenced by pressure, temperature and surface tension at the liquid vapour surface. Bubbles form and collapse on the surface itself.

(iii) Film Boiling : The bubble formation is very rapid, the bubbles blanket the heating surface and prevent the incoming fresh liquid from taking their place. Eventually the bubbles coulesce and form a vapour film which covers the surface completely. Insulating effect of the vapour film overshadows the beneficial effect of liquid agitation and consequently the heat flux drops with growth in temperature excess.

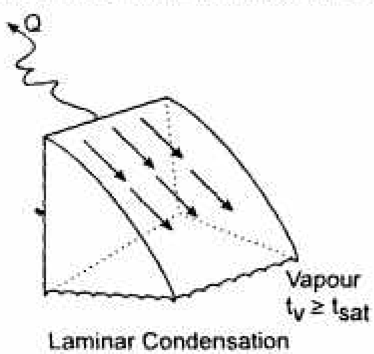


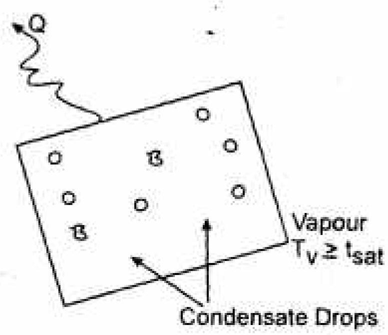
Temperature Excess ($t_s - t_{sat}$)°C

Q. 7. (b) Distinguish between laminar film condensation and dropwise condensation.

Ans. Laminar Film Condensation : The liquid condensate wets the solid surface, spreads out and forms a continuous film over the entire surface. The liquid flows down the cooling surface under the action of gravity and the layer continuously grows in thickness because of newly condensing vapours. The continuous film offers thermal resistance and restricts further transfer of heat between the vapour and the surface. It usually occurs when a vapour, relatively free from impurities, is allowed to condense on a clean surface.

Dropwise Condensation : The liquid condensate collects in droplets and does not wet the solid cooling surface. The droplets develop in cracks and pits on the surface, grow in size, break away from the surface, knock off other droplets and eventually run off the surface without forming a film. A part of the condensation surface is directly exposed to the vapour without an insulating film of condensate liquid. Evidently there is no film barrier to heat flow and higher heat transfer rates are experienced; heat transfer fluxes of the order of 750 kW/m^2 have been obtained with dropwise condensation.



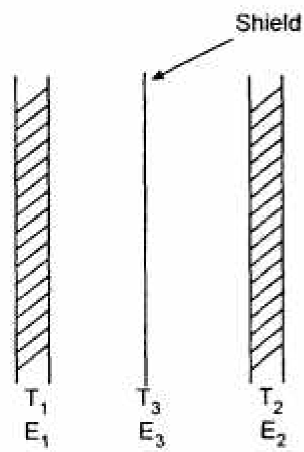


Q. 8. Write short notes on any *two* :

(a) Radiation Shields

(b) Fin effectiveness and Fin efficiency

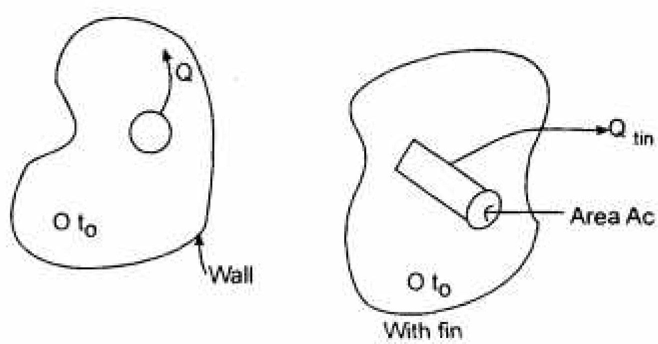
Ans. (a) Radiation Shields : Many situations are encountered where it is desired to reduce the overall heat transfer between two radiating surfaces. The task is accomplished by placing radiation shields between the emitting surfaces. The shields are thin opaque partitions arranged in the direction perpendicular to the propagation of radiated heat and made of materials of very low absorptivity and high reflectivity (aluminium or copper etc.) The shields introduce a sort of additional resistance in the heat flow path and accordingly the net heat flux is reduced.



Comparisons has shown that the heat flow is reduced by using shields.

For n number of shields heat exchange reduces in the ratio of $(n+1)$.

(b) Fin Effectiveness and Fin Efficiency : Fin effectiveness represents the ratio of the fin heat transfer rate (heat dissipation with a fin) to the heat transfer rate that would exist without a fin.



Heat Transfer before fin $\Rightarrow$ $Q = hA_c\,(t_o - t_a)$

Heat Transfer with fin $\Rightarrow$ $Q_{fin} = \sqrt{PhkA_c}\,(t_o - t_a)$

$$\therefore \qquad \text{Fin Effectiveness} = \frac{\sqrt{PhkA_c}\,(t_o - t_a)}{hA_c\,.(t_o - t_a)}$$

$$= \sqrt{\frac{PK}{hA_c}}$$

Fin efficiency relates the performance of an actual fin to that of an ideal or fully effective fin. A fin will be most effective i.e., it would dissipate heat at maximum rate if the entire fin surface area is mantained at the base temperature.

$$\eta_f = \frac{\text{Actual heat transfer rate from the fin}}{\text{Heat would dissipate if whole surface of fin is maintained at base temperature}}$$

Maximization of fin performance with respect to its length does not constitute the design criterion for a pin. The efficiency of fin, forms a criterion for judging the relative merits of fins of different geometrics or materials.