B.E.

Sixth Semester Examination, Dec.-2009

Automatic Controls (ME-308-E)

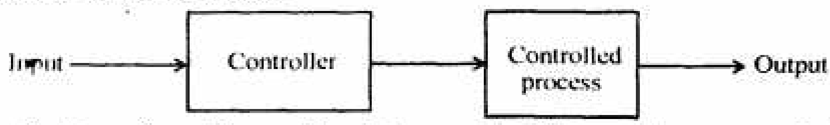
Note : Answer any *five* questions. All questions carry equal marks.

Q. 1. (a) Explain the various types of control systems.

Ans. Types of Control Systems : The control systems can be classified as open loop control system and closed loop control systems.

(i) Open Loop control System : The open loop control system is also known as control system without feedback or non feedback control systems. In open loop systems the control action is independent of the desired output. In this system the output is not compared with the reference input.

The component of the open loop systems are controller and controlled process. The controller may be amplifier filter etc. depends upon the system. An input is applied to the controller and the output of the controller gives to the controlled process.



Example : (i) Automatic washing machine is the example of the open loop systems. In the machine the operating time is set manually. After the completion of set time the machine will stop with the result. We may or may not get the desired (output) amount of cleanliness of washed clothes because there is no feedback provided to the machine for desired output.

Advantages :

(i) Open loop control systems are simple.

(ii) Open loop control systems are economical.

(iii) Less maintenance is required and not difficult.

(iv) Proper calibration is not a problem.

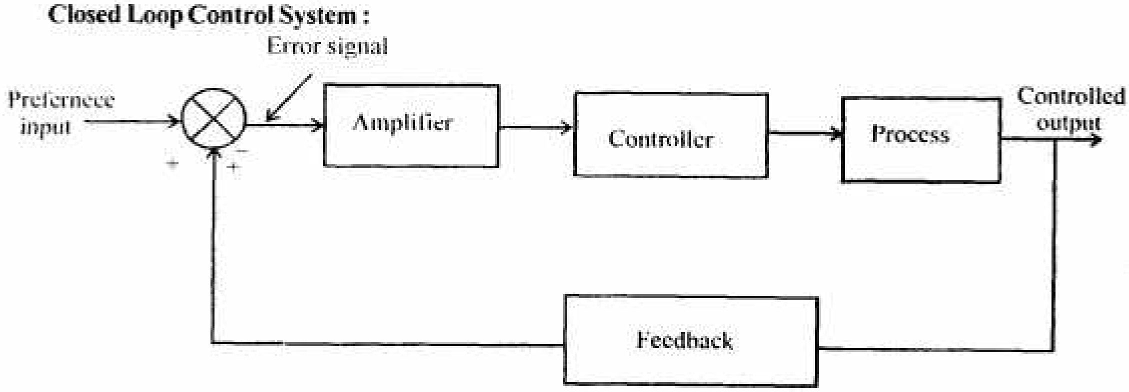
Disadvantages :

(i) Open loop systems are inaccurate.

(ii) These are not reliable.

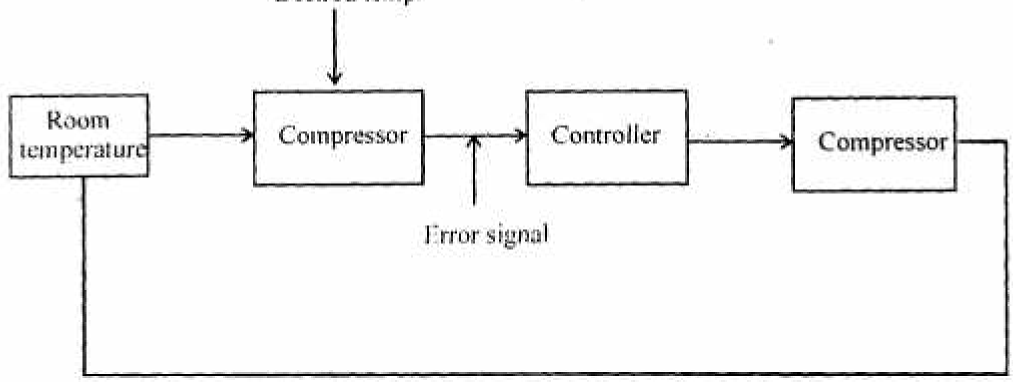
(iii) Optimization is not possible.

Closed Loop Control System :



Closed loop control systems are also known as feedback control systems. In closed loop control systems the control action is dependent on the desired output. If any system having one or more feedback paths forming a closed loop system. In closed loop systems the output is compared with the reference input and error signal is produced. The error signal is fed to the controller to reduce the error and desired output is obtained.

Example : In a room to regulate the temperature & humidity for comparatable living. Air conditions are provided with thermostate. By measuring the actual room temperature & compared it with desired temperature, an error signal is product the thermostat turns ON the compressor or OFF the compressor.



Advantages :

(i) These systems are more reliable.

(ii) Closed loop systems are faster.

(iii) A number of variables can be handled simultaneously.

(iv) Optimization is possible.

