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Candidates must write the Code on the title page of the answer-book.

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- Please check that this question paper contains **11** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate ..
- Please check that this question paper contains 29 questions.
- Please write down the Serial Number of the question before attempting it.
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer script during this period.
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MATHEMATICS

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Time allowed : 3 hours

Maximum Marks: 100

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General Instructions :

- (i) All questions are compulsory.
- (ii) The question paper consists of **29** questions divided into three sections A, B and C. Section A comprises of **10** questions of **one mark** each, Section B comprises of **12** questions of **four marks** each and Section C comprises of **7** questions of **six marks** each.
- (iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
- (iv) There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
- (v) Use of calculators is **not** permitted.

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SECTION A

~31

Question numbers 1 to 10 carry 1 mark each.

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- Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B . State whether f is one-one or not.

"iRT A = {n, 2, 3}, B = {4, 5, 6, 7} 02IT11RT f = {(I, 4), (2, 5), (3, 6)}
A ~ B 1R ~ ~ ~ l ~ ~ ~ cp;rr f~ ~ 3=IWJJ -;r@ l

- What is the principal value of $\cos^{-1}(\cos 23^\circ) + \sin^{-1}(\sin 7^\circ)$?

$\cos^{-1}(\cos 23^\circ) + \sin^{-1}(\sin 7^\circ)$ Cft ~ IIR' cp;rr q' ...

- Evaluate:

$$\cos 15^\circ \sin 15^\circ$$

$$\sin 75^\circ \cos 75^\circ$$

IIR'~~:

$$\cos 15^\circ \sin 15^\circ$$

$$\sin 75^\circ \cos 75^\circ$$

- If $A = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$, write A^{-1} in terms of A .

- If a matrix has 5 elements, write all possible orders it can have.

6. Evaluate:

$$\int (ax + b)^3 dx$$

If $R \sim \sim$:

$$\int (ax + b)^3 dx$$

7. Evaluate:

8. Write the direction-cosines of the line joining the points (1, 0, 0) and (0, 1, 1).

9. Write the projection of the vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$.

10. Write the vector equation of a line given, $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$

$$\frac{x-5}{3}$$

SECTION B

~Gf

Question numbers 11 to 22 carry 4 marks each.

JTR ~ 11 ~ 22 ncr; ~ JTR ~ 4 ~ 3fq; ; /

11. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = 10x + 7$. Find the function $g: \mathbb{R} \rightarrow \mathbb{R}$ such that $g \circ f = f \circ g = I_{\mathbb{R}}$.

OR

A binary operation $*$ on the set $\{0, 1, 2, 3, 4, 5\}$ is defined as :

$$a * b = \begin{cases} a + b, & \text{if } a + b < 6 \\ a + b - 6, & \text{if } a + b \geq 6 \end{cases}$$

Show that zero is the identity for this operation and each element 'a' of the set is invertible with $6 - a$, being the inverse of 'a'.

Let $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = 10x + 7$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ such that $g \circ f = f \circ g = I_{\mathbb{R}}$.

A binary operation $*$ on the set $\{0, 1, 2, 3, 4, 5\}$ is defined as :

$$a * b = \begin{cases} a + b, & \text{if } a + b < 6 \\ a + b - 6, & \text{if } a + b \geq 6 \end{cases}$$

Show that zero is the identity for this operation and each element 'a' of the set is invertible with $6 - a$, being the inverse of 'a'.

12. Prove that :

$$\tan^{-1} \left[\frac{1+x}{1-x} \right] = \frac{1}{2} \tan^{-1} x, \quad x \in (-1, 1)$$

or

$$\tan^{-1} \left[\frac{1+x}{1-x} \right] = \frac{1}{2} \tan^{-1} x, \quad x \in (-1, 1)$$

Using properties of determinants, solve the following for x :

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$

14. Find the relationship between 'a' and 'b' so that the function 'f' defined by:

$$f(x) = \begin{cases} \frac{1}{x} + 1, & \text{if } x \leq 3 \\ bx + 3, & \text{if } x > 3 \end{cases} \text{ is continuous at } x = 3.$$

OR

If $x^y = e^{x \cdot y}$, show that $\frac{dy}{dx} = \frac{\log x}{\{\log(xe)\}^2}$.

$$f(x) = \begin{cases} ax + 1, & \text{if } x \leq 3 \\ bx + 3, & \text{if } x > 3 \end{cases}$$

Find the relationship between 'a' and 'b' so that f is continuous at x = 3.

If $x^y = e^{x \cdot y}$, show that $\frac{dy}{dx} = \frac{1}{\{\log(xe)\}^2}$.

15. Prove that $y = \frac{4 \sin e}{(2 + \cos e)} - e$ is an increasing function in $[0, \pi]$.

OR

If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating its surface area.

$$y = \frac{4 \sin e}{(2 + \cos e)} - e, \quad [0, \pi]$$

Find the approximate error in calculating its surface area.

16. If $x = \tan \left(\frac{1}{2} \log y \right)$, show that

$$(1 + x^2) \frac{d^2y}{dx^2} + (2x - a) \frac{dy}{dx} = 0$$

~ $x = \tan \left(\frac{1}{2} \log y \right)$ ~, $m \sim$ fcfi

$$(1 + x^2) \frac{d^2y}{dx^2} + (2x - a) \frac{dy}{dx} = 0$$

17. Evaluate:

$$\int_0^{\pi/2} \frac{x + \sin x}{1 + \cos x} dx$$

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$$\int_0^{\pi/2} \frac{x + \sin x}{1 + \cos x} dx$$

18. Solve the following differential equation :

$$x dy - y dx = x^2 + y^2 dx$$

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$$x dy - y dx = x^2 + y^2 dx$$

19. Solve the following differential equation:

$$(y + 3x^2) \frac{dx}{dy} = x$$

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$$(y + 3x)^2 \frac{dx}{dy} = x$$

20. Using vectors, find the area of the triangle with vertices A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5).

~ qjf **m** ~, ~ ~ qjf ~ ~ ~ ~ ~ -wf A(1, 1,2), B(2, 3, 5)
 "(*2T C(1, 5, 5) ~ l

$$\int_{t/6}^{t/3} (1+anX)^{3l} dt$$

1Wr~~:

$$\int \frac{6x+7}{(x-5)(x-4)} dx$$

26. Sketch the graph of $y = 1x + 3$ and evaluate the area under the curve $Y = 1x + 3$ above x-axis and between $x = -6$ to $x = 0$.

$$y = 1x + 3 \quad \text{mtn} \quad \text{O} \quad y = 1x + 3 \quad x = -6 \quad x = 0$$

27. Find the distance of the point $(-1, -5, -10)$, from the point of intersection of the line $\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$.

$$\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$$

$$\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$$

28. Given three identical boxes I, II and III each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold?

$$P(\text{Gold} | \text{Box I}) = \frac{1}{2}$$

$$P(\text{Gold} | \text{Box II}) = 0$$

$$P(\text{Gold} | \text{Box III}) = \frac{1}{2}$$

A merchant plans to sell two types of personal computers - a desktop model and a portable model that will cost Rs. 25,000 and Rs. 40,000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs. 70 lakhs and his profit on the desktop model is Rs. 4,500 and on the portable model is Rs. 5,000. Make an L.P.P. and solve it graphically.

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