

Series SOS I

Code No. **65/1**  
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fR;r -;t.



Candidates must write the Code on the title page of the answer-book.  
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- Please check that this question paper contains **11** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate ..
- Please check that this question paper contains 29 questions.
- Please write down the Serial Number of the question before attempting it.
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer script during this period.
- ~ ~ ~ ~ 1% ~ ~ ~ ~ ~ **11** ~ |
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### MATHEMATICS

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Time allowed : 3 hours

Maximum Marks: 100

~.3fq;: 100

**General Instructions :**

- (i) All questions are compulsory.
- (ii) The question paper consists of **29** questions divided into three sections A, B and C. Section A comprises of **10** questions of **one mark** each, Section B comprises of **12** questions of **four marks** each and Section C comprises of **7** questions of **six marks** each.
- (iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
- (iv) There is no overall choice. However, 'internal choice' has been provided in **4** questions of **four marks** each and **2** questions of **six marks** each. You have to attempt only one of the alternatives in all such questions.
- (v) Use of calculators is **not** permitted.

(i) ~ JfR' 3lf.:rcrr4 ff /

(ii) W JfR' q;r '# **29** JfR' ~. Iiff' rft;r ?9Uif if ~ ~: 3T, tif #PJT 'fr /  
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et 3fq; q;r "! /

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(iu) l[Uf JfR' q;r '# fqq;c;q ;rff ~. / fiR '1ft rm: afrif ~ 4 **m** '# #PJT lf:"  
3fqff ~ 2 **m** '# 3f1,qRCII fqq;c;q "! / #r ~ **m** '# '# 3WFlit 7l:Cfi tr  
~ CjIBT "! /

(v) i/;<1j c12< it JPirIf ctt 31pTfrr ;r.ft. "! /

SECTION A

~31

Question numbers 1 to 10 carry 1 mark each.

Jf"R' r&IT 1 "# 10 rrq; wifq; Jf"R' 1 m q;r "ff l

1. Let  $A = \{1, 2, 3\}$ ,  $B = \{4, 5, 6, 7\}$  and let  $f = \{(1, 4), (2, 5), (3, 6)\}$  be a function from  $A$  to  $B$ . State whether  $f$  is one-one or not.

"iIRT A = {1, 2, 3}, B = {4, 5, 6, 7} 02IT11RT f = {(1, 4), (2, 5), (3, 6)}  
A ~ B 1R ~ ~ ~ l ~ ~ cp;rr f~ ~ 3=IWJJ -;r@ l

2. What is the principal value of  $\cos^{-1}(\cos 237^\circ) + \sin^{-1}(\sin 7^\circ)$ ?

$\cos^{-1}(\cos 237^\circ) + \sin^{-1}(\sin 7^\circ)$  CHT ~ IIR' cp;rr q' (...)

3. Evaluate:

$$\cos 15^\circ \sin 15^\circ$$

$$\sin 75^\circ \cos 75^\circ$$

IIR'~~:

$$\cos 15^\circ \sin 15^\circ$$

$$\sin 75^\circ \cos 75^\circ$$

4. If  $A = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ , write  $A^{-1}$  in terms of  $A$ .

5. If a matrix has 5 elements, write all possible orders it can have.

6. Evaluate:

$$\int (ax + b)^3 dx$$

If  $R \sim \sim$ :

$$\int (ax + b)^3 dx$$

7. Evaluate:

8. Write the direction-cosines of the line joining the points  $(1, 0, 0)$  and  $(0, 1, 1)$ .

9. Write the projection of the vector  $\hat{i} - \hat{j}$  on the vector  $\hat{i} + \hat{j}$ .

10. Write the vector equation of a line given  $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$

$$\frac{x-5}{3}$$

SECTION B

-Gf

Question numbers 11 to 22 carry 4 marks each.

JTR ~ 11 ~ 22 ncr; ~ JTR iii 4 3fq; ; /

11. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined as  $f(x) = 10x + 7$ . Find the function  $g: \mathbb{R} \rightarrow \mathbb{R}$  such that  $g \circ f = f \circ g = I_{\mathbb{R}}$ .

OR

A binary operation  $*$  on the set  $\{0, 1, 2, 3, 4, 5\}$  is defined as :

$$a * b = \begin{cases} a + b, & \text{if } a + b < 6 \\ a + b - 6, & \text{if } a + b \geq 6 \end{cases}$$

Show that zero is the identity for this operation and each element 'a' of the set is invertible with  $6 - a$ , being the inverse of 'a'.

Let  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = 10x + 7$  and  $g: \mathbb{R} \rightarrow \mathbb{R}$  such that  $g \circ f = f \circ g = I_{\mathbb{R}}$ .

Let  $*$  be a binary operation on the set  $\{0, 1, 2, 3, 4, 5\}$  defined as :

$$a * b = \begin{cases} a + b, & \text{if } a + b < 6 \\ a + b - 6, & \text{if } a + b \geq 6 \end{cases}$$

Show that zero is the identity for this operation and each element 'a' of the set is invertible with  $6 - a$  being the inverse of 'a'.