Disadvantages :

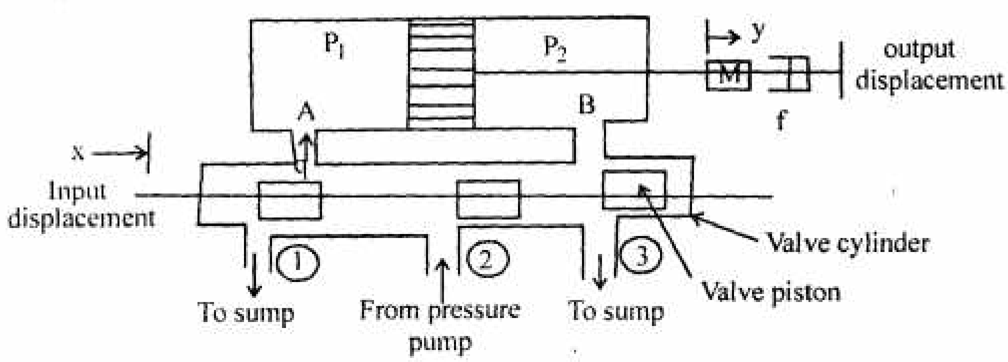
(i) Closed loop systems are expensive.

(ii) Maintenance difficult.

(iii) Complicated installation.

Q. 1. (b) Explain the operation of hydraulic controller.

Ans. Hydraulic Controller :



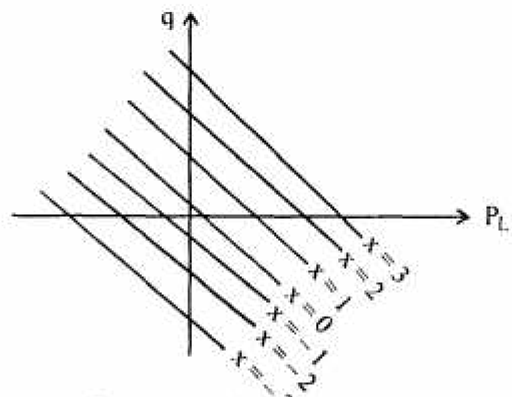
Hydroulic Actuator

Control large displacement power at the shaft of main piston. The input power requirement is very small and provided by the displacement of the valve piston. The linear motion of the valve piston controls the flow of oil to either side of the main piston.

The pressure P_1 on the left side of the main piston is higher than the pressure P_2 on the right side of the piston. The pressure difference $(P_1 - P_2)$ causes the main piston to more from left to right and the resulting displacement of the mass attached to main piston in is y.

The mathematical relation between the input displacement x and the output displacement y gives the model of the system.

The linearized performance characteristics of hydraulic actuator relating the oil from $q(m^3/s)$ and the pressure difference $(P_1 - P_2) = P_L(N/m^2)$ acting on the main piston as a function of valve displacement x.



Linearised performance characteristics of hydraulic actuator

The oil flow q is a function of valve displacement x and pressure difference P_L and can be expressed,

$$q = f(x, P_L) \quad ...(i)$$

Performing partial differentiation on relation (i),

$$dq = \frac{\partial q}{\partial x}dx + \frac{\partial q}{\partial P_L}dP_L \quad ...(ii)$$

Measuring x, P_L and q from initial zero values and assuming partial derivatives as constant,

$$q = \left(\frac{\partial q}{\partial x}\right)x + \left(\frac{\partial q}{\partial P_L}\right)P_L \quad ...(iii)$$

In equation (iii),

Put
$$K_1 = \left(\frac{\partial q}{\partial x}\right)$$

$$K_2 = \left(-\frac{\partial q}{\partial P_L}\right)$$

$$q = K_1 x - K_2 P_L \quad ...(iv)$$

If main piston displacement is dy in time dt and assuming oil,

$$q = A\frac{dy}{dt} \quad ...(v)$$

Where A is the area of piston.
Substituting equation (v)

$$A\frac{dy}{dt} = Kx_1 - K_2P_L \quad ...(vi)$$

Equation. (vi) following equation in obtained.

$$P_L = \frac{1}{K}\left(K_1x_1 - A\frac{dy}{dt}\right) \quad ...(vii)$$

The force developed by the piston in AP_L. Therefore multiplying equation (viii) by A

$$F = \frac{A}{K_2}\left(K_1x - A\frac{dy}{dt}\right) \quad ...(ix)$$

The force F is applied to load having a mass M and coefficient of viscous friction at the load is f,

$$F = M\frac{d^2y}{dt^2} + f\frac{dy}{dt} \quad ...(x)$$

Equations (x) & (ix),

$$\frac{d^2y}{dt^2} + f\frac{dy}{dt} = \frac{A}{K_2}\left(K_1x - A\frac{dy}{dt}\right) \quad ...(xi)$$

Assuming initial conditions as zero, taking Laplace transform on both sides of equation (xi),

$$Ms^2y(s) + f(s)y(s) = \frac{A}{K_2}\left[K_1X(s) - A_sY(s)\right] \quad ...(xii)$$

From equation (xii), the transfer function of hydraulic actuator relating Y(s) and X(s) is obtained,

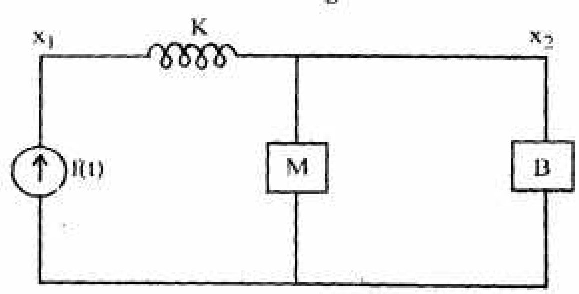
$$\frac{Y(s)}{X(s)} = \frac{AK_1/K_2}{S\left[M_s + \left(f + A^2\right)\right]} \quad ...(xiii)$$

