B.E.
Sixth Semester Examination, 2010
Automatic Controls (ME-308-E)

Note : Attempt five questions. All questions carry equal marks.

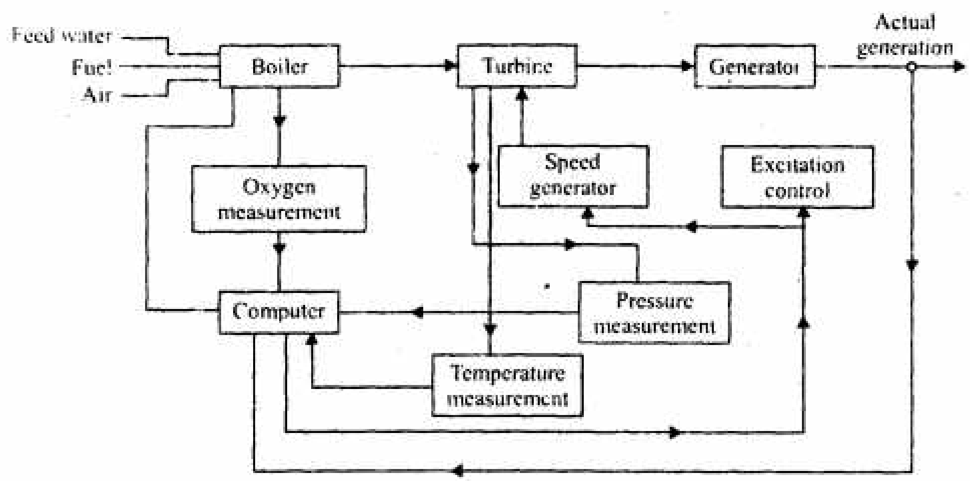
Q. 1. (a) Discuss the applications of control system in machine tool control and boiler control.

Ans. Control System : It's basically a mechanism by which the other equipment is maintained or altered in a desired manner.

There are a lot of applications of control systems even, we are using very frequently them in our daily life.

Such that control system in maintaining tank level, example of a position control system, use in missile launching, use in automatic frequency control, in microwave communication transmitter, electromagnetic balance system, beam balance system etc.

Boiler Generator Control :



The boiler control system is shown in above figure, it has :

(i) Watts, vars, line voltage.

(ii) Temperature and pressure of steam inlet to turbine.

(iii) Oxygen content in furnance air.

These all the inputs are processed by the control transfer function of computer to produce few signals to produce signal to adjust throttle level, signal to adjust fuel, feed water and air.

Q. 1. (b) Explain the various types of control actions.

Ans. Various Types or Stages of Control System : The stages of development of control system can be classified into four categories :

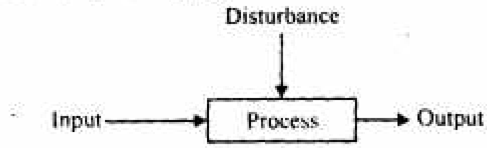
(i) Open loop system

(ii) Closed loop system

(iii) Utilisation of adaptive controller in closed loop system.

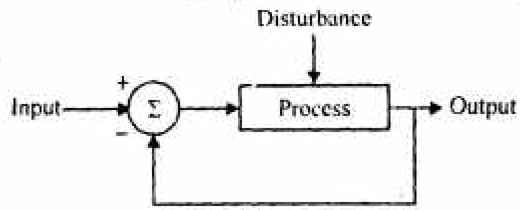
(iv) Learning system.

Open Loop System : Here the open loop system is shown where we have applieded



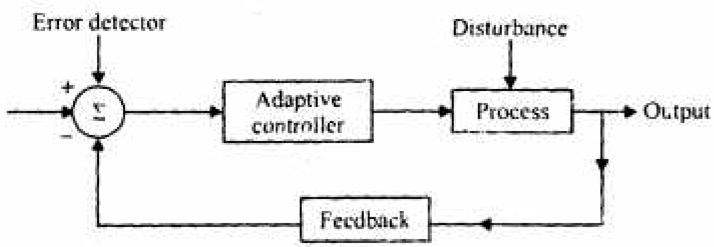
Input and by process we get output respectives. Here no feedback is provided so open loop.

