B.E.

Sixth Semester Examination, Dec.-2007

Automatic Controls (ME-308-E)

Note : Attempt any *five* questions.

Q. 1. Draw root loci for a system with $GH(s) = K / [s(s+2)(s+3)]$ and find its intersect on the imaginary axis. Also find the value of K for which this system will be unstable.

Ans. We have, $G(s)H(s) = \dfrac{K}{s(s+2)(s+3)}$

1. The finite poles of open loop transfer function are at $s = 0$, $s = -2$ and $s = -3$.
2. As the system has no zero, so the root loci would terminate at the zeroes located at infinity.
3. The number of root loci, P = 3.
4. The given transfer function is rational, so the root loci is symmetrical about real axis.
5. The angles of asymptotes for K > 0 are :

$$K = 0, \qquad \theta_1 = \frac{\pi}{n-m} = \frac{\pi}{3-0} = \frac{\pi}{3} = 60^\circ$$

$$K = 1, \qquad \theta_2 = \frac{3\pi}{3} = 180^\circ$$

$$K = 2, \qquad \theta_3 = \frac{5\pi}{3} = 300^\circ$$

6. Intercepts of asymptotes at real axis,

$$= \frac{(P_1 + P_2 + P_3 + ...P_n) - (Z_1 + Z_2 + Z_3 + ...-Z_m)}{n-m}$$

$$= \frac{(0-2-3)-0}{3-0} = \frac{-5}{3} = -1.66.$$

7. Root locus on real axis :
   (a) Between $s = 0$ to $s = -2$ and
   (b) Between $s = -3$ to $-\infty$.
8. Intersection with imaginary axis :

The characteristic equation is,

$$s(s+2)(s+3) + K = 0$$

$$\Rightarrow \qquad s^3 + 5s^2 + 6s + K = 0$$

Put $s = jw$, we have

$$-jw^3 - 5w^2 + 6jw + K = 0$$

Equating imaginary parts equal to zero,

$$-jw^3 + 6jw = 0$$

$$\Rightarrow \quad w = \pm\sqrt{6} \approx \pm 2.45$$

Now, equating real parts equal to zero and putting the value of $w = \pm\sqrt{6}$, gives

$$K = 5 \times 6 = 30$$

So the loci intersect the imaginary axis at $\pm\sqrt{6}$ with $K = 30$.

9. Break away point : The characteristic equation is,

$$s^3 + 5s^2 + 6s + K = 0$$

$$\Rightarrow \quad K = -\left(s^3 + 5s^2 + 6s\right)$$

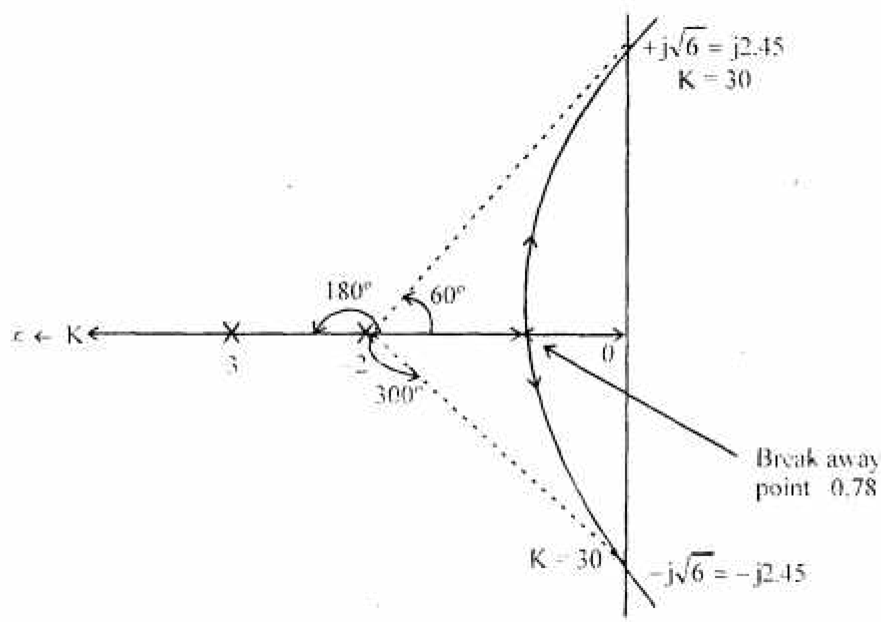
So, $$\frac{dK}{ds} = -\left(3s^2 + 10s + 6\right) = 0$$

Hence, $$s = \frac{-10 \pm \sqrt{100 - 72}}{6}$$

$$= -2.55 \text{ and } -0.78$$

The value -2.55 is not lying on root locus. So the valid value is $= -0.78$.