Or
$$\frac{Y(s)}{X(s)} = \frac{K}{s(1+sT)}$$

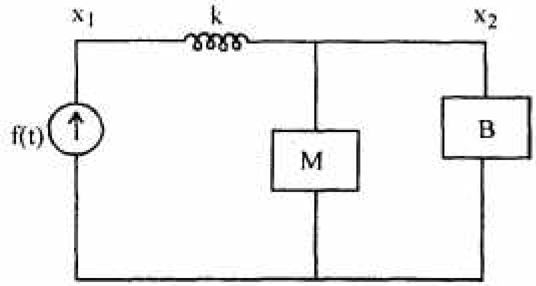
$$K = \frac{AK_1}{K_2\left(f + A^2\right)}$$

$$T = \frac{M}{\left(f + A^2\right)}.$$

Q. 2. (a) Derive the transfer function as shown in fig.



Ans.



At node 1, $$f(t) = K(x_1 - x_2) + M_1 \frac{d^2x_1}{dt^2} + B\frac{d(x_1 - x_2)}{dt} \quad ...(i)$$

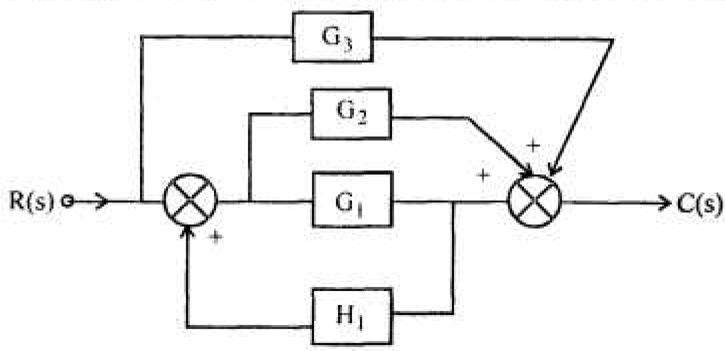
At node 2, $$(x_2 - x_1)K + M\frac{dx_2^2}{dt^2} + B\frac{d}{dt}(x_2 - x_1) = 0 \quad ...(ii)$$

Assuming initial conditions as zero, taking laplace transform on both sides equation (i) & (ii),

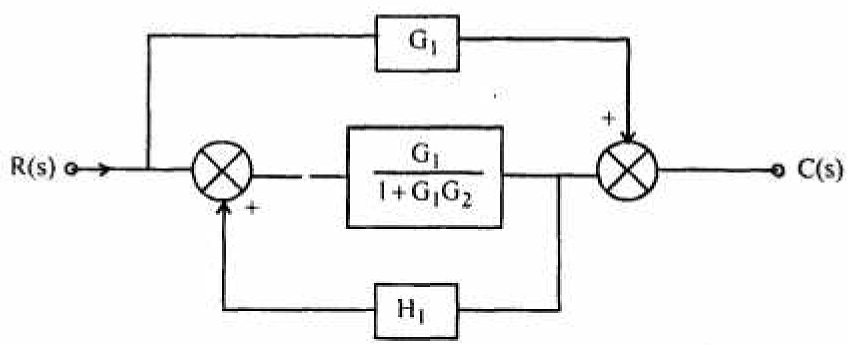
$$F(s) = Ms^2X_1(s) + K[X_1(s) - X_2(s)] + B[X_1(s) - X_2(s)] \quad ...(iii)$$

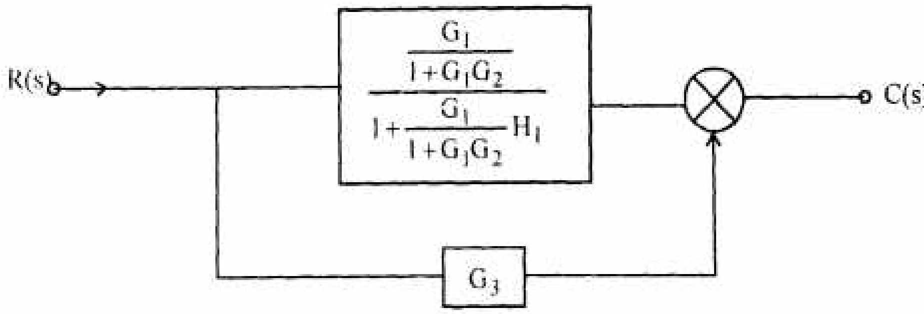
$$K[X_2(s) - X_1(s)] + Ms^2X_2(s) + B[X_2(s) - X_1(s)] = 0 \quad ...(iv)$$

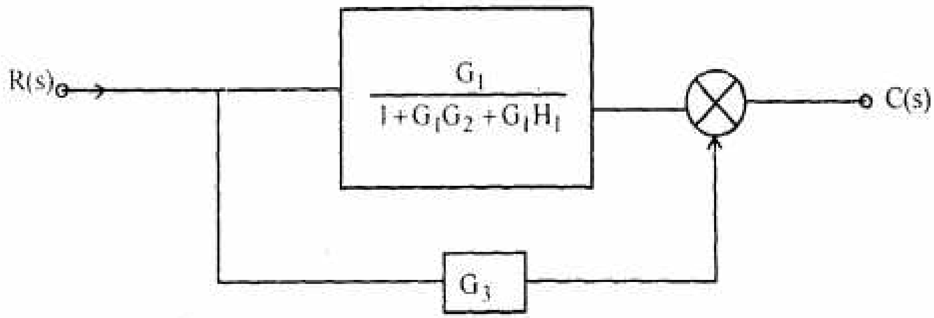
Q. 2. (b) Using Block diagram reduction techniques, calculate the transfer function as shown in fig.