Closed Loop System : Here the following figure shows the closed loop system.



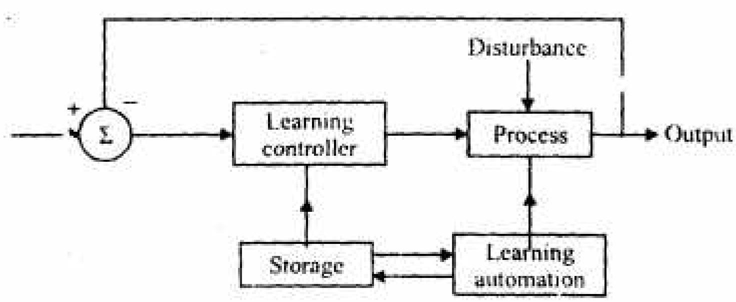
Here it contains feedback so that whatever be the output it's feeded back to the next input hence error is measured.

Adaptive Controller :



Here the adaptive controller is shown so that we can use any alternative controller i.e., proportional or integral or derivative or their combinations are used.

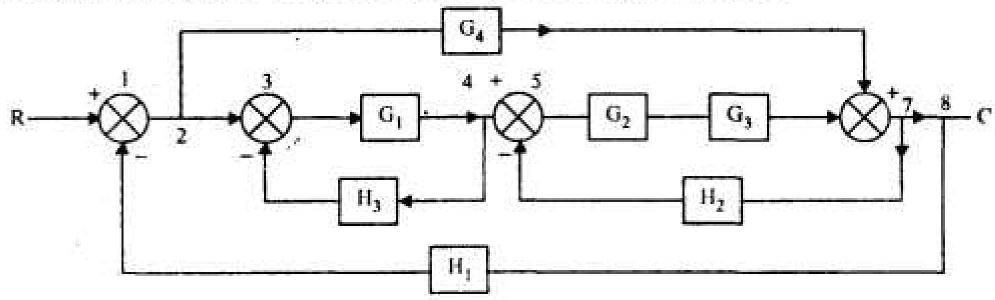
Learning System :



Here the figure for the learning system is shown we use an extra controller called as learning controller here.

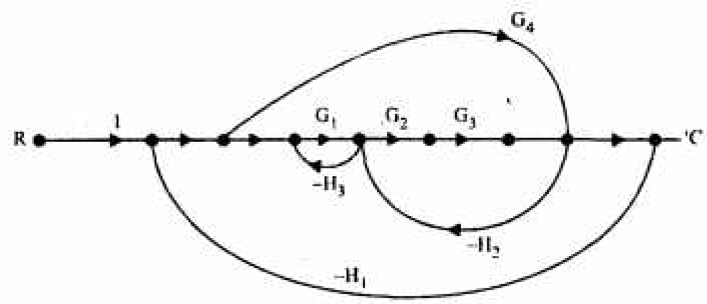
& another storing device is used here while the negative feedback is provided.

Q. 2. For the block diagram in figure, draw the signal flow diagram and derive expression for c/r using Mason's formula and also check by block diagram algebra.



Ans. Block Diagram Reduction Techniques & Mason Gain Formula :

(i) Mason Gain Formula :