The root loci are given below :

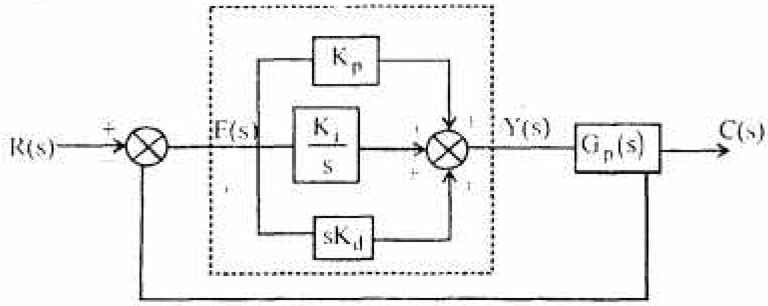


Value of K for unstable system.

Since the root locus intercects the imaginary axis at $K = 30$. So for $K > 30$, the root locus lies in the right half of s-plane. So the system is unstable for $K > 30$.

Q. 2. Derive an expression for transfer function of a PDI type hydraulic controller.

Ans. The block diagram of a PDI type hydrarulic controller is shown below :



In the diagram, the PDI type hydrarulic controller is shown inside the dotted lines.

Let, K_p be the proportional gain of the controller

K_i be the integral gain of the controller

K_d be the derivative gain of the controller

and $G_p(s)$ be the transfer function of the system, whose output is to be controlled by the PDI controller

$E(s)$ be input signal to the controller or actuating signal

$Y(s)$ be the output of the controller

As from the block diagram, the output of the controller is given by (in time domain)

$$Y(t) = K_p e(t) + k_i \int_0^t e(t)dt + K_d \frac{de(t)}{dt}$$

Taking Laplace transformation, we have

$$Y(s) = K_p E(s) + \frac{K_i}{s} E(s) + sK_d E(s)$$

So, the transfer function of the controller is

$$\frac{Y(s)}{E(s)} = K_p + \frac{K_i}{s} + sK_d$$

$$= K_p \left[1 + \frac{K_i}{sK_p} + \frac{sK_d}{K_p} \right]$$

Or
$$\frac{Y(s)}{E(s)} = K_p \left[1 + \frac{1}{sK_p / K_i} + \frac{sK_d}{K_p} \right]$$

If we write
$$\frac{K_p}{K_i} = T_i$$

and
$$\frac{K_d}{K_p} = T_d$$

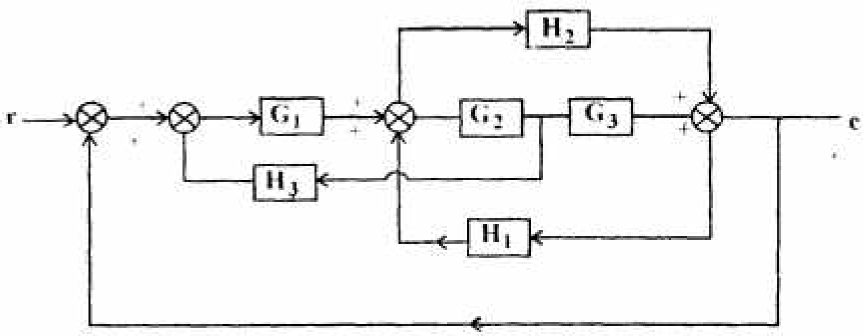
Then the transfer function of a PDI type hydraulic controller is given by