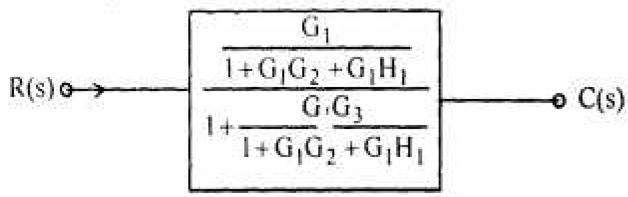


Ans.







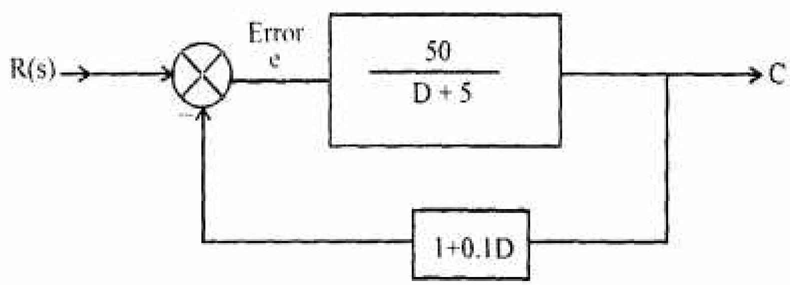


$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1}{1+G_1G_2+G_1H_1+G_1G_3}}$$

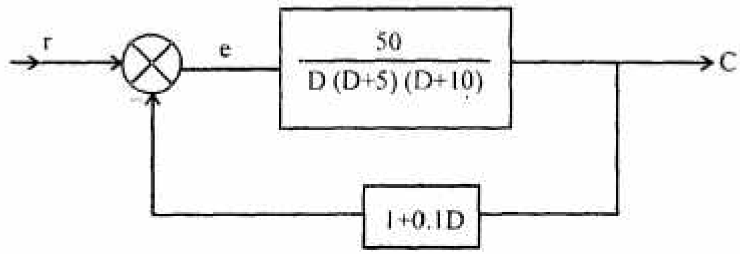
Q. 3. For a system with block diagram as in fig., find steady state error due to :

(i) Unit step reference input

(ii) Unit ramp reference input



Ans.



$$\frac{C(s)}{R(s)} = \frac{\dfrac{50}{D+(D+5)(D+10)}}{1+\dfrac{50(1+0.1D)}{D(D+5)(D+10)}}$$

$$= \frac{50}{D(D+5)(D+10)+50(1+0.1D)}$$

$$= \frac{50}{(D^2+5D)(D+10)+50+5D}$$

$$= \frac{50}{D^3+10D^2+5D^2+50D+50+5D}$$

$$= \frac{50}{D^3+15D^2+55D+50}$$

$$K_P = \lim_{D\to 0} C(s)R(s) = \lim_{D\to 0} \frac{50}{D^3+15D^2+55D+50}$$

$$\boxed{K_P = \infty}$$

$$K_v = \lim_{s\to 0} DC(s)R(s) = \lim_{s\to 0} D.\frac{50}{D^3+15D^2+55D+50}$$

$$\boxed{K_v = \infty}.$$

Q. 4. Plot the root locus of the system having the loop transfer function

$$G(s)H(s) = \frac{K}{s(s+2)(s^2+4s+8)}.$$

Also calculate the value of K.

Ans. $$G(s)H(s) = \frac{K}{s(s+2)(s^2+4s+8)}$$

Step 1 : Plot the poles and zeros.

Poles are at $$S_1 = 0,\ S_2 = -2,\ S_3;\ S_4 = \frac{-4 \pm \sqrt{16 - 4 \times 8}}{2}$$

$$= \frac{-4 \pm \sqrt{16 - 32}}{2}$$

$$= \frac{-4 \pm 4j}{2}$$

$$= -2 \pm 2j$$

$$S_1 = 0,\ S_2 = -2,\ S_3 = -2 + 2j,\ S_4 = -2 - 2j$$

Step 2 : The segment on the real axis between S = 0 & S = –2 is the part of the root locus.

Step 3 : Number of root locii

Number of poles $P = 2$

Number of zero $Z = 0$

Number of root locii $N = P = 2$

Step 4 : Centroid of the asymptotes.

$$\sigma_A = \frac{\text{Sum of poles} - \text{Sum of zeros}}{P - Z}$$

$$= \frac{0 - 2 - 2 + 2j - 2 - 2j - 0}{2 - 0} = \frac{-6}{2} = -3$$

Step 5 : Angle of asymptotes.

$$\phi = \frac{2k + 1}{p - z} 180^\circ$$

K = 0

$$\phi_1 = \frac{2 \times 0 + 1}{2} 180^\circ = 90^\circ$$

K = 1

$$\phi_2 = \frac{2 \times 1 + 1}{2} \times 180^\circ = 270^\circ$$

K = 2

$$\phi_3 = \frac{2 \times 2 + 1}{2} \times 180^\circ = 450^\circ$$

K = 3

$$\phi_4 = \frac{2 \times 3 + 1}{2} \times 180^\circ = 630^\circ$$

Step 6 : Breakaway Point.

