B.Tech. 2nd Semester F Scheme

Examination, May–2014

MATHEMATICS-II

Paper–Math–102–F

Common for all branches

Time allowed : 3 hours]　　　　[Maximum marks : 100

Note : Question No. 1 is compulsory. Attempt five questions in total.

1.　(a)　Define constant vector.

　　(b)　Find value of curl (grad ϕ).

　　(c)　Solve $(3x^2 + 6xy^2)\,dx + (6x^2y + 4y^3)\,dy = 0$.

　　(d)　Solve $\dfrac{d^2x}{dt^2} + 6\dfrac{dx}{dt} + 9x = 0$.

　　(e)　Find the Laplace transform of $e^{-3t}\cos^2 t$.

　　(f)　Define Laplace transform of periodic function.

　　(g)　Solve $\dfrac{\partial^2 u}{\partial x\,\partial t} = e^{-t}\cos x$.

　　(h)　Solve $z = px + qy - 2\sqrt{pq}$.

(b) A vector field is given by

$\vec{A} = (x^2 + xy^2)\ \hat{i} + (y^2 + x^2y)\ \hat{j}$. Show that the field is irrotational and find the scalar potential.

3. (a) Find the circulation of $\vec{F}$ round the curve c, where $\vec{F} = e^x \sin y\ \hat{i} + e^x \cos y\ \hat{j}$ and c is the rectangle whose vertices are

$(0, 0),\ (1, 0),\ \left(1, \dfrac{\pi}{2}\right)$ and $\left(0, \dfrac{\pi}{2}\right)$.

(b) Verify Stoke's theorem for the vector field $\vec{F} = (2x - y)\hat{i} - yz^2\ \hat{j} - y^2 z\ \hat{k}$ over the upper half surface of $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy-plane.

Section–B

4. (a) Solve $y\ (xy + 2x^2 y^2)\ dx + x\ (xy - x^2 y^2)\ dy = 0$.

(b) When a switch is closed in a circuit containing a battery E, a resistance R and an inductance to the

5. (a) Apply the method of variation of parameters to solve $\dfrac{d^2y}{dx^2} - y = \dfrac{2}{1+e^x}$.

    (b) Solve $x^2 \dfrac{d^2y}{dx^2} + 3x \dfrac{dy}{dx} + y = \dfrac{1}{(1-x)^2}$.

Section–C

6. (a) Find the inverse Laplace transform of

    $$\dfrac{1}{s^3 (s^2 + a^2)}.$$

    (b) Find the inverse Laplace transform of

    $$\cot^{-1}\left(\dfrac{s+a}{b}\right).$$

    (c) Apply convolution theorem to evaluate

    $$L^{-1}\left[\dfrac{s^2}{(s^2 + 4)^2}\right].$$

(b) Solve $\int_0^t \dfrac{y(u)}{\sqrt{t-u}}\, du = \sqrt{t}$ by Laplace transform.

Section–D

8. (a) Form the partial differential equation by eliminating the arbitrary functions from $f(x^2 + y^2 + z^2, z^2 - 2xy) = 0$.

   (b) Solve $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$.

   (c) Solve $2zx - px^2 - 2q\,xy + pq = 0$.

9. (a) Solve the equation $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}$ with boundary conditions

   $u(x, 0) = 3 \sin n \pi x, \ u(0, t) = 0, \ u(1, t) = 0,$
   where $0 < x < 1, t > 0$.

   (b) Solve $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$ within the rectangle

   $0 \leqslant x \leqslant a, \ 0 \leqslant y \leqslant b$ given that

   $u(0, y) = u(a, y) = u(x, b) = 0$ and