

Series: SSO/1

Code.No. **6511/1**

Roll No.

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Candidates must write the Code on the title page of the answer-book.

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- Please check that this question paper contains 8 printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 29 questions.
- Please write down the serial number of the question before attempting it.
- 15 Minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the student will read the question paper only and will not write any answer on the answer script during this period.
- ~-CR~fcnw~-q;r.q~"T08t I
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MATHEMATICS

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Time allowed: 3 hours I
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[Maximum marks: 100
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General Instructions :

- All questions are compulsory.
- The question paper consists of 29 questions divided into three Sections A, Band C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.
- All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question. .
- There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
- Use of calculators is not permitted.

- (i) $"R"It JIR a\#tncf \sim I$
- (ii) $W JIR-q;\{ '429 JIR \sim ;;ff r\{t:((gugf '4 \sim \sim : 3T, 'if, (fP\{I r\{I \sim 3T '410 JIR \sim f.ir;rlf "\# \sim rp\#) .3fq; CfiT \mathbf{t} I \sim (if '412 JIR \sim f.ir;rlf "\# \sim Tm" .3fq; CfiT \mathbf{t} I \sim rr '4 7 JIR \sim f.ir;rlf "\# \sim 'f\$: .3fq; CfiT \mathbf{t} I$
- (iii) $\sim 3T '4 "R"It \mathbf{m} cl; "3fiT rp\#) \sim rp\#) CfiTikT 3\{PIqf JIR qft 311C1:t<1Cfi(1f \sim \sim ifT \mathbf{N} \rightsquigarrow$
- (iv) $rp\{ JIR q;\{ '4 \sim ::rtf \sim I \mathbf{m} \sim Tm" 3fqff CfiTff 4 \mathbf{m} '4 (fP\{I 'f\$: 3fqff CfiTff 2 \mathbf{m} '43fi"dlRCfi \sim \sim I \mathbf{\#} \sim \mathbf{m}'4" \# 3TTfiCfiT rp\#) tt \sim cnRT \mathbf{t} I$
- (v) $4lH1H2 < cl;wWrqft \sim o:rft I$

SECTION-A

~3T

Question numbers 1 to 10 carry one mark each.

~~1 ~10(fCf;~~~3lq;cnrt I

- Find the projection of \mathbf{i} on \mathbf{l} if $\mathbf{i} \cdot \mathbf{l} = 8$ and $\mathbf{l} = 2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$.
 $\frac{-7}{a} \frac{-:-7}{b} IR \sim m \sim \frac{-7}{a \cdot b} = 8om \quad \frac{-:-7}{b} = 2i^{\wedge} + 6j^{\wedge} + 3kt^{\wedge} I$
- Write a unit vector in the direction of $\mathbf{l} = 2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$.
 $\sim \frac{-7}{a} = 2i^{\wedge} - 6j^{\wedge} + 3k^{\wedge} Cfft \sim ll"QCfi \sim \sim I$
- Write the value of p for which $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} + 9\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + p\mathbf{j} + 3\mathbf{k}$ are parallel vectors.
- If matrix $A = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$, write AA' , where A' is the transpose of matrix A .
 $\sim \sim A = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix} tm \sim AA' \sim \sim A', \sim A cnr - qft cfflt I$
- Write the value of the determinant

$$\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$$

$$\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix} \quad cnr llR \sim I$$

$$\sin^{-1}(\sin 3) \\ \sim \text{IIR Cf;TWWr Cjij f.r.r;r Cf;Tt:fR} \sim \sim \quad | \\ \sin^{-1}(\sin 3)$$

7. Evaluate: $\int \frac{\sec 2x}{3 + \tan x} dx$.

"t:fR"~~: $\int \frac{\sec^2 x}{3 + \tan x} dx.$

8. If $\int (3x^2 + 2x + k) dx = 0$, find the value of k.

$$\int_0^1 (3x^2 + 2x + k) \, dx = 0 \quad \text{Find } k \text{ if } T: \mathbb{R} \rightarrow \mathbb{R} \quad \sim$$

9. If the binary operation $*$ on the set of integers \mathbb{Z} , is defined by $a * b = a + 3b^2$, then find the value of $2 * 4$.

$$\sim \text{it} \sim \sim^* \sim \sim \text{'Wl-c;q;q} \text{ Z } 1\text{R} \sim \sim \sim \sim \text{fcf; a}^* \text{ b} = \text{a} + 3\text{b}^2 \sim, \text{'ffi}$$

10. If A is an invertible matrix of order 3 and $|A| = 5$, then find $|\text{adj. } A|$.

~ A ~py;&lJ40n4 ~t~Cfi1fc 3~, ~ |A|=5~m 1adj. A|~~ I

SECTION -B

 $\sim\text{-if}$

Question number 11 to 22 carry 4 marks each.

