B.E.

Sixth Semester Examination, December-2008

Automatic Controls (ME-308-E)

Note : Attempt any **five** questions. All questions carry equal marks.

Q. 1. Draw root loci for a system with GH(s) = K/[s (s + 2)(s + 3)] and find its intersect on the imaginary axis. Also find the value of K for which this system will be unstable.

Ans. Given $$G(s)H(s) = \frac{K}{s(s+2)(s+3)}$$

Step I : $P = 3$, $Z = 0$, as $P > Z$ so, $N = P = 3$ branches

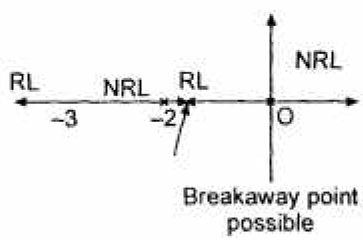
$P - Z = 3$ branches approaching to ∞

Starting points $= 0, -2, -3$ .... open loop poles

Terminating points $= \infty, \infty, \infty$ ... no open loop zeros so ∞

Step II : Section of real axis

One breakaway point between 0 and -2 exists.



Step III : Angle of asymptotes

$$\theta = \frac{(2q+1)180°}{P-Z} \qquad q = 0, 1, 2$$

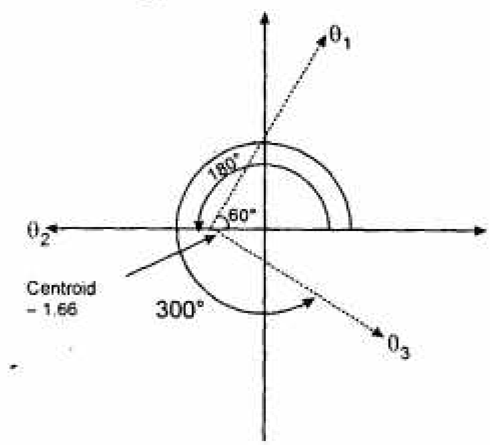
$$\theta_1 = \frac{180°}{3} = 60°, \qquad \theta_2 = \frac{3 \times 180°}{3} = 180°, \qquad \theta_3 = \frac{5 \times 180°}{3} = 300°$$

Step IV : Centroid

$$\sigma = \frac{\Sigma \text{ R. P. of Poles} - \Sigma \text{ R. P. of Zeros}}{P-Z}$$

$$= \frac{0-2-3-0}{3}$$

$$= \frac{-5}{3} = -1.66$$



Step V : Breakaway point

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K}{s(s+2)(s+3)} = 0$$

$$\therefore \quad s^3 + 5s^2 + 6s + K = 0$$

$$K = -s^3 - 5s^2 - 6s \qquad \dots(1)$$

$$\frac{dK}{ds} = -3s^2 - 10s - 6 = 0$$

$$\therefore \quad 3s^2 + 10s + 6 = 0$$

$$s = \frac{-10 \pm \sqrt{100-72}}{6}$$

$$= \frac{-10 \pm \sqrt{28}}{6} = \frac{-10 \pm 5.29}{6} = \mathbf{-7.54, -0.785}$$

Hence the breakaway point is $\mathbf{s = -0.785}$

Substituting in equation (1)

$$K = -(-0.785)^3 - 5(-0.785)^2 - 6(-0.785)$$

$$K = -0.483 - 3.08 + 4.71$$

$$K = 1.147 \text{ for } s = -0.785$$

As K is positive, $s = -0.785$ is valid breakaway point.

Step VI : Intersection with negative real axis.

$$s^3 + 5s^2 + 6s + K = 0$$

$$\begin{array}{c|cc} s^3 & 1 & 6 \\ s^2 & 5 & K \\ s^1 & \frac{30-K}{5} & 0 \\ s^0 & K & \end{array}$$

$$30 - K = 0$$

$$\boxed{K_{mar} = 30}$$

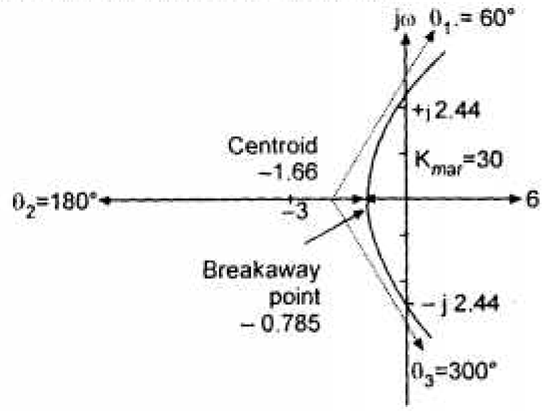
$$A(s) = 5s^2 + K = 0$$

$$5s^2 + 30 = 0$$

$$s^2 = \frac{-30}{5} = -6$$

$s = \pm j\sqrt{6} = \pm j\,2.44$ ... Intersection with imaginary axis.