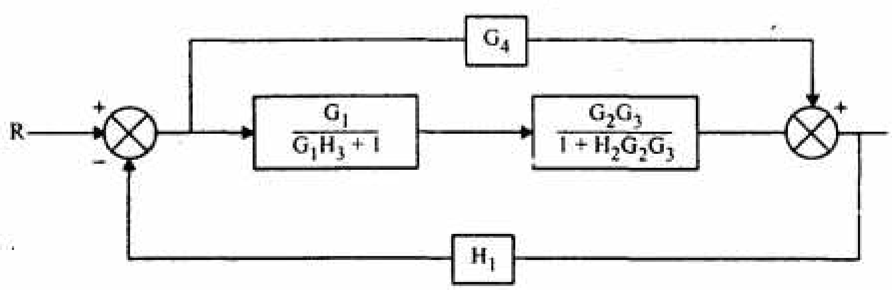
Hence forward paths $= G_1G_2G_3;\ G_4$

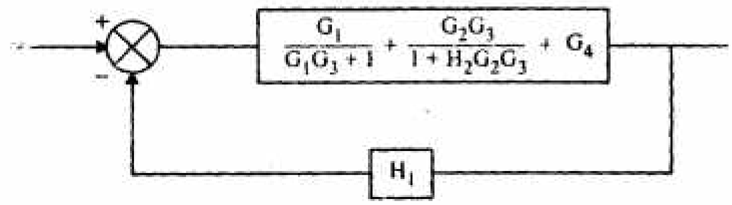
& loops $= -G_1H_3;\ -G_2G_3H_2;\ -G_4H_1;\ -G_1G_2G_3H_1$

Two non touching loops gain $= G_1H_3.G_4H_1$

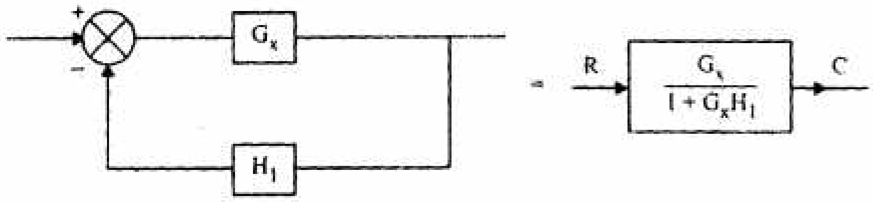
Hence $$\frac{C}{R} = \frac{G_1G_2G_3 + G_4}{1 + G_1H_3 + G_2G_3H_2 + G_4H_1 + G_1G_2G_3H_1}$$

By Block Diagram Reduction :





Let $$G_x = \frac{G_1}{G_1H_3+1} + \frac{G_2G_3}{1+H_2G_2G_3} + G_4$$



Hence $$\frac{C}{R} = \frac{G_1G_2G_3+G_4}{1+G_1H_3+G_2G_3H_2+G_4H_1+G_1G_2G_4H_1}$$

On simplifying we get the same transfer function as that by Meason gain.

Q. 3. What are the time domain quantities which characterise a transient response? Derive an expression for percentage overshoot of a second order.

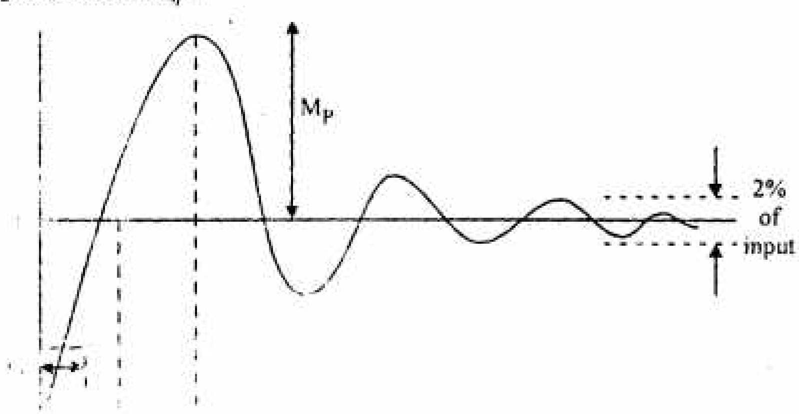
Ans. Time domain quantities which characterise a transient response are :

(i) Delay time (t_d) (ii) Rise time (t_r)

(iii) Peak time (t_p) (iv) Settling time (t_s)

(v) Maximum overshoot (M_p)

(vi) Damping coefficient (e_q)



Derivation of Percentage Overshoot :

$\because$ We know that at the peak time the value of the system that is above to 1 is called as overshoot hence

$$M_p = \text{value at } t_p - 1$$

$\because$
$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left(\sin\left[\omega_n\sqrt{1-\xi^2}\,t_p + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right]\right)$$

So
$$M_p = \frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\sin\left[\omega_n\sqrt{1-\xi^2}\,t_p + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right]$$

But
$$t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

$$= \frac{e^{-\xi\omega_n\pi/\omega_n\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}}\sin\left\{\pi + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right\}$$

$$= \frac{e^{-\pi\xi/\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}}\sin\left\{\tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right\}$$