$$\boxed{G_c = K_p\left[1 + \frac{1}{sT_i} + sT_d\right]}$$

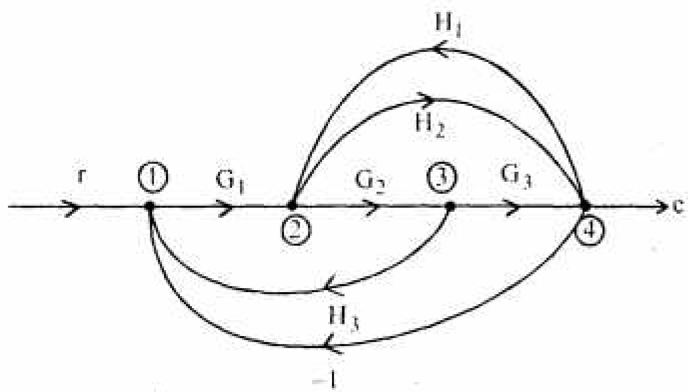
Where, $T_i = \dfrac{K_p}{K_i}$ is called integral time constant of the PDI controller.

and $T_d = \dfrac{K_d}{K_p}$ is called derivative time constant of the PDI controller.

Q. 3. For the block diagram in figure, draw signal flow diagrams and derive expression for c/r, using Mason's formula. Check by block diagram algebra.



Ans. The signal flow graph of the given block diagram is given below :



According to the Mason's formula :

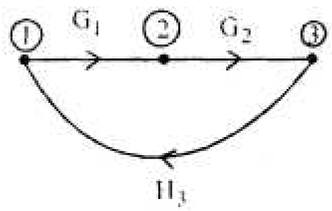
$$\frac{c}{r} = \frac{1}{\Delta}\sum_{K=1}^{P} M_K \Delta_K$$

$$= \frac{1}{\Delta}(M_1\Delta_1 + M_2\Delta_2 + \ldots\ldots\ldots M_P\Delta_P)$$

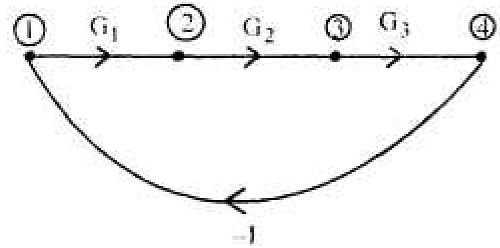
Δ = 1–(Sum of all individual loops) + sum of the products of loop gains of all possible combination of non-touching loops taken two at a time).

There are 5 individual loops :

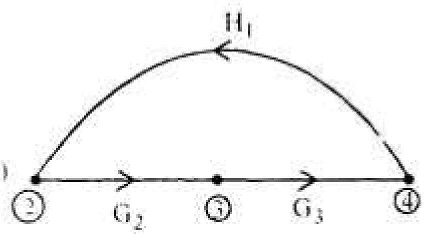
(1)



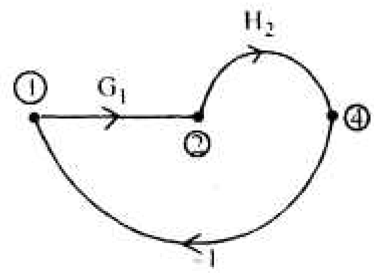
(2)



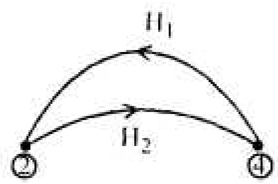
(3)



(4)



(5)



