Roll No. ....

1982

B. E. 1st Semester

Examination – December, 2009

MATHEMATICS

Paper : Math-I

Time : Three hours] *[Maximum Marks : 100*

Before answering the question, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting two questions from each part.

PART – A

1. (a) Discuss the convergence of the series

$$1+\frac{x}{2}+\frac{2!}{3^2}x^2+\frac{3!}{4^3}x^3+\frac{4!}{5^4}x^4+\ldots\ldots\ldots\ldots\infty(x>0)$$

(b) Test the convergence of the series

$$\sum_{n=1}^{\infty}\frac{(-1)^n(x+1)^n}{2^n\, n^2}$$ 10, 10

2. (a) Prove that

$$\log(\sec x+\tan x)=x+\frac{x^3}{6}+\frac{x^5}{24}+\ldots\ldots\ldots\ldots$$

(b) Show that radius of curvature at the end of the major axis of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is equal to the semi-latus rectum of the ellipse. 7, 7

(c) Find all the asymptotes of the curve

$$y^3 - xy^2 - x^2y + x^3 + x^2 - y^2 = 0$$ 6

3. (a) If $r^2 = x^2 + y^2 + z^2$ and $V = r^m$, show that $V_{xx} + V_{yy} + V_{zz} = m(m+1)r^{m-2}$

(b) Of $u = \sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, Evaluate

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial xry}+y^2\frac{\partial^2 u}{\partial y^2}$$ 10, 10

4. (a) In a plane triangle ABC, find the maximum value of Cos A Cos B Cos C.

(b) Prove that $\int_0^{\pi/2}\frac{\log(1+y\sin^2 x)}{\sin^2 x}dx = \pi\left[\sqrt{1+y}-1\right]$

10, 10

PART – B

5. (a) Find the surface of solid generated by revolution of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the x-axis.

(b) Find by double integration, the area lying between the parabola $y = 4x - x^2$ and the line $y = x$. 10,10

6. (a) Find the volume of the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1.$$

(b) Prove that $\Gamma(m).\Gamma\left(m+\frac{1}{2}\right)=\frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$ 10, 10

7. (a) For a solenoidol vector F, show that curl curl curl curl F = ∇^4 F

(b) Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at (2, –1, 2). 10, 10

8. (a) Apply Green's theorem to evaluate

$\int_C \left[(2x^2-y^2)\,dx+(x^2+y^2)\,dy\right]$ where C is the boundry of the area enclosed by x-axis and the upper half of the circle $x^2+y^2=a^2$.

(b) Evaluate by Stokes theorem

$\oint_C (yz\,dx+zx\,dy+xy\,dz)$ where C is the curve $x^2+y^2=1, z=y^2$. 10,10