Let
$$\theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$$

So
$$\sin\theta = \sqrt{1-\xi^2}$$

$$= \frac{e^{-\pi\xi/\sqrt{1-\xi^{-2}}}\times\sqrt{1-\xi^2}}{\sqrt{1-\xi^2}}$$

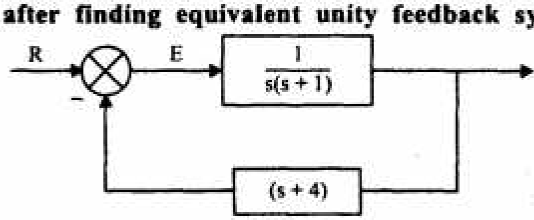
$$= e^{-\pi\xi/\sqrt{1-\xi^2}}$$

Hence
$$\boxed{\%M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100\%}$$ **Proved**

Q. 4. For the system as shown in figure, find the peak value of 1M' and the frequency at which it occurs. Use :

(i) M-circle

(ii) Nichols charts after finding equivalent unity feedback system. Check the results calculations.



Ans. $\because$ $\dfrac{C(s)}{R(s)}$ of the given network is as

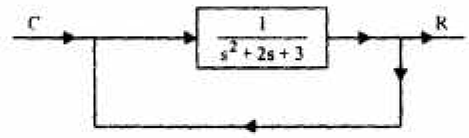
$$\frac{C}{R}=\frac{\dfrac{1}{s(s+1)}}{1+\dfrac{(s+4)}{s(s+1)}}=\frac{1}{s(s+1)+(s+4)}=\frac{1}{s^2+2s+4}$$

Hence if we use the equivalent feedback system then

$$\frac{1}{s^2+2s+4}=\frac{G'(s)}{1+G'(s)}$$

$$\Rightarrow \qquad 1+G'(s)=G'(s)[s^2+2s+4]$$

$$\Rightarrow \qquad G'(s)=\frac{1}{s^2+2s+3}=1$$



Hence the equivalent unity feedback system is as shown in figure.

Here the peak value $\qquad M_p=e^{-\pi\xi/\sqrt{1-\xi^2}}$

$\because \qquad \omega_n=2; \quad 2\times\xi\times 2=2$

So $\qquad \xi=\frac{1}{2}=.5$

$$M_p=e^{-\pi\times .25/\sqrt{1-(.0625)}}=e^{-\pi\times .25/.9682}$$

$$=4$$

$$\%M_p=e^{-\pi\xi/\sqrt{1-\xi^2}}=e^{-\pi\times .5/\sqrt{1-.25}}$$

$$=e^{-\pi\times .5/\sqrt{.75}}=e^{-1.8138}=.163\times 100$$

$$=16\%$$

and as we see by the Nicholas chart the value comes out to be 15.6% and the small variation we get due to the improperty.

Q. 5. Draw the root locus for a system whose open loop transfer function is given by

$$G(s)H(s)=\frac{K}{s(s+4)(s^2+4s+20)}$$

Show all the salient points on the locus.

Ans. $\qquad G(s)H(s)=\dfrac{K}{s(s+4)(s^2+4s+20)}$

(i) Poles $=0; \ -4; \ -2\pm 4j$

(ii) Existence on real axis $(0, \ -4)$

(iii) Centroid $= \dfrac{\Sigma P - \Sigma Z}{P - Z} = \dfrac{0-4-2-2}{4} = -2$

(iv) Angle of asymptotes $= \dfrac{(2K+1)180^\circ}{45} = 45^\circ;\ 135^\circ;\ 225^\circ;\ 315^\circ$

(v) Break away point $\dfrac{dk}{ds} = \dfrac{d}{ds}[s^4 + 8s^3 + 36s^2 + 80s]$

$\Rightarrow \quad 4s^3 + 24s^2 + 72s + 80 = 0$

$\Rightarrow \quad s^3 + 6s^2 + 18s + 20 = 0$

$\Rightarrow \quad s^3 + 2s^2 + 4s^2 + 8s + 18s + 20 = 0$

$\Rightarrow \quad (s^2 + 4s + 10)(s + 2) = 0$

$$s = -2,\ -2 \pm j\sqrt{6}$$

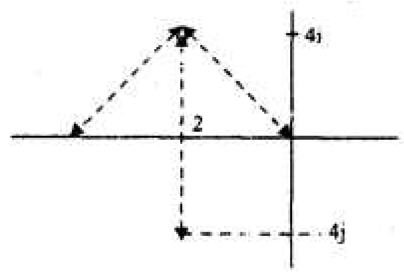
So

(vi) Angle of departure $\phi = -\tan^{-1}(2) + 180^\circ + \tan^{-1}(2) + 90^\circ$

