24041

B.Tech. 3rd Semester (Information Technology)

(Branch–VI) Examination, December–2013

DISCRETE STRUCTURES

Paper–CSE-203-F

Time allowed : 3 hours] *[Maximum marks : 100*

Note : ***Question number 1 is compulsory. Attempt one question from each section. All questions carry equal marks.***

1. (a) What do you understand by Algebra of sets ? 2.5×8

(b) Let R be the relation on the set {1, 2, 3, 4, 5 } containing the ordered pairs (1, 1), (1, 2), (1, 3), (2, 3), (2, 4), (3, 1), (3, 4) (3, 5), (4, 2), (4, 5), (5, 1), (5, 2), and (5, 4). Find

(i) R^2 (ii) R^3 (iii) R^4.

(c) Draw the Hasse diagram representing the partial ordering {(a, b) | a divides b} on

a. {1, 2, 3, 4, 6, 8, 12}.

(d) What do you understand by Hamilton path and circuit ?

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(e) Consider the function f, g: $R \to R$ defined by $f(x) = x^2 + 3x + 1$, $g(x) = 2x - 3$. Find the composition functions (a) fof (b) fog

(f) Define tautologies and prove that the statement $(P \to (q \to r)) \to ((p \to q) \to (p \to r))$ is a tautology.

(g) A group of 30 people have been trained as astronauts to go on the first mission to Mars. How many ways are there to select a crew of six people to go on this mission (assuming that all crew members have the same job) ?

(h) For which values of m and n does the complete bipartite graph Km, n have an

(i) Euler circuit

(ii) Euler path.

Section–A

2. (a) Let $f(x) = ax + b$ and $g(x) = cx + d$, where a, b, c and d are constants. Determine necessary and sufficient conditions on the constants a, b, c, and d so that

$f \circ g = g \circ f$. 8

(b) Suppose that f is a function from A to B, where A and B are finite sets with $|A| = |B|$. Show that f is one-to-one if and only if it is onto. 6

(c) Use De Morgan's laws to find the negation of each of the following statements.

(i) Jan is rich and happy.

(ii) Carlos will bicycle or run tomorrow. 6

3. (a) Suppose that R1 and R2 are equivalence relations on the set S. Determine whether each of these combinations of R1 and R2 must be an equivalence relation.

(i) R1 □ R2

(ii) R1 $\cap$ R2

(iii) R1 □ R2 8

(b) What is the covering relation of the partial ordering $\{(A, B) \mid A \subseteq B\}$ on the power set of S, where $S = \{a, b, c\}$? 6

(c) Show that 6

(i) $p \to q$ and its contrapositive $(\sim q \to \sim p)$ are logically equivalent.

(ii) $\neg (p \oplus q)$ and $p \leftrightarrow q$ are logically equivalent.

Section–B

4. (a) Let $a_n = 2n + 5 . 3n$ for $n = 0, 1, 2, ...$ 8

(i) Find $a_0, a_1, a_2, a_3,$ and a_4.

(ii) Show that $a_2 = 5a_1 - 6a_0$, $a_3 = 5a_2 - 6a_1$, and $a_4 = 5a_3 - 6a_2$.

(iii) Show that $a_n = 5a_{n-1} - 6a_{n-2}$ for all integers n with $n \geq 2$.

(b) How many bit strings contain exactly eight 0s and 10 1s if every 0 must be immediately followed by a 1 ? 6

(c) Let n and r be integers with $1 \leq r < n$. Show that

$C(n, r-1) = C(n+2, r+1) - 2C(n+1, r+1) + C(n, r+1)$. 6

5. (a) Solve the recurrence relation $T(n) = nT2(n/2)$ with initial condition $T(1) = 6$ when $n = 2k$ for some integer k. 8

(b) Find the sum of the following series :

$0.7 + 0.77 + 0.777 + 0.7777 + \ldots\ldots\ldots\ldots$ 6

(c) How many 4 – permutations of the positive integers not exceeding 100 contain three consecutive integers k, k + 1, k + 2, in the correct order

(i) where these consecutive integers can perhaps be separated by other integers in the permutation ?

(ii) where they are in consecutive positions in the permutation ? 6

Section–C

6. (a) Let G be the set of all non-zero real numbers.

(i) Define Binary operation

$*$ on G as a, b $\in$ G, $a*b = (ab/2)$.

(ii) Show that (G, *) is an Abelian group. 10

(b) State and prove Lagrange's theorem. 10

7. (a) Differentiate between the following :

(i) Isomorphism, Homomorphism and Automorphism

(ii) Monoid, Group and Ring. 10

(b) Define the following with suitable examples :

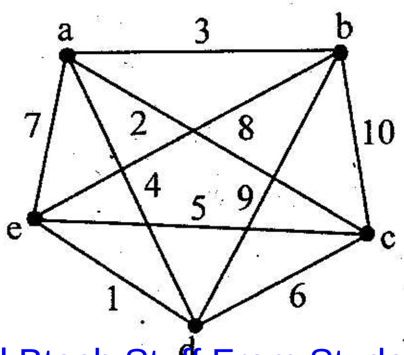
(i) Integral Domain

(ii) Cosets

(iii) Field. 10

Section–D

8. (a) Solve the Travelling salesperson problem for this graph by finding the total weight of all Hamilton circuits and determining a circuit with minimum total weight. 10



(b) Let G is a connected planar simple graph with e edges and v vertices contains no simple circuits of length 4 or less. Show that $e \le (5/3)v - (10/3)$ if $v \ge 4$. 4

(c) Let G be a simple graph with n vertices. Show that

(i) G is a tree if and only if it is connected and has $n - 1$ edges.

(ii) G is a tree if and only if G has no simple circuits and has $n - 1$ edges. 6

9. Write short notes on : 5×4

(a) Spanning Tree in a weighted graph.

(b) Planar graph

(c) Weighted graph

(d) Hamilton path and Circuit.