~~11 tt22(fCf;~~4~Cf;Tt

11. If $\overline{ax} \parallel \overline{b}$ and $\overline{ax} \parallel \overline{c}$, show that $\overline{a} - \overline{d}$ is parallel to $\overline{b} - \overline{c}$.

where a, d and b are constants.

$$\sim \frac{-t}{a} \times \frac{-7}{b} = \frac{-t}{c} \times \frac{-t}{d} \sim \frac{-t}{a} \times \frac{-t}{c} = \frac{-7}{b} \times \frac{-t}{d} \quad \mathbf{t}_{m \sim fcf}; \quad \frac{-t}{a} - \frac{-t}{d} \sim \frac{-7}{b} - \frac{-t}{c}, \sim$$

$$\sim t, \sim \quad \begin{array}{cc} \text{---t} & \text{---t} \\ \text{a} & \text{d} \end{array} \sim \quad \begin{array}{cc} \text{---7---} & \text{---} \\ \text{b} & \text{c} \end{array} \sim \text{I}$$

12. Prove that: $\sin^{-1}(\sin x) + \sin^{-1}(\sin 3x) + \sin^{-1}(\sin 5x) = \pi$

OR

Solve for x: $\tan^{-1} 3x + \tan^{-1} 2x = \frac{\pi}{4}$

Ans: $\sin^{-1}(\sin x) + \sin^{-1}(\sin 3x) + \sin^{-1}(\sin 5x) = \pi$,

~

13. Find the value of λ so that the lines

$\frac{x-1}{-3} = \frac{y-7}{2} = \frac{z-10}{11}$ and $\frac{x-7}{1} = \frac{y-6}{-5} = \frac{z-1}{-5}$

are perpendicular to each other.

14. Solve the following differential equation:

$$\frac{dy}{dx} + y = \cos x - \sin x.$$

Ans: $y = \cos x - \sin x + \frac{1}{2} \sin 2x$

$$\frac{dy}{dx} + y = \cos x - \sin x.$$

15. Find the particular solution, satisfying the given condition, for the following differential equation :

$y'' + y = 0$; $y = 0$ when $x = 1$.

Ans: $y = \sin(x-1)$

$y'' + y = 0$; $y = 0$ if $x = 1$.

16. By using properties of determinants, prove the following:

$$\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$$

17. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.

$$\frac{1}{6} \times \left(\frac{5}{6}\right)^5 = \frac{5^5}{6^6}$$

18. Differentiate the following function w.r.t. x :

$$x^{\sin x} + (\sin x) \cos x$$

19. Evaluate: $\int_{-5}^5 e^{x-2} dx$.
- OR

$$\int \frac{(x-4)e^x}{(x-2)^3} dx$$

$$\int \frac{f(x-4)e^x}{(x-2)^3} dx$$

20. Prove that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : [a - b] \text{ is even}\}$, is an equivalence relation.

$$A = \{1, 2, 3, 4, 5\}, R = \{(a, b) : [a - b] \text{ is even}\}$$

21. Find y if $(x^2 + y^2)^2 = xy$.

OR

If $y = 3 \cos(\log x) + 4 \sin(\log x)$, then show that $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$

$$(x^2 + y^2)^2 = xy \implies x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$$

$$y = 3 \cos(\log x) + 4 \sin(\log x), \quad x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$$

22. Find the equation of the tangent to the curve $y = 3x^2 - 2$ which is parallel to the line $4x - 2y + 5 = 0$.

OR

Find the intervals in which the function f given by $f(x) = x^3 + \frac{3}{x}$, $x \neq 0$ is

(i) increasing (ii) decreasing.

$$y = 3x^2 - 2 \quad \text{COT} \quad 4x - 2y + 5 = 0$$

$$f(x) = x^3 + \frac{3}{x}, \quad x \neq 0, \quad (i) \text{ increasing } (ii) \text{ decreasing}$$

SECTION-C

Question number 23 to 29 carry 6 marks each.

23 to 29 carry 6 marks each.