$= 270^\circ$

Hence $\phi_b = 180^\circ - 270^\circ = -90^\circ$

& other $= +90^\circ$



Intersection with $j\omega$ axis

$$1 + G(s)H(s) = 0$$

$$s^4 + 8s^3 + 36s^2 + 80s + K = 0$$

s^4	1	36	K
s^3	8	80	0
s^2	26	K	0
s	$\dfrac{26 \times 80 - 8K}{26}$		0
s^0	K		0

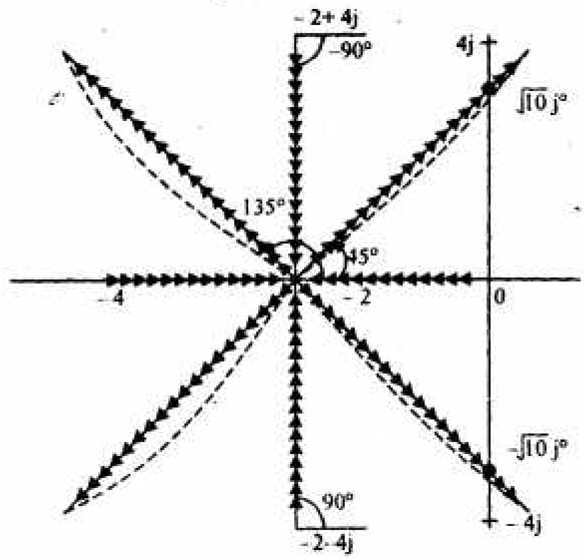
$$K = 260$$

$$26s^2 + 260 = 0$$

$$s^2 = -10$$

$$s = \pm\sqrt{10}$$

$$\omega = \pm\sqrt{10} = -3$$



Q. 6. Draw Nyquist plot for a control system with open loop transfer function of

$$G(s)H(s) = \frac{50}{(1+0.2s)(s^2+10s+20)}$$

and find if the system is stable or not.

Ans.
$$G(s)H(s) = \frac{50}{(1+0.2s)(s^2+10s+20)}$$

Then
$$G(s)H(s) = \frac{50}{\sqrt{1+(.2\omega)^2}\sqrt{(20-\omega^2)^2+(10\omega)^2}} \angle -\tan^{-1}(.2\omega) - \tan^{-1}\left(\frac{10\omega}{20-\omega^2}\right)$$

Now at $\omega = 0$ $\quad$ gain $= \frac{50}{20} = 2.5$ phase $= 0°$

$\omega = \infty$ $\quad$ gain $= 0$ $\quad$ phase $= -90°$

Type $= 0$

Order $= 4$

$$G(j\omega)H(j\omega) = \frac{50}{(1+.2j\omega)(20-\omega^2+10j\omega)}$$

$$= \frac{50}{(20-\omega^2)-2\omega^2+j(10\omega+.2\omega(20-\omega^2))}$$

Intersection with x axis

$$10\omega + .2\omega(20-\omega^2) = 0$$

$$\Rightarrow \quad 100 + 2(20-\omega^2) = 0$$

$$\Rightarrow \qquad 50+20-\omega^2=0$$

$$\Rightarrow \qquad \omega^2=70$$

$$\Rightarrow \qquad \omega=\sqrt{70}=8.366$$

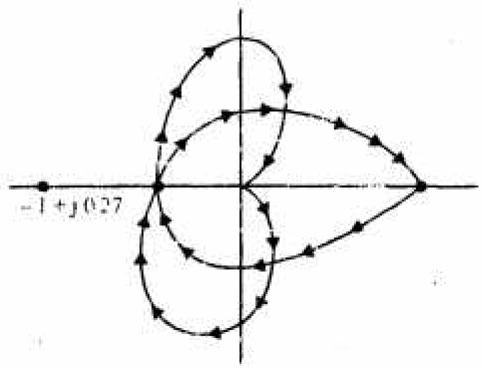
& gain at that frequency

$$=\frac{50}{\sqrt{1+(.2\times 8.36)^2}\sqrt{(20-8.366^2)^2+(83.66)^2}}$$

$$=\frac{50}{\sqrt{3.8}\sqrt{9487.96}}=\frac{50}{1.9\times 97.4}=\frac{50}{185.0717}$$

