Roll No. .....

24018

B. Tech. 2nd Semester (F Scheme) Examination – May, 2010

MATHEMATICS–II

Math-102-F

Time : Three hours] *[Maximum Marks : 100*

Before answering the question, candidate should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Answer *five* questions in all, Question No. **1** is compulsory.

1. (a) If $\vec{a}$ is a constant vector and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, prove that curl $(\vec{a} \times \vec{r} = 2\vec{a}$. 3

(b) Find inverse L.T. of $\tan^{-1}\left(\frac{2}{s^2}\right)$. 3

(c) Find the L. T. of the function : 3

$$f(t) = \begin{cases} \sin wt, & 0 < t < \pi/w \\ 0 & \pi/w < t < 2\pi/w \end{cases}$$

(d) Solve p.d. equation : 3

$(x^2 - y^2 - z^2)\, p + 2xyq = 2xz$.

(e) Solve $\frac{\partial^2 z}{\partial x^2}+z=0$, given that when $x = 0$, $z=e^y$ and $\frac{\partial z}{\partial x}=1$. 3

(f) Solve $ydx-xdy+3x^2y^2e^{x^3}dx=0$. 2½

(g) Prove that $\frac{1}{D-a}X=e^{ax}\int Xe^{-ax}dx$. 2½

UNIT – I

2. (a) Find the work done in moving a particle once round the circle $x^2+y^2=9$ in the xy-plane if the field of force is $\vec{F}=(2x-y-z)\hat{i}+(x+y-z^2)\hat{j}+(3x-2y+4z)\hat{k}$. If possible, find its scalar potential. 7

(b) Find the values of a, b, c for which the vector $\vec{V}=(x+y+az)\hat{i}+(bx+3y-z)\hat{j}+(3x+cy+z)\hat{k}$ is irrotational. 7

(c) Evaluate $\iint_s \vec{r}.\hat{n}\,ds$, where s is a closed surface. 6

3. (a) Apply Stoke's theorem to evaluate $\int_c (ydx+zdy+xdz)$ where c is the curve of intersection of $x^2+y^2+z^2=a^2$ and $x+z=a$. 10

(b) Using Divergence theorem, evaluate $\iint_s \vec{F}.d\vec{s}$ where $\vec{F}=4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$ and s is the surface bounding the region $x^2+y^2=4$, $z=0$ and $z=3$. 10

UNIT – II

4. (a) Solve the d. e. $(x^2y^2+xy+1)ydx+(x^3y^2-x^2y+x)dy=0$ 7

(b) If the temperature of the air is 30°C and the substance cools from 100°C to 70°C in 15 minutes, find when the temperature will be 40°C. 7

(c) Find orthogonal trajectories of hyper-bolas $y^2=4ax$. 6

5. (a) Solve $x^3\frac{d^3y}{dt^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$. 7

(b) Solve simultaneous equations :

$t\frac{dx}{dt}+y=0, t\frac{dy}{dt}+x=0$ given $x(1)=1, y(-1)=0$. 7

(c) Find how many second a clock would lose per day, if the length of its pendulum were increased in the ratio 900 : 901. 6

UNIT – III

6. (a) Find L:Transform of the function $f(t)$ defined as $f(t)=|t-1|+|t+1|+|t+2|+|t-2|, t\geq 0$. 6

(b) Find the inverse laplace transform of $\tan^{-1}\left(\frac{2}{s^2}\right)$. 7

(c) Apply Convolution theorem to evaluate $L^{-1}\left[\frac{s^2}{(s^2+4)^2}\right]$. 7

7. (a) Solve the equation by Laplace transform :

$$\frac{dx}{dt}-\frac{dy}{dt}-2x+2y=1-2t, \frac{d^2x}{dt^2}+\frac{2dy}{dt}+x=0$$

subject to the condition $x=0, y=0, \frac{dx}{dt}=0$, when $t=0$. 10

(b) Solve the integral equation by L. T. method : 10

$$F(t)=t+\int_0^t F(t-u)\cos u\, du \;1\; F(0)=4$$

UNIT – IV

8. (a) Solve $\frac{\partial^2 z}{\partial x^2}+z=0$ given that when $x = 0$, $z=e^y$ and $\frac{\partial z}{\partial x}=1$. 6

(b) Solve the differential equation : 7

$$x^2(y-z)p+y^2(z-x)q=z^2(x-y)$$

(c) Solve the equation $p^2+q^2=(x^2+y^2)z$. 7

9. (a) Solve the equation by Charpit's method : 6

$$2z+p^2+qy+2y^2=0$$

(b) Solve the equation by the method of separation of variables : 6

$$\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial t}+u, \text{ where } u(x,0)=6e^{-3x}$$

(c) Solve $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ which satisfies the conditions : $u(0, y) = u(x, 0) = 0$ and $u(x, a) = \sin\left(\frac{n\pi x}{l}\right)$. 8