23. Find the volume of the largest cylinder that can be inscribed in a sphere of radius r .

OR

A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m^3 . If building of tank costs Rs. 70 per sq. metre for the base and Rs. 45 per sq. metre for sides, what is the cost of least expensive tank?

$$\text{COT} \quad r \text{ m} \quad 1 \text{ ft} \quad t$$

$$14 \text{ m} \times 3 \text{ m} \times 2 \text{ m} \quad 8 \text{ m}^3 \quad 70 \text{ Rs/m}^2 \quad 45 \text{ Rs/m}^2$$

24. A diet is to contain at least 80 units of Vitamin A and 100 units of minerals. Two foods F1 and F2 are available. Food F1 costs Rs. 4 per unit and F2 costs Rs. 6 per unit. One unit of food F1 contains 3 units of Vitamin A and 4 units of minerals. One unit of food F2 contains 6 units of Vitamin A and 3 units of minerals. Formulate this as a linear programming problem and find graphically the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements.

Let x_1 and x_2 be the units of food F1 and F2 respectively. Then the linear programming problem can be formulated as follows:

$$\text{Minimize } Z = 4x_1 + 6x_2$$

$$\text{Subject to } 3x_1 + 6x_2 \geq 80$$

$$4x_1 + 3x_2 \geq 100$$

$$x_1 \geq 0, x_2 \geq 0$$

25. Three bags contain balls as shown in the table below:

| Bag | Number of White balls | Number of Black balls | Number of Red balls |
|-----|-----------------------|-----------------------|---------------------|
| I | 1 | 2 | 3 |
| II | 2 | 1 | 1 |
| III | 4 | 3 | 2 |

A bag is chosen at random and two balls are drawn from it. They happen to be white and red. What is the probability that they came from the III bag?

Let P_1, P_2, P_3 be the probabilities of choosing bags I, II, and III respectively. Then

$$P_1 = \frac{1}{6}, P_2 = \frac{1}{6}, P_3 = \frac{1}{3}$$

| | | | |
|-----|---|---|---|
| I | 1 | 2 | 3 |
| II | 2 | 1 | 1 |
| III | 4 | 3 | 2 |

Let $P_{1W}, P_{1R}, P_{2W}, P_{2R}, P_{3W}, P_{3R}$ be the probabilities of drawing a white ball and a red ball from bags I, II, and III respectively. Then

26. Using matrices, solve the following system of equations:

$$2x - 3y + 5z = 11$$

$$3x + 2y - 4z = -5$$

$$x + y - 2z = -3$$

Let $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, and $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$. Then the system of equations can be written as $AX = B$.

$$2x - 3y + 5z = 11$$

$$3x + 2y - 4z = -5$$

$$x + y - 2z = -3$$

27. Evaluate: $\int_0^{7\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx$

OR

Evaluate: $\int_0^{7\pi} (2 \log \sin x - \log \sin 2x) dx$

Ans: $\int_0^{7\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx$

Ans: $\int_0^{7\pi} (2 \log \sin x - \log \sin 2x) dx$

28. Using the method of integration, find the area of the region bounded by the lines

$$2x + y = 4, 3x - 2y = 6 \text{ and } x - 3y + 5 = 0$$

Ans

$$2x + y = 4, 3x - 2y = 6 \text{ and } x - 3y + 5 = 0$$

Ans

29. Find the equation of the plane passing through the point $(-1, 3, 2)$ and perpendicular to each of the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.

Ans: $(-1, 3, 2)$ is a point on the plane. The normal vector to the plane is the cross product of the normal vectors of the two planes. The normal vector of the plane $x + 2y + 3z = 5$ is $\vec{n}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$ and the normal vector of the plane $3x + 3y + z = 0$ is $\vec{n}_2 = 3\hat{i} + 3\hat{j} + \hat{k}$. The cross product of \vec{n}_1 and \vec{n}_2 is $\vec{n} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & 3 & 1 \end{vmatrix} = \hat{i}(2 \cdot 1 - 3 \cdot 3) - \hat{j}(1 \cdot 1 - 3 \cdot 9) + \hat{k}(1 \cdot 3 - 2 \cdot 9) = -8\hat{i} + 26\hat{j} - 15\hat{k}$. The equation of the plane is $-8(x + 1) + 26(y - 3) - 15(z - 2) = 0$, which simplifies to $-8x + 26y - 15z = 1$.

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