The sum of gains of these 5 loops,

$$= G_1G_2H_3 - G_1G_2G_3 + G_2G_3H_1 - G_1H_2 + H_1H_2$$

So, $$\Delta = 1 - G_1G_2H_3 + G_1G_2G_3 - G_2G_3H_1 + G_1H_2 - H_1H_2$$

There are 2 Nos. of forward paths,

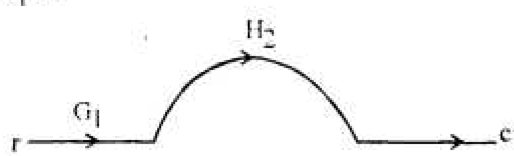
$P_1 =$

Path gain, $M_1 = G_1G_2G_3$

There is no non-touching loop

So, $\Delta_1 = 1$

$P_2 =$



Path gain $= G_1H_2 = M_2$

There is no non-touching loop. So,

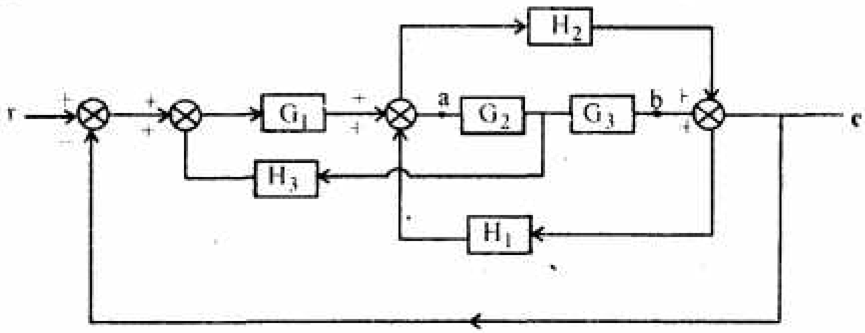
$$\Delta_2 = 1$$

Hence, $$\frac{c}{r} = \frac{1}{\Delta}(M_1\Delta_1 + M_2\Delta_2)$$

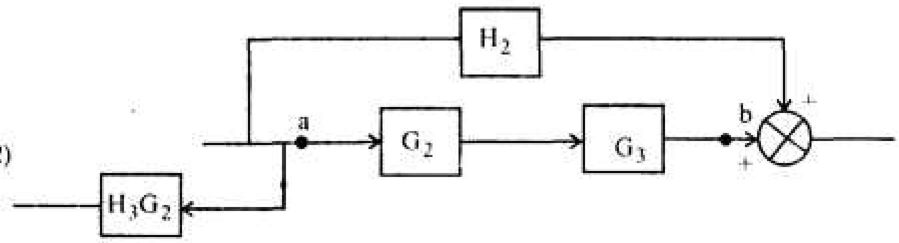
$$= \frac{G_1 G_2 G_3 + G_1 H_2}{1 - G_1 G_2 H_3 + G_1 G_2 G_3 - G_2 G_3 H_1 + G_1 H_2 - H_1 H_2}$$

Block diagram reduction : The successively reduced block diagrams are given below :

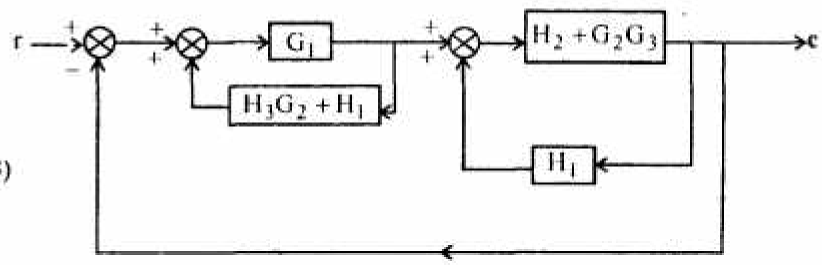
(1)



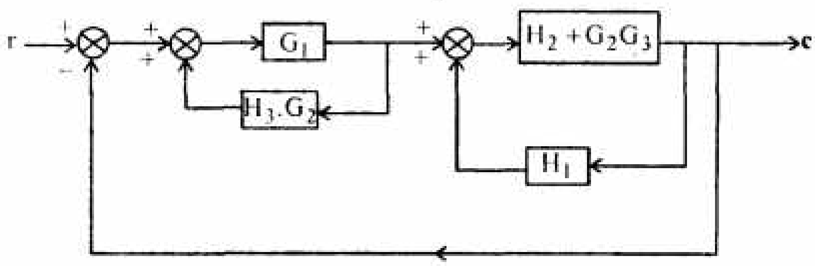
(2)

