Roll No. ....

1991

B. E. 2nd Semester

Examination – December, 2011

MATHEMATICS-II

Paper : Math-102-E

Time : Three hours] *[Maximum Marks : 100*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting at least *one* question from each Part.

PART – A

1. (a) Reduce the following matrix to normal form, hence find its rank :

$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 3 & -1 & -2 & -4 \\ 1 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

(b) For what values of a and b do the equations $x+2y+3z=6$, $x+3y+5z=9$, $2x+5y+az=b$ have (i) no solution (ii) a unique solution (iii) more than one solution ?

2. (a) Are the following vectors linearly dependent ? If so, find a relation between them : $X_1=(1,1,-1,1)$, $X_2=(1,-1,2,-1)$, $X_3 = (3,1,0,1)$.

(b) Verify caley-Hamilton theorem for the matrix A and hence find A^{-1} when A is :

$$A=\begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$$

PART – B

3. (a) Solve the following differential equation :

$$(2xy\cos x^2 - 2xy + 1)\ dx + (\sin x^2 - x^2)dy = 0.$$

(b) A cup of coffee at temperature 100°C is placed in a room whose temperature is 15°C and it cools to 60°C in 5 minutes. Find its temperature after a further interval of 5 minutes.

4. (a) Solve the following differential equation :

$$\frac{d^2y}{dx^2}+\frac{3dy}{dx}+2y=e^{e^x}$$

(b) In an e.m.f. $E \sin wt$ applied to a circuit containing a resistance R, an inductance L and a condenser of capacity C, the charge on the condenser satisfies the equation :

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = E \sin wt$$

If $R = 2\sqrt{LC}$, solve the differential equation for q.

5. (a) Solve by the method of variation of parameters $\frac{d^2y}{dx^2} - y = e^{-x}(\sin e^{-x}) + \cos(e^{-x})$.

(b) Solve the simultaneous equations :

$$t\frac{dx}{dt} + y = 0,\ t\frac{dy}{dt} + x = 0, \text{ given } x(1) = 1, y(-1) = 0$$

PART – C

6. (a) (i) Find the Laplace transform of $te^{at} \sin at$.

(ii) Find the increase Laplace transform of :

$$\frac{21s - 33}{(s+1)(s-2)^3}$$

(b) Apply convolution theorem to evaluate :

$$L^{-1}\left\{\frac{s^2}{(s^2 + a^2)(s^2 + b^2}\right]$$

7. (a) Solve the simulaneous equations :

$$(D^2 - 3)x - 4y = 0,\ x + (D^2 + 1)y = 0$$

for t > 0 given that $x = y = \frac{dy}{dt} = 0$ and $\frac{dx}{dt} = 2$ at $t = 0$.

(b) Constact voltage E is applied at $t = 0$ to a circuit with an inductance L, capacitance c and resistance R. Find the current I at time t, if the initial current and charge are zero.

8. (a) Solve the following differential equation :

$$(x^2 - y^2 - z^2)p + 2xyq = 2xz$$

(b) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ within the rectangle $0 \le x \le a$, $0 \le y \le b$ given that $u(0, y) = u(a, y) = u(x, b) = 0$ and $u(x, 0) = x(a - x)$