Step VII : The complete root locus is shown in below fig.



Step VIII : Comment on stability

For $0 < K < 30$, system is stable as entire root locus is in the left half of s-plane.

For $30\ K < \infty$, the system is unstable.

Q. 2. Prove that the polar plots of the sinusoidal transfer function G(jω) = jωT/(1 + jωT) for $0 \le \omega \le \infty$ is a semicircle. Find the corner and radius of the circle.

Ans. Given $$G(j\omega) = \frac{j\omega T}{(1 + j\omega T)}$$

Here $$H(j\omega) = 1$$

So, $$|G(j\omega)| = M = \frac{\omega T}{\sqrt{1 + \omega^2 T^2}}$$

$$\angle G(j\omega) = \phi = \frac{\tan^{-1}\left(\frac{\omega T}{0}\right)}{\tan^{-1}(\omega T)} = 90° - \tan^{-1}(\omega T)$$

Starting Point :

$\omega \to 0$ $\qquad\qquad$ $\phi \to 90°$

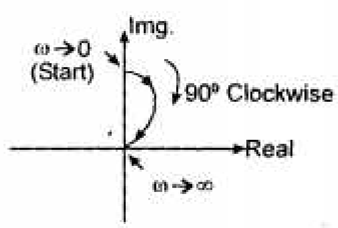
Terminating Point :

$\omega \to \infty$ $\qquad\qquad$ $\phi \to 0°$

Rotation of Plot :

$= 0 - 90° = -90°$ (clockwise)

The corresponding polar plot is :



Q. 3. Determine the range of K for stability for the characteristic equation :

$s^4 + K s^3 + s^2 + s + 1 = 0$; using Routh's criterion.

Ans. Given, $s^4 + Ks^3 + s^2 + s + 1 = 0$

$$F(s) = s^4 + Ks^3 + s^2 + s + 1 = 0$$

$$\begin{array}{c|ccc} s^4 & 1 & 1 & 1 \\ s^3 & K & 1 & 0 \\ s^2 & \frac{K-1}{K} & 1 & \\ s^1 & \frac{\frac{K-1}{K} - K}{\frac{K-1}{K}} & 0 & \\ s^0 & 1 & & \end{array}$$

For the system to be stable there should not be sign change in the first column.

$$\frac{K-1}{K} > 0 \quad \text{from} \quad s^2$$

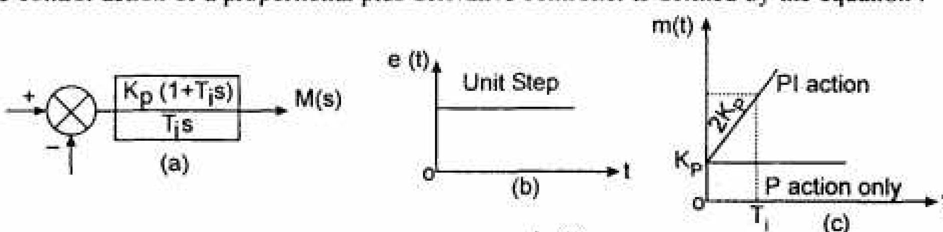
$$K - 1 > 0 \quad \Rightarrow \boxed{K > 1}$$

Q. 4. Explain Pneumatic Proportional – plus – Derivative Controller and obtain its transfer function.

Ans. Proportional Plus – Derivative Control Action

(PD Action) – Pneumatic Controllers :

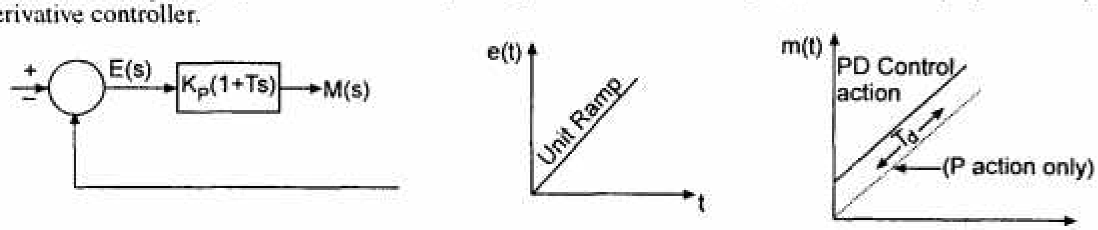
The control action of a proportional plus derivative controller is defined by the equation :



$$m(t) = K_p\ e(t) + K_p\ T_d\ \frac{d_e(t)}{dt}$$

$$\frac{M(s)}{E(s)} = K_p\ (1 + T_d\ s)$$

Where K_p is proportional sensitivity & T_d derivative time. Fig. (b) shows the proportional plus derivative controller.