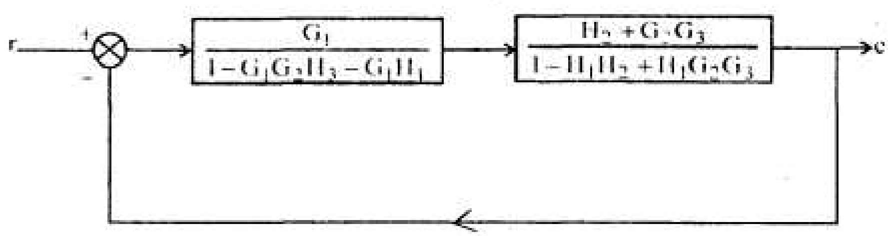


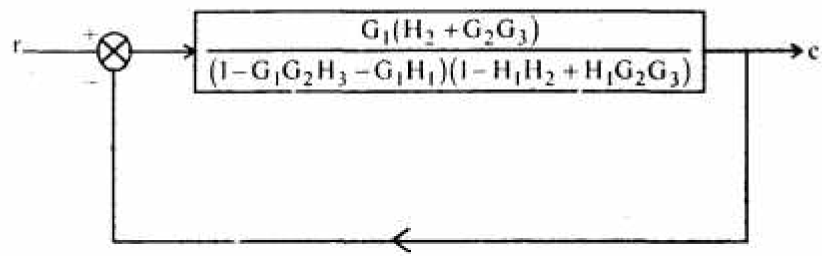
(3)

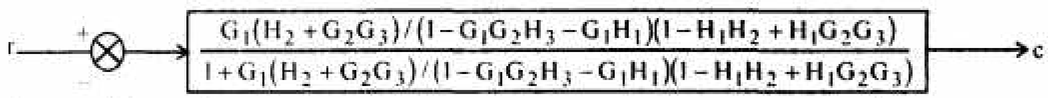


(4).







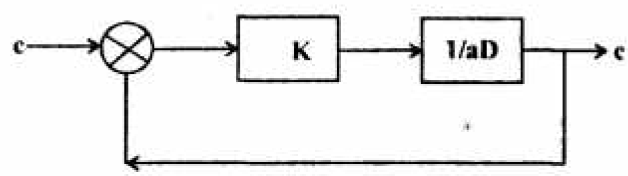


So, on solving we have

$$\frac{c}{r} = \frac{G_1G_2G_3 + G_1H_2}{1 - G_1G_2H_3 + G_1G_2G_3 - G_2G_3H_1 + G_1H_2 - H_1H_2}$$

which is same as found by Mason's formula.

Q. 4. Draw polar plots of the following first order system :



Ans. The open loop transfer function of the given first order system is given by :

$$G_o(t) = \frac{K}{aD}$$

Taking Laplace transform, we have

$$G_o(s) = \frac{K}{as}$$

The closed loop transfer function is :

$$G(s) = \frac{K}{sa + K}$$

Putting $s = jw$, we have

$$G(jw) = \frac{K}{jaw + K} = \frac{K}{K^2 + a^2w^2} - j\frac{awK}{K^2 + a^2w^2}$$

Now, $$|G(jw)| = \frac{K}{\sqrt{K^2 + a^2w^2}} \qquad ...(1)$$

and $$\phi = -\tan^{-1}\frac{aw}{K} \qquad ...(2)$$

(i) Now, when w = 0, $|G(jw)| = \frac{K}{\sqrt{K^2}} = 1$

and $\phi = -\tan^{-1} 0° = 0$

It is also the intersecting point with the real axis of the plot.

(ii) When w tends ∞, then

$$|G(jw)| = 0$$

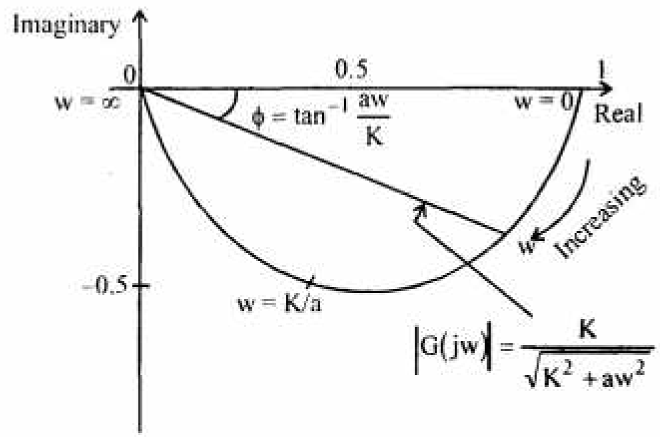
and $\phi = -90°$

This is the point where the plot crosses the imaginary axis.

