24018

B. Tech. 2nd Semester Examination, May–2011

MATHEMATICS - II

Paper - Math-102-F

Time allowed : 3 hours] *[Maximum marks : 100*

Note : *Attempt five questions in total, question No. 1 is compulsory.*

1\. (a) Find Div (Curl $\vec{V}$) where $\vec{V}$ is a vector function. 3

(b) Find the inverse Laplace transform of $\cot^{-1}\left(\frac{s}{2}\right)$ 3

(c) Find the Laplace transform of the function

$$f(t) = \begin{cases} \sin wt, & 0 < t < \pi/w \\ 0 & \pi/w < t < 2\pi/w \end{cases}$$ 3

(d) Solve the partial differential equation : 3

$x^2p^2 + y^2q^2 = z^2$

(e) Solve the p.d. equation : $(p-q)(z-px-qy)=1$ 3

(f) Solve $\frac{dy}{dx} + \frac{y \cos x + \sin y + y}{\sin x + x \cos y + x} = 0$ 2½

(g) Show that the frequency of free vibrations in a closed electrical circuit with inductance L and capacity C in series is $30/\pi\sqrt{LC}$ per minute. 2½

Unit - I

2. (a) For the curve $x=\cos t\, i + t \sin t$, $y = \sin t - t \cos t$ find the tangential and the normal components of the acceleration at any time t. 7

(b) If $r^2 = x^2+y^2+z^2$, then show that $\nabla\phi(r) = \frac{\phi'(r)}{r}\vec{r}$

and hence, show that $\nabla(\int r^n dr) = r^{n-1}\vec{r}$ 7

(c) Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ 6

3. (a) Use the divergence theorem to evaluate

$\iint_S \vec{F}.\hat{N}\, ds$ where $\vec{F} = x^2z\,\hat{i} + y\,\hat{j} - xz^2\,\hat{k}$ and

S is the boundary of the region bounded by the paraboloid $z=x^2+y^2$ and the plane $z=4y$ 10

(b) Using Stoke's theorem to evaluate $\oint_C (2y^3dx+x^3dy+zdz)$.

Where C is the trace of the cone $z = \sqrt{x^2+y^2}$, intersected by the plane $z = 4$ and the surface of the cone below $z = 4$. 10

Unit - II

4. (a) Solve the differential equation :

$$\left(y + \frac{y^3}{3} + \frac{x^2}{2}\right) dx + \frac{1}{4}(x+xy^2)\, dy = 0$$ 7

(b) A body originally at 80°C cools down to 60°C in 20 minutes, the temperature of the air being 40°C. What will be the temperature of the body after 40 minutes from the original? 7

(c) Find the orthogonal trajectories of the family of parabola $y = ax^2$ 6

5. (a) Solve the differential equation :

$$\frac{d^2y}{dx^2} + 4y = 4\tan(2x)$$ 7

(b) Solve by the method of variation of parameters :

$y'' - 2y' + y = e^x \log x$ 6

(c) Solve the simultaneous equations :

$$\frac{dx}{dt} + 2x+3y = 0,\ 3x + \frac{dy}{dt} + 2y = 2e^{2t}$$ 7

Unit - III

6. (a) Find Laplace transforms of $\left(\sqrt{t} - \frac{1}{\sqrt{t}}\right)^3$ 6

(b) Find inverse L.T. of $\log\left(\frac{s^2+4}{s(s+1)}\right)$ 7

(c) Apply convolution theorem to evaluate

$$L^{-1}\left(\frac{s^2}{s^4-a^4}\right)$$ 7

7. (a) Solve the equations by Laplace transform :

$$\frac{dx}{dt} - y = e^t, \quad \frac{dy}{dt} + x = \sin t$$

given $x(0) = 1$, $y(0) = 0$ 10

(b) Solve the integral equation by L.T. method :

$$y(t) = 2e^{-t} - 2\int_0^t y(u)\cos(t-u)\,du.$$ 10

Unit-IV

8. (a) Solve the equation :

$$\frac{\partial^2 z}{\partial x^2} = a^2 z \text{ given that,}$$

when $x=0$, $\frac{\partial z}{\partial x} = a \sin y$ and $\frac{\partial z}{\partial y} = 0$ 6

(b) Solve the differential equation :

$(mz - ny)\,p + (nx - lz)\,q = ly - mx$ 7

(c) Solve the equation : $p^2+q^2 = z^2(x+y)$ 7

9. (a) Solve the equation by Charpit's method :

$2z + p^2 + qy + 2y^2 = 0$ 6

(b) Solve the equation by the method of separation of variables :

$$x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} = 0$$ 6

(c) The ends A and B of a rod 20 cm long have the temperature at 30°C and 80°C until steady - state prevails. The temperature of the ends are changed to 40° and 60° respectively. Find the temperature distribution in the rod at time t. 8