24018

B.Tech. 2nd Semester Examination,

May–2013

MATHEMATICS–II

Paper–Math–102–F

Time allowed : 3 hours] *[Maximum marks : 100*

Note : Question No. 1 is compulsory. Attempt total five questions selecting one question from each unit. All questions carry equal marks.

1. (a) If $\vec{F} = \text{grad}\,(x^3 + y^3 + z^3 - 3xyz)$. Find Div $\vec{F}$.

(b) Find inverse L.T. of $\dfrac{s-3}{s^2-6s+13}$

(c) Solve $pq = p + q$.

(d) Find Laplace transformation of $\dfrac{e^{-t}\sin t}{t}$

(e) Solve $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = e^{-2t}$

(f) Solve $z = px + qy + pq$

(g) Find the Laplace transformation of $e^{t-2}\,u\,(t-2)$

(h) If $\dfrac{d^2P}{dt^2} = 6tI - 12t^2J + 4\cos t\,K$, find P, given that $\dfrac{dP}{dt} = -1 - 3K$ and $P = 2I + J$ when $t = 0$.

Unit–I

2. (a) Give the geometrical interpretation of gradient, also prove that curl curl F = grad (div F) - ∇^2F.

(b) If $R = xi + yj + zk$ and $r \neq 0$ show that

(i) $\text{grad}\left(\frac{1}{r^2}\right) = \frac{-2R}{r^4}$

(ii) $\text{div}(r^n R) = (n+3)r^n$

3. (a) Apply Green's theorem to evaluate $\oint_C [(y - \sin x)dx + \cos x dy]$ where C is the plane triangle enclosed by the lines $y = 0$, $x = \frac{\pi}{2}$ and $y = \frac{2}{\pi}x$.

(b) Verify Divergence theorem for

$F = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$

Taken over the rectangular parallelepiped

$0 \leq x \leq a;\ 0 \leq y \leq b;\ 0 \leq z \leq c$

Unit–II

4. (a) Solve the following differential equation :

$(2x^2y - 3y^4)dx + (3x^3 + 2xy^3)dy = 0$

(b) A body originally at 80° C cools down at 60°C in 20 minutes, the temperature of the air being 40°C. What will be the temperature of the body after 40 minutes from the original ?

5. (a) Solve the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 8x^2 e^{2x} \sin 2x.$$

(b) In an L-C-R circuit, the charge q on a plate of condenser is given by $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = E \sin pt$. The circuit is tuned to resonance so that $P^2 = \frac{1}{LC}$. If initially the current I and the charge q be zero, then show that for small values of $\frac{R}{L}$, the current in the circuit at time t is given by $\frac{Et}{2L}\sin pt$.

Unit–III

6. (a) Find the Laplace transform of

(i) cosh at cos at

(ii) $\int_0^t t \sin 3t \, dt$.

(b) Evaluate $L^{-1}\left[\frac{s}{(s^2+1)(s^2+4)}\right]$ by convolution.

7. (a) (i) Evaluate $\int_0^\infty \left(\frac{e^{-t}-e^{-3t}}{t}\right) dt$

(ii) Find Laplace Inverse of $\tan^{-1}\left(\frac{2}{s^2}\right)$.

(b) Solve $x'' + 4y' + 3y = e^{-t}$, $y(0) = y'(0) = 1$ using Laplace transform.

Unit–IV

8. (a) Solve the following differential equation

$$(x^2 - y^2 - z^2)\, p + 2xyq = 2xz$$

(b) Solve the equation by Char pit's method

$$(p^2 + q^2)\, y = qz$$

9. (a) Find the differential equation of all planes which are at a constant distance 'a' from the origin.

(b) Using method of separation of variables, solve

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u\text{, where } u(x, 0) = 6e^{-3x}.$$

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