& $\qquad = 0.27017$

Hence



$\therefore N=$ No. of encirclement $=0$ & $P=0$

Hence $\qquad Z=0$

$$N=P-Z$$

It means system is stable.

Q. 7. Find free solution for the following state equations :

$$[x(t)]=\begin{bmatrix}0 & 1\\ -3 & -4\end{bmatrix}[x(t)]+\begin{bmatrix}0\\ 2\end{bmatrix}u(t)$$

Where initial conditions are

$$[x(0)]=\begin{bmatrix}1\\ 0\end{bmatrix}$$

Ans.

$$x(t)=\begin{bmatrix}0 & 1\\ -3 & -4\end{bmatrix}[x(t)]+\begin{bmatrix}0\\ 2\end{bmatrix}u=\begin{bmatrix}1\\ 0\end{bmatrix}$$

$$A=\begin{bmatrix}0 & 1\\ -3 & -4\end{bmatrix}\quad B=\begin{bmatrix}0\\ 2\end{bmatrix}$$

$$\because \quad e^{At} = I + At + \frac{1}{2!}A^2t^2 + \frac{1}{3!}A^3t^3$$

$$A = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} \quad A^2 = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} = \begin{bmatrix} -3 & -4 \\ 12 & 1 \end{bmatrix}$$

$$\because \quad (sI - A) = \begin{bmatrix} s & -1 \\ 3 & s+4 \end{bmatrix}$$

$$Adj(sI - A) = \begin{bmatrix} s+4 & 1 \\ -3 & s \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{1}{(s^2 + 4s + 3)}\begin{bmatrix} (s+4) & 1 \\ -3 & s \end{bmatrix}$$

$$\phi(t) = L^{-1}\{(sI - A)^{-1}\} = \begin{bmatrix} \frac{(s+4)}{(s+3)(s+4)} & \frac{1}{(s+3)(s+1)} \\ -\frac{3}{(s+3)(s+1)} & \frac{s}{(s+3)(s+1)} \end{bmatrix}$$

$$\phi(t) = L^{-1}\begin{bmatrix} \frac{3/2}{(s+1)} - \frac{1/2}{(s+3)} & \frac{1/2}{(s+1)} - \frac{1/2}{(s+3)} \\ \frac{3/2}{(s+3)} - \frac{3/2}{(s+1)} & \frac{3/2}{(s+3)} - \frac{1/2}{(s+1)} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{2}e^{-t} - \frac{1}{2}e^{-3t} & \frac{1}{2}e^{-t} - \frac{1}{2}e^{-3t} \\ \frac{3}{2}e^{-3t} - \frac{3}{2}e^{-t} & \frac{3}{2}e^{-3t} - \frac{1}{2}e^{-t} \end{bmatrix}$$

$$X(t) = \phi(t)\left[X_0 + \int_0^t \phi(-\tau)Budt\right]$$

$$\phi(-\tau)Bu = \begin{bmatrix} \frac{3}{2}e^{\tau} - \frac{1}{2}e^{3\tau} & \frac{1}{2}e^{\tau} - \frac{1}{2}e^{+3\tau} \\ \frac{3}{2}e^{3\tau} - \frac{3}{2}e^{\tau} & \frac{3}{2}e^{3\tau} - \frac{1}{2}e^{\tau} \end{bmatrix}\begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} e^{\tau} - e^{3\tau} \\ 3e^{3\tau} - e^{\tau} \end{bmatrix}$$

$$\int_0^t \phi(-\tau)Budt = \begin{bmatrix} \int(e^{\tau} - e^{3\tau})dt \\ \int(3e^{3\tau} - e^{\tau})dt \end{bmatrix}$$