12. Prove that :

$$\tan^{-1} \left[ \frac{1+x}{1-x} \right] + \tan^{-1} \left[ \frac{1-x}{1+x} \right] = \frac{\pi}{4} - \frac{\pi}{2} \cos^{-1} x, \quad x \in (-1, 1)$$

or

$$\tan^{-1} \left[ \frac{1+x}{1-x} \right] + \tan^{-1} \left[ \frac{1-x}{1+x} \right] = \frac{\pi}{4} - \frac{\pi}{2} \cos^{-1} x, \quad x \in (-1, 1)$$

Using properties of determinants, solve the following for x :

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$

~RfOlcbi it ~ Cft m Cfk f.lkir-fujd c.n) x ~ ~ ~ ~

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$

14. Find the relationship between 'a' and 'b' so that the function 'f' defined by:

$$f(x) = \begin{cases} r + 1, & \text{if } x \leq 3 \\ bx + 3, & \text{if } x > 3 \end{cases} \text{ is continuous at } x = 3.$$

OR

If  $x^y = e^{x \cdot y}$ , show that  $\frac{dy}{dx} = \frac{\log x}{\{\log(xe)\}^2}$ .

~ tg;R 'f', m f(x) =

$$f(x) = \begin{cases} ax + 1, & \text{if } x \leq 3 \\ bx + 3, & \text{if } x > 3 \end{cases}$$

~, m 'a' ~ 'b' ~ iilTq Cft ~ ~ ~

3IWCff

~  $x^y = e^{x \cdot y}$  ~, m ~, fen  $\frac{dy}{dx} = \frac{1 - \log x}{\{\log(xe)\}^2}$ .

15. Prove that  $y = \frac{4 \sin e}{(2 + \cos e)} \cdot e'$  is an increasing function in  $[0, \pi]$ ,

OR

If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating its surface area.

ft.& ~ fen  $y = \frac{4 \sin e}{(2 + \cos e)} \cdot e, [0, \pi]$  il ~ ~ ~ ~

3IWCff

~ ~ ~ Cft ~ 9 W:ft lftfi ~, ~ 0.03 W:ft Cft We ~, m ~ ~

~ ~ qRCf<?H il ~"Cft2 We ~ ~

16. If  $x = \tan^{-1}(\log y)$ , show that

$$(1 + x^2) \frac{d^2y}{dx^2} + (2x - a) \frac{dy}{dx} = 0$$

~  $x = \tan^{-1}(\log y)$  ~,  $m \sim$  fcfi

$$(1 + x^2) \frac{d^2y}{dx^2} + (2x - a) \frac{dy}{dx} = 0$$

17. Evaluate:

$$\int_0^{\pi/2} \frac{x + \sin x}{1 + \cos x} dx$$

||R" ~ ~

$$\int_0^{\pi/2} \frac{x + \sin x}{1 + \cos x} dx$$

18. Solve the following differential equation :

$$x dy - y dx = x^2 + y^2 dx$$

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$$x dy - y dx = x^2 + y^2 dx$$

-'if':'

19. Solve the following differential equation:

$$(y + 3x^2) \frac{dx}{dy} = x$$

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$$(y + 3x^2) \frac{dx}{dy} = x$$

20. Using vectors, find the area of the triangle with vertices A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5).

~ qjf  $m \sim$ , ~ ~ qjf ~ ~ ~ ~ ~ -wf A(1, 1, 2), B(2, 3, 5)  
 ("12π C(1, 5, 5) ~ 1

--- ~  
 ""n1 ~n\I ~11^SI~ :

$$\int_{1/6}^{1/3} 1 + \sqrt{x} \, dx$$

1Wr~~:

$$\int \frac{6x + 7}{(x - 5)(x - 4)} \, dx$$

26. Sketch the graph of  $y = 1x + 31$  and evaluate the area under the curve  $Y = 1x + 31$  above x-axis and between  $x = -6$  to  $x = 0$ .

$y = 1x + 31$  CfiT mtn ~  $O \sim$  Cfsifi  $y = 1x + 31 \sim x \sim \sim \sim x = -6 \sim x = 0$  CliCfiT ~~~ I

27. Find the distance of the point  $(-1, -5, -10)$ , from the point of intersection of the line  $\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$  and the plane  $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$ .

$\sim (-1, -5, -10)$  ctt  $\sim \vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k}) \sim \vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$  q. J'R1-G ~ ~ ~ ~ ~ I

28. Given three identical boxes I, II and III each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold ?

"ffl:f ~ ~ I, II am: III ~ ~ ~ ~ "GT ~ ~ I ~ I~, "GRT ~  
 m ~ ~, ~ II~, ~ ~ ~ ~ am: ~ III ~, ~ ~ CfiT ~  
 ~ CfiT ~ t I ~ "&Tfui IIIi:0111 ~ ~ %ffiT t 3ffi: ~ ~ ~ fuCf&iT  
 ACfil<1('11 t i ~ ~ ~ m CfiT, m ~ J'11-Cfi('11 t, fij; ~ ~ ~ fuCf&iT  
 '4t m CfiT ?

A merchant plans to sell two types of personal computers - a desktop model and a portable model that will cost Rs. 25,000 and Rs. 40,000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs. 70 lakhs and his profit on the desktop model is Rs. 4,500 and on the portable model is Rs. 5,000. Make an L.P.P. and solve it graphically.

Let the number of desktop computers be  $x$  and the number of portable computers be  $y$ .

Objective function: Maximize  $Z = 4500x + 5000y$

Constraints:

$$25000x + 40000y \leq 700000$$

$$x + y \leq 250$$

$$x \geq 0, y \geq 0$$

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