B.E./B.Tech. 2nd Semester E-Scheme

Examination, May–2014

MATHEMATICS-II

Paper–MATH-102-E

Common for all branches

Time allowed : 3 hours] *[Maximum marks : 100*

Note : *Attempt five questions in all taking at least one question from each part. All questions carry equal marks.*

Part–A

1. (a) Find the inverse of $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix}$ by elementary row operations.

(b) Find the rank of the matrix by reducing into the normal form

$$\begin{bmatrix} 0 & 2 & 3 & 4 \\ 2 & 3 & 5 & 4 \\ 4 & 8 & 13 & 12 \end{bmatrix}$$

2. (a) Verify Cayley-Hamilton theorem and hence find A^{-1} for the matrix

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$ 15

(b) Find the eigen values of $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ 5

Part-B

3. (a) Solve :

(i) $(\sec x \tan x \tan y - e^x)\, dx + \sec x \sec^2 y\, dy = 0$

(ii) $(x^2 - 3xy + 2y^2)\, dx + x(3x - 2y)\, dy = 0$

(b) Find the orthogonal trajectory of the cardioids $r = a(1 - \cos\theta)$

4. (a) Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$.

(b) Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2} + 16y = 32 \sec 2x$.

5. (a) Solve

$$(2x+3)^2 \frac{d^2y}{dx^2} - 2(2x+3)\frac{dy}{dx} - 12y = 6x.$$

(b) If an e.m.f. E sin wt is applied to a circuit containing a resistance R, an inductance L and a condenser of capacity C, the charge on the condenser at time t satisfies the equation $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = E \sin wt$. If $R = 2\sqrt{\frac{L}{C}}$, solve the differential equation for q.

Part–C

6. Find the Laplace transforms of the following functions

(a) $t^3 e^{-4t}$

(b) $e^{-t} \cos^2 t$

(c) $\frac{\sinh t}{t}$

(d) $t^2 \sin at$

7. (a) Find the inverse transform of $\frac{1}{s(s^2 - a^2)}$ by convolution theorem.

(b) Solve the equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t}\sin t, \quad x(0) = 0, \; x^1(0) = 1$$

by Laplace transform.

8. (a) Solve $z^2(p^2 z^2 + q^2) = 1$ by Charpit's method.

(b) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If it is set vibrating by giving to each of its points a velocity $\lambda x(l - x)$, find the displacement of the string at any distance x from one end at any time t.