Roll No. ....

1982

B. E. 1st Semester

Examination – December, 2011

MATHEMATICS - I

Paper : Math - I

Time : Three hours] *[Maximum Marks : 100*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions taking at least *two* from each part.

PART – A

1. (a) Test for convergence the following series :

$$1+\frac{1^2.2^2}{1.3.5}+\frac{1^2.2^2.3^2}{1.3.5.7.9}+..........\infty$$

(b) $x^2(\log 2)^q + x^3(\log 3)^q + x^4(\log 4)^q + ...\infty$.

2. (a) Expand $e^{\sin x}$ by Maclaurin's series upto the term containing x^4 .

(b) Use Taylor's series, to prove that :

$$\tan^{-1}(x+h) = \tan^{-1} x + (h\sin z)\frac{\sin z}{1} - (h\sin z)^2 . \frac{\sin 2z}{2} + (h\sin z)^3 \frac{\sin 3z}{3} + \text{.........} \text{ where } z = \cot^{-1} x .$$

3. (a) If ρ_1 and ρ_2 be the radii of curvature at the ends of a focal chord of the parabola $y^2 = 4ax$, then show that $\rho_1^{-2/3} + \rho_2^{-2/3} = (2a)^{-2/3}$.

(b) Find the asymptotes of the curve $x^3 + 3x^2 y - 4y^3 - x + y + 3 = 0$.

4. (a) Expand $e^x \log(1+y)$ in powers of x and y upto terms of third degree.

(b) If u = xyz, $v = x^2 + y^2 + z^2, w = x + y + z$ find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$.

PART – B

5. (a) Evaluate the integrals $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dz\, dy\, dx$.

(b) Change the order of integration in the integral :

$$I = \int_{-a}^{a} \int_{0}^{\sqrt{a^2-y^2}} f(x,y)\, dx\, dy$$

6. (a) Find the volume bounded by paraboloid $x^2 + y^2 = az$, the cylinder $x^2 + y^2 = 2ay$ and the plane $z = 0$.

(b) Prove that $\beta\left(m,\frac{1}{2}\right)=2^{2m-1}\beta(m,m)$.

7. (a) Show that $\nabla^2(r^n)=n(n+1)r^{n-2}$.

(b) Compute the line integral $\int_c (y^2dx - x^2dy)$ about the triangle whose vertices are (1, 0), (0, 1) and (–1, 0).

8. (a) Verify Green's theorem for $\int_c \left[(xy+y^2)dx + x^2dy\right]$, where c is bounded by $y=x$ and $y=x^2$.

(b) Evaluate $\int_s \vec{F}.ds$, where $\vec{F}=4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$ and s is the surface bounding the region $x^2+y^2=4$, $z=0$ and $z=3$.
