

Roll No.

2011

B. E. 3rd Semester (I.T.)
Examination – December, 2009

DISCRETE STRUCTURE

Paper : CSE-203-E

Time : Three hours]

[Maximum Marks : 100

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt any *five* questions. All question carry equal marks.

1. (a) Define Multi-set. Discuss various operations that can be performed on multi-sets. 4
- (b) Determine the power set of the following : 6
 - (i) $\{a\}$
 - (ii) $\{\{a\}\}$
 - (iii) $\{\emptyset\}$
 - (iv) $\{\emptyset, \{\emptyset\}\}$

(c) Prove the following statements analytically, where A, B and C are arbitrary sets : 10

(i) $(A \cap B) - C = (A - C) \cap (B - C)$

(ii) $A - B = A \cap B^C$

(iii) $(A \oplus B)^C = A^C \oplus B^C = A \oplus B^C$

(iv) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

2. (a) Let (A, \leq) be a POSET. Let \leq_R be the binary relation on A such that for a and b in A $a \leq_R b$ if and only if $b \leq a$. 10

(i) Show that \leq_R is partial ordering relation.

(ii) Show that if (A, \leq) is lattice, then (A, \leq_R) is also a lattice.

(b) Draw the Hasse diagram representing the partial ordering $\{(A, B) \mid (A \subseteq B)\}$ on the power set $P(S)$, where $S = \{a, b, c\}$. Find the maximal, minimal, greatest and least elements of the POSET. 6

(c) Describe Equivalence relation and Equivalence Class with the help of examples. 4

3. (a) Explain Conjunctive and disjunctive normal forms. Find the CNF corresponding to the propositions: 10

(i) $(p \wedge \sim(q \wedge r)) \vee (p \rightarrow q)$

(ii) $\sim((p \vee \sim q) \wedge \sim r)$

(b) Write a compound statement which is true : 6

(i) when exactly two of three statements p, q, r are true

(ii) when none or one or two of three statements p, q, r are true.

(c) Without using truth tables, prove the following : 4

$$p \rightarrow (q \rightarrow p) \equiv \sim p \rightarrow (p \rightarrow q)$$

4. (a) How many distinguishable words can be formed from the letters of the word MISSISSIPPI. 6

(b) Suppose repetitions are not permitted : 8

(i) How many three digit numbers can be formed from 6 digits {2, 3, 5, 6, 7, 9}

(ii) How many of them are < 400

(iii) How many of them are even.

(iv) How many of them are < 600 but > 200

(c) Discuss finite, countable infinite and uncountable infinite set with example. 6

5. (a) Determine discrete numeric function for the following generating functions : 8

$$G(x) = \frac{1}{5 - 6x + x^2}$$

(b) Find the sum of the following series : 6

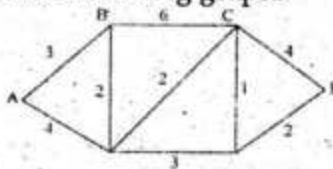
$$0.7 + 0.77 + 0.777 + 0.7777 + \dots$$

(c) Find an explicit formula for the sequence defined by : 6

$$C_n = 3C_{n-1} - 2C_{n-2}, C_1 = 5, C_2 = 3$$

6. (a) A tree has $2n$ vertices of degree 1, $3n$ vertices of degree 2 and n vertices of degree 3. Determine the number of vertices and edges in the tree. 6

- (b) Write and explain any one algorithm to find shortest path in a weighted graph. Use that algorithm to find shortest path between vertices A and F of the following graph. 14



7. (a) Explain isomorphism, homomorphism and automorphism with the help of suitable examples. 10
- (b) State and prove Lagrange's theorem. 10
8. (a) Define Ideal and find it for $(\mathbb{Z}_6, \oplus, \otimes)$. 5
- (b) Discuss Coset and Ring with example. 5
- (c) What is meant by Eulerian and Hamiltonian circuits? Draw a graph which contains: 10
- An Eulerian circuit that is also a Hamiltonian circuit.
 - Neither Eulerian nor Hamiltonian circuit. Justify your answer.