(iii) When $w = K/a$

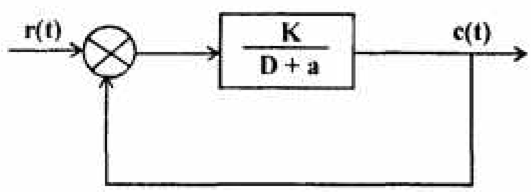
$$|G(jw)| = \frac{K}{\sqrt{K^2 + K^2}} = \frac{1}{\sqrt{2}}$$

$$\phi = -\tan^{-1} 1 = -45°$$



The polar plot is drawn in the figure shown above.

Q. 5. For the first order system shown in figure, derive the solution for output C(t) as a function of time a unit step input r(t) = 1, using time domain analysis.



Ans.

The open loop transfer function of the given system is $\frac{K}{D+a}$.

The closed loop transfer function of this system in s domain is given by,

$$G(s) = \frac{C(s)}{R(s)} = \frac{K}{s+a+K+1}$$

$$= \frac{K}{(1+a+K)+s}$$

Hence, $$C(s) = \frac{K}{(1+a+K)} . R(s) \qquad ...(1)$$

But $r(t) = 1$ given

So, $R(s) = \frac{1}{s}$ (taking Laplace transformation)

Putting this value in equation (1), we have,

$$C(s) = \frac{K}{(1+a+K)} \times \frac{1}{s}$$

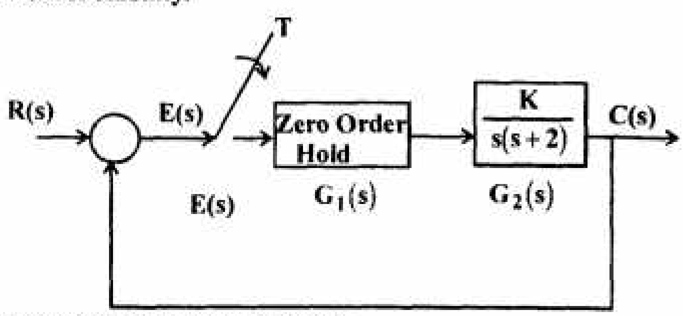
$$= \frac{K}{s[(1+a+K)+s]}$$

Taking inverse Laplace transformation, we have

$$C(t) = K\left[1 - e^{-(1+a+K)t}\right]$$

is the required solution of the given system.

Q. 6. Find, using Routh's criterion if the system is stable or not. Take $G_2(s) = K/[s(s+1)]$, T = 1 sec and K = 1. If K = 10, check for stability.



Ans. The transfer function of the given system is,

$$G(s) = \frac{K}{s(s+1)} \times \frac{K}{s(s+2)}$$

$$= \frac{K^2}{s^2(s+1)(s+2)} \times \frac{K^2}{s^4+3s^3+2s^2}$$

The characteristic equation is given by,

$$s^4 + 3s^3 + 2s^2 + K^2 = 0$$

(i) If K = 1, the characteristic equation is,

$$s^4 + 3s^3 + 2s^2 + 1 = 0$$

Here, as the coefficient of 's' is zero, so the system either unstable or critically stable.

The Routh's array is given by :

s^4	1	2
s^3	3	1
s^2	$\frac{4}{3}$	0
s	1	0
s^0	0	0

There is no change of sign. So no pole lies in right half of s-plane, but as the coefficient of term 's' is missing. So the system is critically stable for K = 1.

(ii) Now, K = 10

The Routh's array is given below :

s^4	1	2
s^3	3	100
s^2	$-\frac{94}{3}$	0
s^1	100	0
s^0	0	0

As the sign changes two times. So two poles lies in the right half of s-plane. So the system is unstable for K = 10.

Q. 7. For the system with transfer function $\frac{Y(s)}{U(s)} = \frac{s^2+2s+1}{s^3+7s^2+14s+8}$. Derive the state-space representation.