Thee characteristic equation $1 + G(s)H(s) = 0$

$$1+\frac{K}{s(s+2)\left(s^2+4s+8\right)}=0$$

$$K+\frac{1}{\left(s^2+2s\right)\left(s^2+4s+8\right)}=0$$

$$K+\frac{1}{s^4+4s^3+8s^2+2s^3+8s^2+16s}=0$$

$$K+\frac{1}{s^4+6a^3+16s^2+16s}=0$$

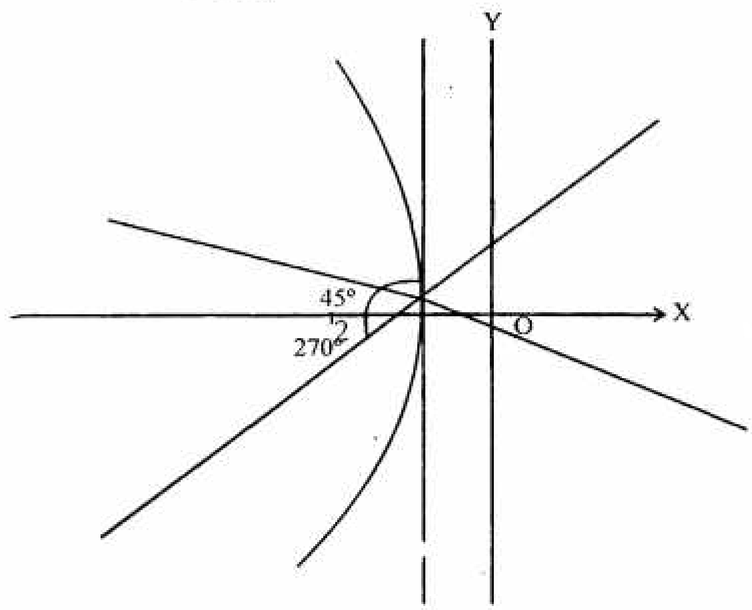
$$K+s^4+6s^3+16s^2+16s=0$$

$$K=-\left(s^4+6s^3+16s^2+16s\right)$$

$$\frac{dk}{ds}=-\left(4s^3+18s^2+32s+16\right)=0$$

By trial & error method.

$$s=0.21j$$



Step 7 : Determination of jw cross-own (by Routh Hurwitz)

$$s^4 + 6s^3 + 16x^2 + 16s + k = 0$$

s^4	1	16	k
s^3	6	16	

Q. 5. For a certain control system

$$G(s)H(s) = \frac{K}{s(s+2)(s+10)},$$

Sketch the Nyquist plot and hence calculate the range of value of K for stability.

Ans. $$G(s)H(s) = \frac{K}{s(s+2)(s+10)}$$

Put $s = j\omega$

$$G(j\omega) = \frac{K}{j\omega(2+j\omega)(10+j\omega)} \quad \text{...(i)}$$

$$G(j\omega) = \frac{-12k\omega^2}{\omega^2(4+\omega^2)(100+\omega^2)} - j\frac{k(20\omega - \omega^3)}{\omega^2(4+\omega^2)} \quad \text{...(ii)}$$

To get the point of intersection in real axis, equate the imaginary part to zero.

$$\frac{K(20\omega - \omega^3)}{\omega^2(4+\omega^2)(100+\omega^2)} = 0$$

$$\omega = 4.47 \text{rad / sec.}$$

$$|G(j\omega)|_{\omega=4.47} = -0.0041K^*$$

Put $$\omega = 4.47 \text{ rad / sec.}$$

$$0.0047K = 0.501$$

$$a = 0.501$$

$$\boxed{K = 1220}$$

(ii) $$PM = \angle 4 - (j\omega) - H(j\omega) + 180°$$

$$45° = -90° - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{10} + 185$$

$$\tan^{-1}\frac{\omega}{2} + \tan^{-1}\frac{\omega}{10} = 45$$

Taking tangent on both sides,

$$\frac{\frac{\omega}{2}+\frac{\omega}{10}}{1-\frac{\omega}{2}\frac{\omega}{10}}=1$$

$$\omega^2+12\omega-20=0$$

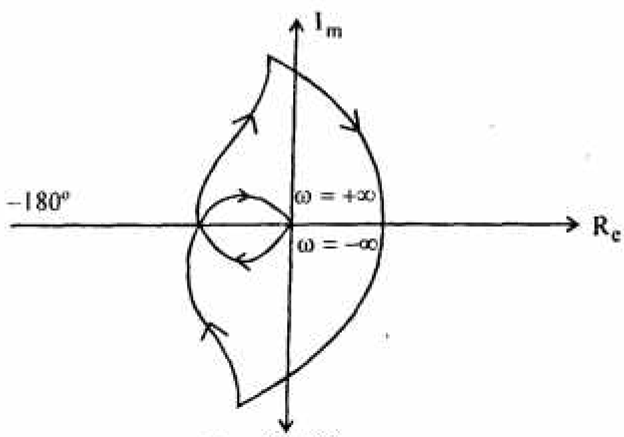
$$\omega=-13.48,\ 1.48$$

Consider the positive value of ω i.e., $\omega = 1.48\ \text{rad/sec}$.

