B.E. 1st Semester Examination, December–2012

MATHEMATICS-I

Paper–Math–I

Time allowed : 3 hours] *[Maximum marks : 100*

Note : *Attempt any five questions taking atleast one from each part. All questions carry equal marks.*

Part–A

1. (a) Discuss the convergence of the series;

$$x+\frac{(2x)^2}{2!}+\frac{(2x)^3}{3!}+\frac{(2x)^4}{4!}+\frac{(2x)^5}{5!}+\dots\dots\dots\infty$$

(b) Prove that the series $\frac{\sin x}{1^3}-\frac{\sin 2x}{2^3}+\frac{\sin 3x}{3^3}\dots\dots\dots$ converges absolutely.

2. (a) Discuss the convergence of the series;

$$1+\frac{a.b}{1.c}x+\frac{a.(a+1).b.(b+1)}{1.2.c.(c+1)}x^2+\frac{a.(a+1).(a+2).b.(b+1).(b+2)}{1.2.3.c.(c+1).(c+2)}x^3+\dots\dots\dots$$

(b) Prove that $\int_0^\infty \frac{\tan^{-1} ax}{(x)(1+x^2)}dx=\frac{\pi}{2}\log(1+a)$, where $a>0$

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3. (a) If ρ_1, ρ_2 be the radii of curvature at the extremities of any chord of the cardioid $r = a(1 + \cos\theta)$ which passes through the pole. Find the value of $\rho_1^2 + \rho_2^2$.

(b) Find the asymptotes of the curve;
$(x + y)^2 (x + y + 2) = x + 9y - 2$

4. (a) $u = \operatorname{cosec}^{-1}\left(\dfrac{(x)^{\frac{1}{2}} + (y)^{\frac{1}{2}}}{(x)^{\frac{1}{3}} + (y)^{\frac{1}{3}}}\right)^{\frac{1}{2}}$ then prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + y^2 \frac{\partial^2 u}{\partial y^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{\tan^2 u}{12}\right)$$

(b) Find the points on the surface $z^2 = xy + 1$, nearest to the origin.

Part–B

5. (a) Evaluate by changing the order of integration

$$\int_0^\infty \int_0^x x . e^{-\frac{x^2}{y}} . dydx$$

(b) Find the double integration, the area lying inside the circle $r = a \sin\theta$ and outside the cardiod $r = a(1 - \cos\theta)$.

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6. (a) Find the value of the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

(b) Define βeta function and Gamma function and prove that $\beta(m, n)=\frac{\Gamma m \Gamma n}{m+n}$

7. (a) Give the geometrical interpretation of gradient, also prove that curl curl $\vec{F} = \text{grad}(\text{div}\,\vec{F}) - \nabla^2 \vec{F}$.

(b) If $\vec{r} = x\,i + y\,j + z\,k$ and $r \neq 0$ show that

(i) $\text{grad}\left(\frac{1}{r^2}\right) = \frac{-2\vec{r}}{r^4}$;

(ii) $\text{div}\,(r^n \vec{r}) = (n+3)\,r^n$

8. (a) Apply Green's theorem to evaluate

$\oint_c [(3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy]$ where C is the boundary of the region defined by the $y = \sqrt{x}$, $y = x^2$

(b) Verify Divergence theorem for $F = (x^2 - yz)\,i + (y^2 - zx)\,j + (z^2 - xy)\,k$ Taken over the rectangular parallelopiped $0 \le x \le a;\ 0 \le y \le b;\ 0 \le z \le c$

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