Roll No. ....

1991

B. E. 2nd Semester

Examination – December, 2009

MATHEMATICS-II

Paper : Math-102-E

Time : Three hours]　　　　　　　[Maximum Marks : 100

Before answering the question, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, by selecting at least one question from each Part.

PART – A

1. (a) Test for consistency the system of linear equations : 　　　　　　10

$$-2x + y + z = a$$
$$x - 2y + z = b$$
$$x + y - 2z = c$$

where a, b, c are constants.

(b) Determine the eigen vectors of the matrix

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}.$$ Also determine.

whether the eigen vectors are orthogonal.　　10

2. (a) Verify Cayley-Hamilton Theorem for the matrix

$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$ and hence obtain its inverse.   **10**

(b) Find the inverse transform of the following :   7

$$y_1 = x_1 + 2x_2 + 5x_3$$
$$y_2 = -x_2 + 2x_3$$
$$y_3 = 2x_1 + 4x_2 + 11x_3$$

(c) Define the following by giving one example in each case :   **3**

(i) Rank of a matrix

(ii) Orthogonal Matrix

PART – B

3. (a) Solve the differential equation :   **10**

$$(x^2y^2 + xy + 1)ydx + (x^2y^2 - xy + 1)xdy = 0$$

(b) Show that the family of parabolas $y^2 = 4cx + 4c^2$ is self orthogonal.   **5**

(c) Find the orthogonal trajectories of :   **5**

$$r^2 = a \sin 2\theta$$

4. (a) Solve the differential equation :   **10**

$$(D^2 - 4D + 3)z = 2ye^{3y} + 3e^y \cos 2y, \text{ where } D \equiv \frac{d}{dy}$$

(b) Solve the differential equation :   10

$$x^2 y'' - 4xy' + 4y = 4x^2 - 6x^3, y(2) = 4, y'(2) = -1$$

5. (a) Solve the following differential equation by the method of variation of parameters :   10

$$(D^2 - 2D)y = e^x \sin x$$

(b) Determine Q and I in the LCR circuit with $L = 0.5H$, $R = 6\Omega$, $C = 0.02F$, $E(t) = 24 \sin 10t$ and initial conditions $Q = I = 0$ at $t = 0$ 10

PART – C

6. (a) Find the Laplace Transform of the following :   5

    (i) $t^{7/2} e^{5t}$

    (ii) $\int_0^t \dfrac{\sin x}{x} dx$

(b) Prove that if $L\{f(t)\} = \bar{f}(s)$ then   7

$$L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} \bar{f}(s), \text{ for } n = 1, 2, 3, \ldots\ldots .$$

(c) Find the inverse Laplace Transform of the following by convolution theorem :   8

$$\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}$$

7. (a) Solve the following differential equation by the method of Laplace transformation :   10

$$y'' - 8y' + 15y = 9te^{2t}, y(0) = 5, y'(0) = 10.$$

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(b) Form the partial differential equations by eliminating arbitrary function from the following : 5

$$x + y + z = f(x^2 + y^2 + z^2)$$

(c) Solve the partial differential equation : 5

$(bz - cy)p + (cx - az)q = ay - bx$

8. (a) Solve the partial differential equation by Charpit's method : $2xz - px^2 - 2qxy + pq = 0$ 8

(b) Solve the one-dimensional heat equation by the separation of variables method, i.e., solve 12

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}, \ x(0,t) = 0 = x(L,t) \text{ and } u(x,0) = f(x),$$

where c^2 denotes thermal diffusivity, L being length of the bar and $u(x, 0)$ the initial temperature.