$$|G(j\omega)|=1$$

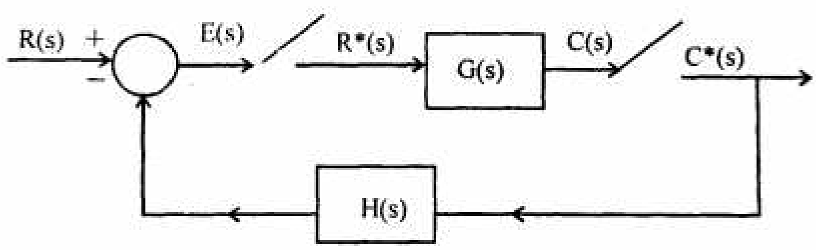
$$\frac{K}{j\omega(2+j\omega)(10+j\omega)}=1$$

$$\frac{K}{j1.48(2+j1.48)(10+j1.48)}=1$$



Q. 6. For the system as shown in fig. find the expression for C(Z).

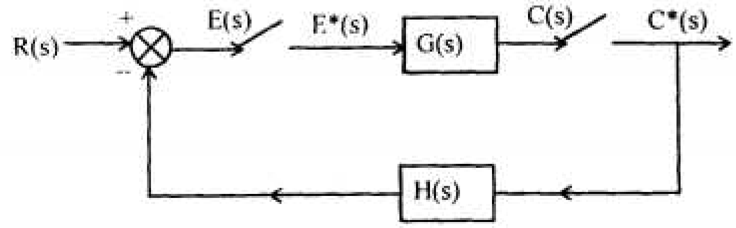
For $G(s)=\frac{1}{s(s+2)}$ and $H(s)=\frac{1}{(s+1)}$, determine the output (CnT) for n = 0, 1, 2, 3, 4. Take sampling time T = 0.5 sec. Check the system for stability using Routh's criterion.



Ans.

$$G(s) = \frac{1}{s(s+2)}$$

$$H(s) = \frac{1}{(s+1)}$$



$$C(s) = G(s)E^*(s) \quad \text{...(i)}$$

$$E(s) = R(s) - C^*(s)H(s) \quad \text{...(ii)}$$

Equation (i),

$$C^*(s) = [G(s)E^*(s)] = G^*(s)E^*(s)$$

$$C^*(s) = G^*(s)E^*(s) \quad \text{...(iii)}$$

Substituting for $C^*(s)$ in equation (ii) from (iii),

$$E(s) = R(s) - G^*(s)E^*(s)H(s) \quad \text{...(iv)}$$

Equation (iv),

$$E^*(s) = [R(s) - G^*(s)E^*(s)H(s)]^*$$

$$= R^*(s) - G^*(s)E^*(s)H^*(s)$$

$$E^*(s)[1 + G^*(s)H^*(s)] = R^*(s)$$

$$E^*(s) = \frac{R^*(s)}{1 + G^*(s)H^*(s)} \quad \text{...(v)}$$

Substituting equation (v) in (iv),

$$C^*(s) = G^*(s).\frac{R^*(s)}{1 + G^*(s)H^*(s)}$$

$$\frac{C^*(s)}{R^*(s)} = \frac{G^*(s)}{1 + G^*(s)H^*(s)} \quad \text{...(vi)}$$

In terms of z-transfotm relation (vi) can be expressed as the required pulse transfer function.

$$\frac{C(z)}{R(z)} = \frac{G(z)}{1 + G(z)H(z)}$$

$$R(z) \longrightarrow \boxed{\frac{G(z)}{1+G(z)H(z)}} \longrightarrow C(z)$$

$$F(z) = \frac{\frac{z}{z-1}\cdot\frac{z}{z-e^{-2T}}}{1+\frac{z}{z-1}\,\frac{z}{z-e^{-2T}}} = \frac{\frac{1}{s(s+2)}}{1+\frac{1}{s(s+2)}\,\frac{1}{(s+1)}} = \frac{\frac{1}{s(s+2)}}{\frac{s(s+2)(s+1)+1}{s(s+2)(s+1)}}$$

$$= \frac{z^2}{(z-1)\left(z-e^{-2T}\right)+z^2}$$

$$= \frac{z^2}{z^2 - ze^{-2T} - z + e^{-2T} + z^2}$$

$$= \frac{z^2}{2z^2 - z\left(1+e^{-2T}\right)+e^{-2T}}$$

z^2	2	e^{-2T}
z^1	$-\left(1+e^{-2T}\right)$	0
z^0	e^{-2T}	0

$$-\left(1+e^{-2T}\right)>0$$

$$-\left(1+e^{-2\times 0.5}\right)>0$$

$$-\left(1+e^{-1}\right)>0$$

$$=0$$

$$C\,(nT)=0.$$

Q. 7. Obtain the state-space representation of

$$\frac{y(s)}{u(s)} = \frac{12(1-s)}{(s+2)(s+5)}$$

Also, find expression for output y (t) for a unit step u (t). Take initial conditions as zero.
Ans.