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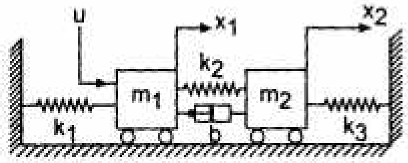
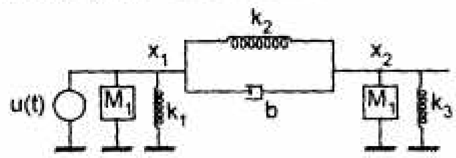
Q. 5. Obtain the transfer functions X_1 (s)/U (s) and X_2 (s)/U (s) of the mechanical system shown in figure.



Ans. The equivalent mechanical system will be as shown :



The equation of equilibrium are

$$u(t) = M_1 \frac{d^2 x_1(t)}{dt^2} + k_1 x_1(t) + k_2 [x_1(t) - x_2(t)] + b\frac{d}{dt}[x_1(t) - x_2(t)] \quad ..(i)$$

$$0 = M_2 \frac{d^2 x_2(t)}{dt^2} + k_3 [x_2(t)] + b\frac{d}{dt}[x_2(t) - x_1(t)] + k_2 [x_2(t) - x_1(t)] \quad ...(ii)$$

Using F-V analogy

$M \to L,\quad b \to R,\quad k \to \frac{1}{C},\quad x \to q,\quad \frac{dx}{dt} \to i$

$\frac{d^2x}{dt^2} \to \frac{di}{dt},\quad x \to q \to \int i dt$

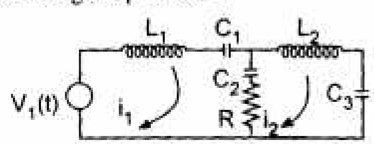
Hence, the replacements,

$$v(t) = L_1 \frac{di_1}{dt} + \frac{1}{C_1}\int i_1 dt + \frac{1}{C_2}\int (i_1 - i_2)\, dt + R(i_1 - i_2)$$

&

$$0 = L_2 \frac{di_2}{dt} + \frac{1}{C_3}\int i_2 dt + R(i_2 - i_1) + \frac{1}{C_2}\int (i_2 - i_1)\, dt$$

Simulating the above equations using loop basis ⇒



Taking the Laplace transformation of equation (i) & equation (ii), we get

$$u(s) = M_1 s^2 x_1(s) + k_1 x_1(s) + k_2 x_1(s) - k_2 x_2(s) + bs\, x_1(s) - bs\, x_2(s) \quad ...(iii)$$

&

$$0 = M_2 s^2 x_2(s) + k_3 x_2(s) + bs\, x_2(s) - bs\, x_1(s) + k_2 x_2(s) - k_2 x_1(s) \quad ...(iv)$$

From equation (iv)

$$x_2(s) = \frac{(k_2 + bs)\, x_1(s)}{M_2 s^2 + k_3 + bs + k_2}$$

Putting value of $x_2(s)$ in equation (iii), we get

$$u(s) = x_1(s)[M_1 s^2 + k_1 + bs + k_2] + x_2(s)[-k_2 - bs]$$

$$= x_1(s)[M_1 s^2 + k_1 + k_2 + bs] - \frac{[k_2 + bs][bs + k_2]x_1(s)}{M_2 s^2 + k_3 + k_2 + bs}$$

$$u(s) = x_1(s)\left[M_1 s^2 + k_1 + k_2 + bs - \frac{(k_2 + bs)^2}{M_2 s^2 + k_3 + k_2 + bs}\right]$$

So,

$$\boxed{\frac{x_1(s)}{u(s)} = \frac{M_2 s^2 + k_2 + k_3 + bs}{(M_2 s^2 + k_2 + k_3 + bs)(M_1 s^2 + k_1 + k_2 + bs) - (k_2 + bs)^2}}$$

