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**B.Tech. 1st Semester (F-Scheme) Examination,****December-2011****MATHEMATICS-I****Paper-Math-101-F**Time allowed : 3 hours ] [ Maximum marks : 100

**Note :** Attempt five questions in all, selecting one question from each section and question number 1 is compulsory.

**1. (Compulsory Question)**

(a) If  $u = \sin^{-1} \left( \frac{x}{y} \right) + \tan^{-1} \left( \frac{y}{x} \right)$  then prove that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$$

(b) Evaluate  $\iint_R xy \, dx \, dy$ , where  $R$  is the first quadrant of the circle  $x^2 + y^2 = a^2$  where  $x, y \geq 0$ .

(c) Find the rank of the matrix :

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

(d) Using Cayley-Hamilton Theorem, find  $A^2$ , where

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

(e) If  $x = r \sin \theta \cos \phi$ ,  $y = r \sin \theta \sin \phi$ ,

$$z = r \cos \theta, \text{ then find } \frac{\partial(r, \theta, \phi)}{\partial(x, y, z)}$$

(f) Test for convergence of the series, where  $n$ th term

$$\text{is } \frac{n^2}{2^n}.$$

(g) Discuss the convergence of the series

$$1 - \frac{1}{3} + \frac{1}{2} - \frac{1}{3^3} + \frac{1}{2^2} - \frac{1}{3^5} + \dots$$

(h) Prove that

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

### Section-A

2. Discuss the convergence of the series below :

(i)  $\frac{x^2}{2 \log 2} + \frac{x^3}{3 \log 3} + \frac{x^4}{4 \log 4} + \dots$  12

(ii)  $1 + \frac{3}{2!} + \frac{5}{3!} + \frac{7}{4!} + \dots$  8

3. Discuss the convergence of the following series :

(i)  $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{n^2}$  10

(ii)  $1 + \frac{1}{2^2} - \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} - \frac{1}{7^2} - \frac{1}{8^2} + \dots$  10

## Section-B

4. (a) Find non-singular matrices P and Q such that PAQ is in the normal form ; where

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 1 \end{bmatrix}$$

- (b) Find the values of  $\lambda$  and  $\mu$  so that the equations

$$2x + 3y + 5z = 9$$

$$7x + 3y - 2z = 10$$

$$2x + 3y + \lambda z = \mu$$

have no solution, a unique solution or an infinite no. of solutions.

5. (a) Show that for any square matrix A, the product of all the eigen values of A is equal to  $\det(A)$ , and the sum of all eigen values of A is equal to the sum of the diagonal elements.

- (b) Using Cayley-Hamilton Theorem find the inverse of the matrix :

$$A = \begin{bmatrix} 1 & 2 & 4 \\ -1 & 0 & 3 \\ 3 & 1 & -2 \end{bmatrix}$$

### Section-C

6. (a) If  $y = \cos(m \log x)$ , then show that

$$x^2 y_{n+2} + (2n+1)x y_{n+1} + (m^2 + n^2) y_n = 0$$

- (b) Expand  $\log x$  in powers of  $(x-1)$  by Taylor's Theorem and hence find the value of  $\log_e^{1.1}$

7. (a) Find the radius of curvature of

$$y = x^2 + 2xy + y^2 + x \text{ at the origin.} \quad 5$$

- (b) Find all the asymptotes of the curve

$$x^3 - 6x^2y + 11xy^2 - 6y^3 + x + y = 0 \quad 5$$

- (c) Prove that if the perimeter of a triangle is constant then its area is maximum when the triangle is equilateral. 10

### Section-D

8. (a) Evaluate the following integrals : 10

(i) 
$$\int_0^1 \frac{dx}{\sqrt{1-x^4}}$$

(ii) 
$$\int_0^{\pi/2} \sin^2 \theta \cos^4 \theta d\theta$$

- (b) Change the order of integration and hence evaluate the integral :

$$\int_0^2 \int_{\sqrt{2y}}^2 \frac{x^2 dx dy}{\sqrt{x^4 - 4y^2}} \quad 10$$

9. (a) Evaluate  $\iiint_R (x^2 + y^2 + z^2) dx dy dz$ , where  $R$  denotes the region bounded by  $x = 0, y = 0, z = 0$  and  $x + y + z = a, (a > 0)$ . 10
- (b) Find the area bounded by  $y^2 = 4 - x$  and  $y^2 = x$ . 10