$$\frac{y(s)}{u(s)} = \frac{12(1-s)}{(s+2)(s+5)}$$

Let $$G_1(s) = \frac{12}{s+2}$$

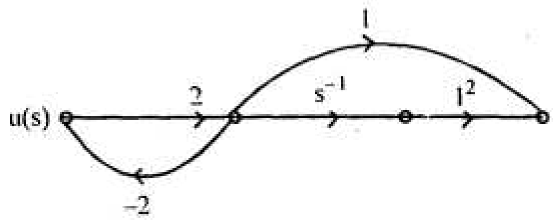
& $$G_2 = \frac{1-s}{s+5}$$

$$\frac{y(s)}{u(s)} = 12(s+2)^{-1}Q(s)$$

$$y(s) = 12$$

$$u(s) = (s+2)^{-1}Q(s)$$

$$Q(s) = 2u(s) - s^{-1}Q(s)$$

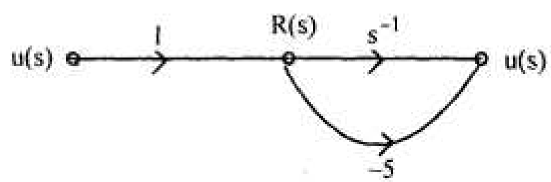


For G_2

$$\frac{y(s)}{u(s)} = \frac{1-s}{s+5} = (1-s) - 1/\left(1+5s^{-1}\right)\frac{Q(s)}{Q(s)}$$

$$y(s) = (1-s)^{-1}Q(s)$$

$$u(s) = \left(1+5s^{-1}\right)Q(s)$$



$$Q(s) = u(s) - 5s^{-1}Q(s)$$

$$\dot{x}_1 = -5x_1 + \dot{x}_2 + 12x_2 \quad ...(i)$$

$$\dot{x}_2 = -2x_2 + u \quad ...(ii)$$

From equation (i) & (ii), put the value x_2.

$$\dot{x}_1 = -5x_1 - 2x_2 + u$$

$$\dot{x}_2 = -2x_2 + u$$

I

$$\begin{bmatrix} \dot{x}_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -5 & -2 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

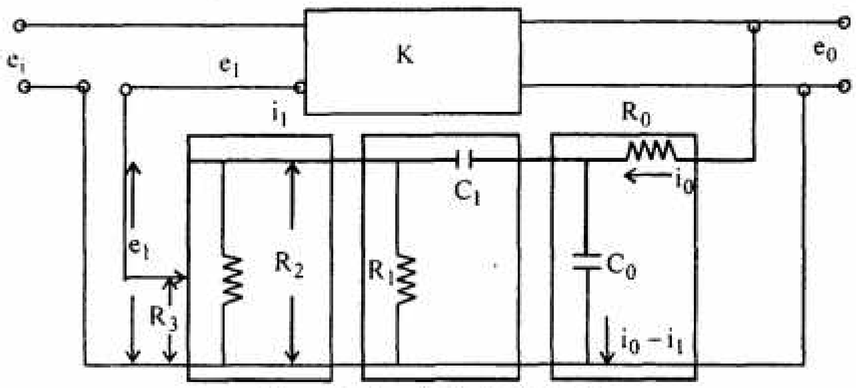
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Q. 8. Discuss the following :

(i) PID controllers

(ii) Signal flow graphs.

Ans. (i) PID Controllers :



Proper Integral Derivatutional

$$e_0 = R_0 i_0 + \frac{1}{C_0}\int (i_0 - i_1)\,dt \qquad \text{...(i)}$$

Laplace transform,

$$E_0(s) = R_0 I_0(s) + \frac{1}{SC_0}\left[I_0(s) - I_1(s)\right]$$

$$E_0(s) = I_0(s)\left[R_0 + \frac{1}{SC_0}\right] - \frac{1}{SC_0} I_1(s) \qquad \text{...(ii)}$$

$$R_1 i_1 + \frac{1}{C_1}\int i_1\,dt - \frac{1}{C_0}\int (i_0 - i_1)\,dt = 0 \qquad \text{...(iii)}$$

Laplace transform of equation (iii),

$$R_1 I_1(s) + \frac{1}{SC_1} I_1(s) - \frac{1}{C_0}\left[I_0(s) - I_1(s)\right] = 0 \qquad \text{...(iv)}$$

Simplify equation (iv),

$$I_0(s) = I_1(s)\left[\frac{R_1 C_1 C_0 S^2 + SC_0 + SC_1}{SC_1}\right] \qquad \text{...(v)}$$

Put the value of $I_0(s)$ from equations (v) in (ii),

$$E_0(s) = I_1(s)\left[\frac{R_1C_1R_0C_0S^2 + R_0C_0S + R_0C_1S + R_1GS + 1}{SC1}\right] \quad ...(vi)$$

Also $\quad e_1 = R_1 i_1 \quad ...(vii)$

Laplace transform of (vii),

$$E_1(s) = R_1 I_1(s) \quad ...(viii)$$

From equations (vi) & (vii),

$$\frac{E_1(s)}{E_0(s)} = \frac{R_1C_1S}{R_1C_1R_0C_0S^2 + (R_0C_0 + R_0C_1 + R_1C_1)S + 1} \quad ...(viii)$$

$$e_0 = k(e_i - e_2) \quad ...(viii)$$

Laplace transform of equation (viii),

$$E_0(s) = K[E_i(s) - E_2(s)] \quad ...(ix)$$

$$e_2 = e_1\left(\frac{R_3}{R_2}\right) \quad ...(x)$$

Laplace transform of equation (x),

$$E_2(s) = E_1(s).\frac{R_3}{R_2} \quad ...(xi)$$

Put the value of $E_2(s)$ from equations (xi) to (ix)

$$E_0(s) = \left[E_i(s) - E_i(s)\frac{R_3}{R_2}\right]k \quad ...(xii)$$

From equations (vii) and (xii), put the value of $E_1(s)$ from equations (vii) in (xii) and solve for $\frac{E_0(s)}{E_i(s)}$.