Ans. The transfer function of the given system is,

$$\frac{Y(s)}{U(s)} = \frac{s^2+2s+1}{s^3+7s^2+14s+8}$$

Breaking the T.F. into two parts, we have

$$\frac{Y(s)}{U(s)} = \frac{X_1(s)}{U(s)} \times \frac{Y(s)}{X_1(s)} = \frac{1}{s^3+7s^2+14s+8} \times \left(s^2+2s+1\right)$$

Now consider,
$$\frac{X_1(s)}{U(s)} = \frac{1}{s^3+7s^2+14s+8}$$

$\Rightarrow \quad (s^3 + 7s^2 + 14s + 8) X_1(s) = U(s)$

Taking inverse Laplace transformation, we have

$$\frac{d^3x_1}{dt^3} + 7\frac{d^2x_1}{dt^2} + 14\frac{dx_1}{dt} + 8x_1 = u(t)$$

$\Rightarrow \quad \dddot{x}_1(t) + 7\ddot{x}_1(t) + 14\dot{x}(t) + 8x_1(t) = u(t)$

Hence, $\quad \dddot{x}_1(t) = -7\ddot{x}_1(t) - 14\dot{x}(t) - 8x_1(t) + u(t)$

Now select the state variables.

$$\dot{x}_1 = x_2$$

$$\ddot{x}_1 = \dot{x}_2 = x_3$$

$$\dddot{x}_1 = \dot{x}_3$$

Now, $\quad \dot{x}_3 = \dddot{x}_1 = -7\ddot{x}_1(t) - 14\dot{x}(t) - 8x_1(t) + u(t)$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -7 & -14 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

Now consider, $\quad \dfrac{Y(s)}{X_1(s)} = (s^2 + 2s + 1)$

$\Rightarrow \quad Y(s) = (s^2 + 2s + 1) X_1(s)$

Taking inverse Laplace transformation, we have

$$y(t) = \frac{d^2x_1}{dt^2} + 2\frac{dx_1}{dt} + x_1$$

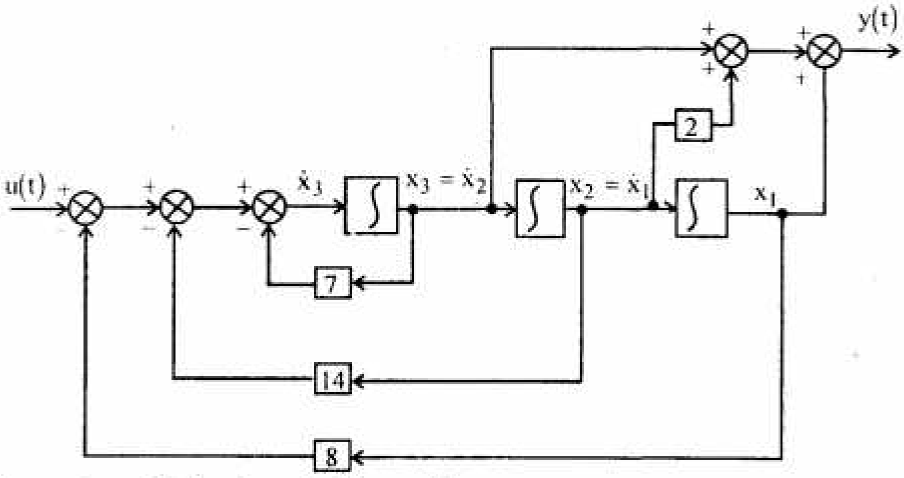
$$= \ddot{x}_1(t) + 2\dot{x}_1(t) + x_1(t)$$

$$= x_3 + 2\dot{x}_2 + x_1$$

Hence, $\quad y(t) = [1,2,1]x(t)$

$$= [1,2,1]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

The block diagram is given below :



Q. 8. For a system with the characteristic equation :

$s^3 - 4s^2 + s + 6 = 0$, find the number of roots, if any with positive real parts.

Ans. The given characteristic equation is,

$$s^3 - 4s^2 + s + 6 = 0$$

The roots of this characteristic equation with positive real parts can be found by Routh's Hurwitz criterion as follows :

The Routh's array is given below :

s^3	1	1
s^2	-4	6
s^1	$-\frac{5}{2}$	0
s^0	6	0

In the first column of the Routh array the sign changes two times. So, there are two roots with positive real parts.