$$= \begin{bmatrix} e^t - \frac{e^{3t}}{3} \\ e^{3t} - e^t \end{bmatrix}$$

I

Then solution is given by $$X = [\phi(t)]\left\{\begin{bmatrix}1\\0\end{bmatrix} + \begin{bmatrix}e^{t} - \dfrac{e^{3t}}{3}\\ e^{3t} - e^{t}\end{bmatrix}\right\}$$

$$X(t) = \begin{bmatrix}\frac{3}{2}e^{-t} - \frac{1}{2}e^{-3t} & \frac{1}{2}e^{-t} - \frac{1}{2}e^{-3t}\\ \frac{3}{2}e^{-3t} - \frac{3}{2}e^{-t} & \frac{3}{2}e^{-3t} - \frac{1}{2}e^{-t}\end{bmatrix}\begin{bmatrix}1 + e^{-t} - \dfrac{e^{3t}}{3}\\ e^{3t} - e^{t}\end{bmatrix}$$

Q. 8. Discuss the following :

(i) Pulse Transfer function

(ii) Hydraulic controllers

(iii) Open loop control system.

Ans. (i) Pulse Transfer Function : Actually every transfer function has some input and produces output with respect to input and so

$$H(s) = \frac{Y(s)}{X(s)}$$

If the input taken is unit step

$$u(t) = \begin{Bmatrix}1, & t > 0\\ 0, & t < 0\end{Bmatrix}$$

Then $$X(s) = \frac{1}{s}$$

So $$H(s) = \frac{Y(s)}{1/s}$$

$\Rightarrow$ $$Y(s) = \frac{H(s)}{s}$$

Or if the input taken is $x(t) = \delta(t)$ then

$$\delta(s) = 1$$

So $$Y(s) = H(s)$$

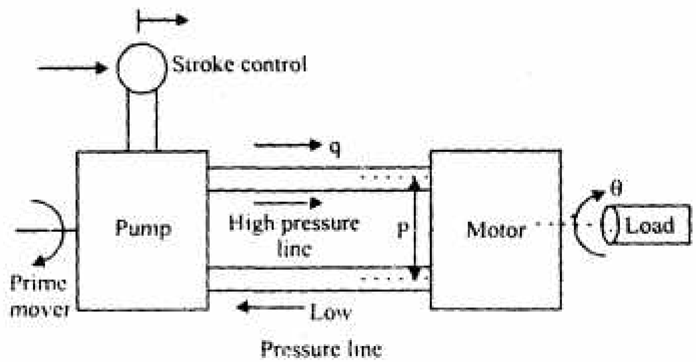
$$\boxed{y(t) = h(t)}$$

It means that if pulse is input then at that condition the impulse response is the output of the system.

(ii) Hydraulic Controllers : It consist of the variable stroke hydraulic pump and a fixed stroke hydraulic motor control of the motor is exercised by varing amount of oil delivered to load (pump).

Like a dc generator or motor, there is no essential difference between hydraulic pump and motor.

In a pump, the input is mechanical power and output, hydraulic power and in a motor, input is hydraulic and output mechanical.

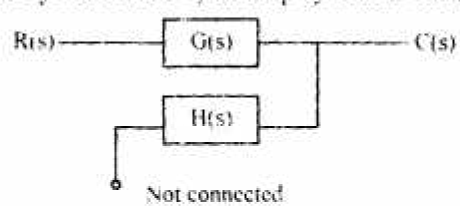


The transfer function of the system is given by

$$G(s) = \frac{\theta(s)}{X(s)} = \frac{K_p}{s\left[\frac{k_e \tau}{k_m} s + \left(k_m + \frac{k_e f}{k_m}\right)\right]}$$

$$= \frac{k}{s(\tau s + 1)}$$

(iii) Open Loop Control System : The open loop system is shown in figure



Where once you given input to the transfer system function the output is obtained.

and here $H(s) = G(s)$

Or $G(s)H(s) \rightarrow$ open loop. because no part of output is going back to input so called open loop (without loop) it's stable system.

Contains ideally no error. Here the gain is maximum. The error value is very less for the practical systems.