$\therefore$ For $\frac{x_2(s)}{u(s)}$

$$x_1(s) = \left[\frac{M_2 s^2 + k_3 + k_2 + bs}{(bs + k_2)}\right]x_2(s)$$

Putting this value in equation (iii), we get

$$u(s) = \left[\frac{M_2 s^2 + k_2 + k_3 + bs}{bs + k_2}\right]x_2(s)[M_1 s^2 + k_1 + k_2 + bs] - x_2(s)[k_2 + bs]$$

$$u(s) = x_2(s)\left[\frac{(M_1 s^2 + k_1 + k_2 + bs)(M_2 s^2 + k_2 + k_3 + bs) - (bs + k_2)^2}{(bs + k_2)}\right]$$

So,

$$\boxed{\frac{x_2(s)}{u(s)} = \frac{bs + k_2}{(M_1 s^2 + k_1 + k_2 + bs)(M_2 s^2 + k_2 + k_3 + bs) - (bs + k_2)^2}}$$

Q. 6. Obtain a state-space equation and output equation for the system defined by :
$Y(s) / U(s) = [2s^3 + s^2 + s + 2] / [s^3 + 4s^2 + 5s + 2]$

Ans. Given $\frac{Y(s)}{U(s)} = \frac{2s^3 + s^2 + s + 2}{s^3 + 4s^2 + 5s + 2}$

As the above expression has same order both numerator & denominator hence, in order to find out the partial fraction we will divide the N^r by D^r.

$$\begin{array}{r|l} & \quad 2 \\ s^3 + 4s^2 + 5s + 2 & 2s^3 + s^2 + s + 2 \\ & 2s^3 + 8s^2 + 10s + 4 \\ \hline & -6s^2 - 8s - 2 \end{array}$$

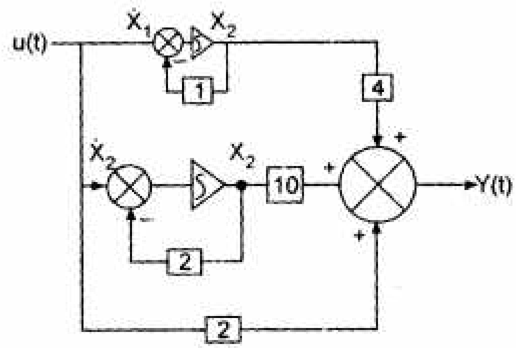
Hence, $\frac{Y(s)}{U(s)} = 2 - \left[\frac{6s^2 + 8s + 2}{s^3 + 4s^2 + 5s + 2}\right]$

$$= 2 - \left[\frac{6s^2 + 8s + 2}{(s+1)^2\,(s+2)}\right]$$

$$= 2 - \left[\frac{A}{(s+1)^2} + \frac{B}{(s+1)} + \frac{C}{(s+2)}\right]$$

$$= 2 - \left[\frac{0}{(s+1)^2} - \frac{4}{(s+1)} + \frac{10}{(s+2)}\right]$$

$$= 2 + \frac{4}{(s+1)} + \frac{10}{(s+2)}$$



The standard state model can be represented as,

$$\dot{X}(t) = A\,X(t) + BU(t)\ \&$$

$$Y(t) = C\,X(t) + D\,U(t)$$

$$\dot{X}_1 = U(t) - X_1(t)$$

$$\dot{X}_2 = U(t) - 2X_2(t)$$

$$Y(t) = 4\,X_1(t) + 10\,X_2(t) + 2$$

Hence, $\dot{X} = AX + BU$

$Y = CX + DU$

Where, $A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$ $\quad C = [4 \;\; 10]$

$B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\quad D = [2]$

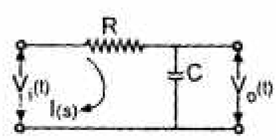
Q. 7. Derive the equations for time response of a first order system subjected to step input. Draw the response curve and find steady state error. Give some example.

Ans. Time Response of a First Order System :