$$\frac{E_0(s)}{E_i(s)} = \frac{K\left[R_1R_2C_1R_0C_0S^2 + R_2(R_0C_0 + R_0C_1 + R_1C_1)S + R_2\right]}{R_1R_2C_1R_0C_0^2 + R_2(R_0C_0 + R_0C_1 + R_1C_1)S + R_2 + KR_1R_3C_1S} \quad ...(xiii)$$

If K>> 1

$$\frac{E_0(i)}{E_i(s)} = R_1\left[ST_0 + \left(1 + \frac{R_0}{R_1} + \frac{T_0}{T_1}\right) + \frac{1}{5T_1}\right] \quad ...(xiv)$$

Where, $\quad R_1 = \frac{R_2}{3}$

$$T_0 = R_0 C_0$$

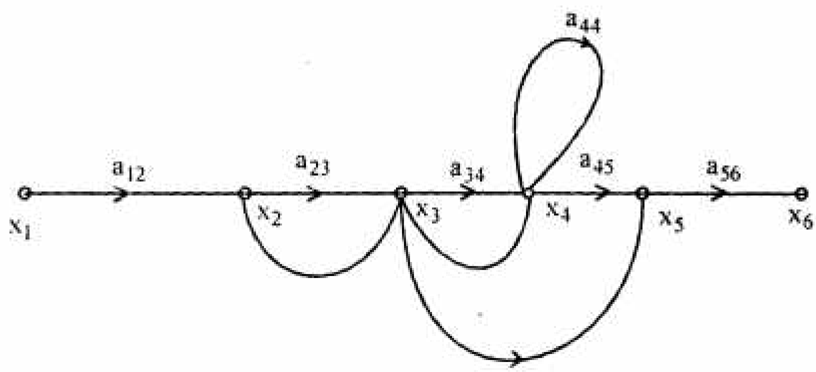
$$T_1 = R_1 C_1$$

$$\frac{E_0(s)}{E_i(s)} = K_1 \propto \left[1 + \frac{T}{\alpha} S + \frac{1}{\alpha T_1 S}\right] \quad \text{...(xv)}$$

Where, $\alpha = 1 + \frac{R_0}{R_1} + \frac{T_0}{T_1}$

(ii) Signal Flow Graphs (SFG) :

The process of block diagram reduction technique is time consuming because at every stage modified block diagram is to be redrawn. A simple method was developed by S.J. Mason which is known as signal flow graph.



Construction of Signal Flow Graph From Equations :

Consider the following sets of equation,

$$y_2 = t_{21}y_1 + t_{23}y_3$$

$$y_3 = t_{32}y_2 + t_{33}y_3 + t_{31}y_1$$

$$y_4 = t_{43}y_3 + t_{42}y_2$$

$$y_5 = t_{54}y_4$$

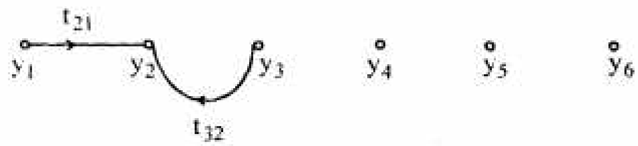
$$y_6 = t_{65}y_5 + t_{64}y_4$$

Where y_1 is the input and y_0 output in the output.

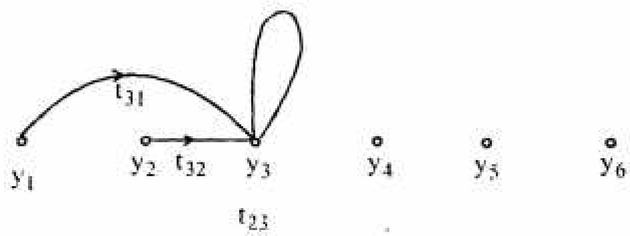
Step 1 : Draw the nodes,

0	0	0	0	0	0
y_1	y_2	y_3	y_4	y_5	y_6

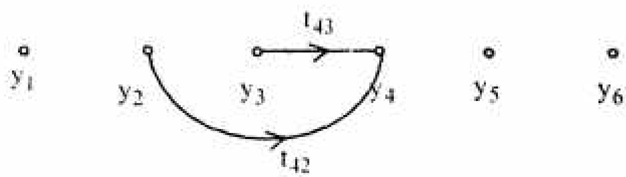
Step 2 : Draw the SFG for equation (i),



Step 3 : Draw the SFG for equation (ii),

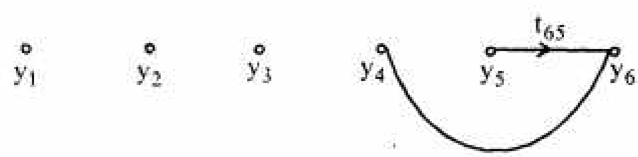


Step 4 : Draw SFG for equation (iii),



Step 5 : Draw SFG equation (iv),

Step 6 : Draw SFG for equation (v),



Step 7 : Draw the complete signal flow graph with the help of above graph.