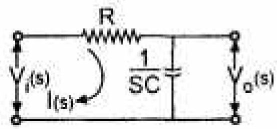
$V_i(t) = 1 \qquad t \geq 0$

$= 0 \qquad t < 0$

$V_i(s) = \frac{1}{s}$



Now first calculate T.F., the laplace network is shown as,



$$V_i(s) = I(s)R + \frac{1}{SC}I(s) \quad \text{...(1)}$$

$$V_o(s) = \frac{1}{SC}I(s) \quad \text{...(2)}$$

$$\therefore \quad \frac{V_o(s)}{V_i(s)} = \frac{1}{1+sRC}$$

Substituting $V_i(s) = \frac{1}{s}$

$$V_o(s) = \frac{1}{s(sRC+1)} = \frac{A'}{s} + \frac{B'}{1+sRC}, \qquad A' = 1 \,\&\, B' = -RC$$

$$V_o(s) = \frac{1}{s} - \frac{RC}{1+sRC}$$

$$= \frac{1}{s} - \frac{1}{s+\frac{1}{RC}}$$

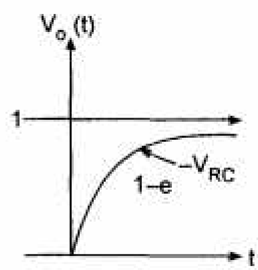
Taking Laplace Inverse,

$$V_o(t) = 1 - e^{-t/RC} \Rightarrow C_{ss} + C_t(t) \text{ form}$$

$$C_{ss} = 1 \,\&\, C_t(t) = e^{-t/RC}$$

The response will be shown as,

t	$V_o(t)$
0	0
RC	0.632
$2RC$	0.860
$3RC$	0.950
$4RC$	0.982
⋮	⋮
∞	1



The response is purely exponential

Now suppose input is changed to step of 'A' units.

Then, $V_i(s) = \frac{A}{s}$

$$V_o(s) = \frac{A}{s(1+sRC)} = \frac{A'}{s} + \frac{B'}{1+sRC}$$

$$A = A'(1+sRC) + sB'$$

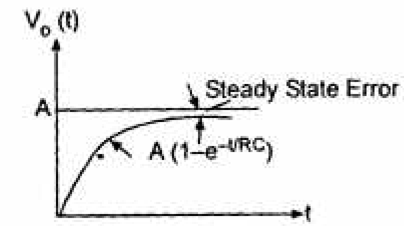
$$A'RC + B' = 0 \quad \& \quad A' = A$$

$$\boxed{B' = -ARC}$$

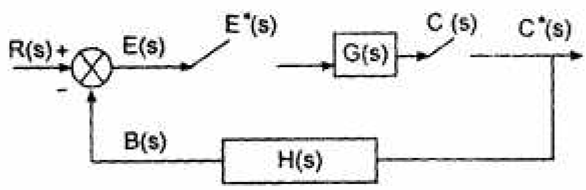
$\therefore$
$$V_o(s) = \frac{A}{s} - \frac{ARC}{HSRC} = \frac{A}{s} - \frac{A}{s+(1/RC)}$$

$$\boxed{V_o(t) = A[1 - e^{-t/RC}]}$$

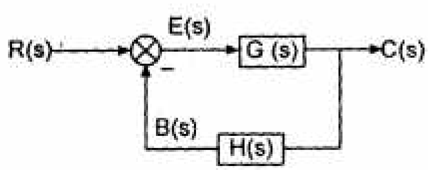
So the rate of decay is not changed but the steady state value has changed. The corresponding response can be shown as in fig. below.



Q. 8. Write expression for C(z) in terms of R(z) for the system as shown in figure.



Ans.



Here, $R(s) \rightarrow$ Laplace of reference input $r(t)$

$C(s) \rightarrow$ Laplace of controlled output $c(t)$

$E(s) \rightarrow$ Laplace of error signal $e(t)$

$B(s) \rightarrow$ Laplace of feedback signal $b(t)$

$G(s) \rightarrow$ Equivalent forward path transfer function

$H(s) \rightarrow$ Equivalent feedback path transfer function

Derivation of T.F. of Simple Closed Loop System : Referring to the above figure, we can write following equations as :

$$E(s) = R(s) - B(s) \quad \ldots\text{(i)}$$

$$B(s) = C(s)\,H(s) \quad \ldots\text{(ii)}$$

$$C(s) = E(s)\,G(s) \quad \ldots\text{(iii)}$$

$B(s) = C(s)\,H(s)$ & substituting in equation (i)

$$E(s) = R(s) - C(s)\,H(s)$$

$$E(s) = \frac{C(s)}{G(s)}$$

So,

$$\frac{C(s)}{G(s)} = R(s) - C(s)\,H(s)$$

$$C(s) = R(s)\,G(s) - C(s)\,G(s)\,H(s)$$

$$C(s)\,[1 - G(s)\,H(s)] = R(s)\,G(s)$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)\,H(s)}}$$

This can be represented as,

$$R(s) \longrightarrow \boxed{\frac{G(s)}{1 - G(s)\,H(s)}} \longrightarrow C(s)$$

Closed loop T.F.