

Series SSR/11

Code No. **6511/1**
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Candidates must write the Code on the title page of the answer-book.

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- Please check that this question paper contains 11 printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 29 questions.
- Please write down the Serial Number of the question before attempting it.
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MATHEMATICS

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Time allowed : 3 hours J

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General Instructions :

1. 1/ questions are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each. Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of 6 marks each.
3. All questions in Section B are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However; internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each.

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you have 10 attempt only one of the alternatives in all such questions.

5. *Use of calculators is not permitted,*

[P.T.O.

$R_{IIII}^4 f\#^w :$

1. $R^t JII R .3Jf.}i;1/4 \sim I$
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Section A

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1. If $\{x\} = x + 7$ and $g(x) = x - 7, x \in \mathbb{R}$, find $(f \circ g)(7)$
 $\sim \{x\} = x + 7$ if $g(x) = x - 7, x \in \mathbb{R}$. ill $(f \circ g)(7) = m0 \sim I$

2. Evaluate $i \sin [;-Sln-I(-\sim)J$

3. Find the value of X and Y if $2[\sim$
 $XCTmy \sim \sim m \sim \sim :2[\sim$
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4. Evaluate: $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

Ans: $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix} = (a+ib)(a-ib) - (c+id)(-c+id)$

5. Find the co-factor of M_{22} in the following:

$$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 5 & 7 & 1 \end{vmatrix}$$

Ans: If M_{22} is the element in the 2nd row and 2nd column, then the co-factor is $(-1)^{2+2} \times \det \begin{vmatrix} 2 & 5 \\ 5 & 1 \end{vmatrix} = 1 \times (2 \times 1 - 5 \times 5) = 1 \times (2 - 25) = -23$

$$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

6. Evaluate: $\int_0^1 \frac{x^2}{1+x^2} dx$

7. Evaluate: $\int_0^1 \frac{dx}{1+x^2}$

Ans: $\int_0^1 \frac{dx}{1+x^2} = \left[\tan^{-1} x \right]_0^1 = \tan^{-1} 1 - \tan^{-1} 0 = \frac{\pi}{4} - 0 = \frac{\pi}{4}$

8. Find a unit vector in the direction of $\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

Ans: $\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

Unit vector $\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{3\hat{i} - 2\hat{j} + 6\hat{k}}{\sqrt{3^2 + (-2)^2 + 6^2}} = \frac{3\hat{i} - 2\hat{j} + 6\hat{k}}{\sqrt{9 + 4 + 36}} = \frac{3\hat{i} - 2\hat{j} + 6\hat{k}}{\sqrt{49}} = \frac{3\hat{i} - 2\hat{j} + 6\hat{k}}{7}$

9. Find the angle between the vectors $a = i - j + k$ and $b = i + j - k$

$$a = i - j + k \quad \text{and} \quad b = i + j - k$$

10. For what value of P are the vectors $a = 2i + pj + k$ and $b = i - 2j + 3k$ perpendicular to each other?

$$a = 2i + pj + k$$

$$b = i - 2j + 3k$$

Section B

- II. (i) Is the binary operation $*$, defined on set N given by $a * b = \frac{a+b}{2}$ for all $a, b \in N$, commutative?
- (ii) Is the above binary operation $*$ associative?

12. Prove the following:

$$\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{8} = \frac{\pi}{4}$$

15. Differentiate the following with respect to x :

$$\frac{d}{dx} \left(\frac{x^2 - 1}{x^2 + 1} \right)$$

16. Find the equation of tangent to the curve $x = \sin 3t, y = \cos 2t$ at $t = \pi/4$

$$\text{Cfif } x = \sin 3t, y = \cos 2t \text{ at } t = \pi/4 \text{ then } \frac{dy}{dx} = \frac{-2 \sin 2t}{3 \cos 3t} = \frac{-2 \sin(\pi/2)}{3 \cos(3\pi/4)} = \frac{-2}{3 \cdot (-\frac{1}{\sqrt{2}})} = \frac{2\sqrt{2}}{3}$$

17. Evaluate : $\int_0^{\pi/2} x \sin x \cos^2 x \, dx$

$$\int_0^{\pi/2} x \sin x \cos^2 x \, dx$$

18. Solve the following differential equation ;

$$(x^2 - 1) \frac{dy}{dx} + 2xy = 0$$

given that $y = 1$ when $x = 1$

OR

Solve the following differential equation:

$$\frac{dy}{dx} = x(2 - x), \text{ if } y = 1 \text{ when } x = 1$$

$$f(x, y) = \frac{1}{2} (x^2 + y^2) \quad \text{and} \quad \nabla f(x, y) = (x, y)$$

$$(x^2 - y^2) dx + 2xy dy = 0$$

$$x^2 + y^2 = 1$$

$$\frac{dy}{dx} = \frac{x(2y-x)}{x(2y+x)}$$

19. Solve the following differential equation :

$$\cos^2 x \frac{dy}{dx} = \tan x$$

$$\cos^2 x \frac{dy}{dx} = \tan x$$

$$\cos^2 x \frac{dy}{dx} = \tan x$$

20. If $\vec{a} = i + j + k$ and $\vec{b} = j - k$, find a vector \vec{c} such that $\vec{a} \times \vec{c} = \vec{b}$

$$\vec{a} \cdot \vec{b} = 3$$

OR

\vec{a} and \vec{b} is 60° ,

$$\rightarrow \rightarrow$$

$$G.C = 3$$

21. Find the shortest distance between the following lines:

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \quad \text{and} \quad \frac{x+1}{7} = \frac{y-1}{-Q} = \frac{z+11}{1}$$

Or

The point on L_C is $\frac{1'+2}{5} = \frac{1'+1}{-2} = \frac{z-3}{2}$ at a distance 3, 2 from the point (1, 1, 1)

f.rq ~ ?li ofrq ~ ~ -Wa ~ :

$$\frac{1:-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \quad (1m \frac{x+1}{7} = \frac{y-1}{-Q} = \frac{z+11}{1})$$

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$$\sim .9T \frac{x+1}{3} = \frac{y+1}{2} = \frac{z-3}{1} \quad \text{en: } \sim \sim \quad \text{WO, f,rl~fl.J,} \sim \sim \quad (J. 2. 3) \sim ' 3 // \text{ ofit}$$

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22. A pair of dice is thrown 4 times, [1 getting a doublet is considered a success. find the probability distribution of number of successes.

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Section C

23. Using properties of determinants prove the following:

$$\begin{vmatrix} x & y & z \\ y & z & x \\ z & x & y \end{vmatrix} = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$

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$$\begin{vmatrix} x & y & z \\ y & z & x \\ z & x & y \end{vmatrix} = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$

24. Show that the rectangle of maximum area that can be inscribed in a circle is a square.

OR

Show that the height of the cylinder of maximum volume that can be inscribed in a cone of height h is $\frac{2}{3}h$.

$$\begin{vmatrix} x & y & z \\ y & z & x \\ z & x & y \end{vmatrix} = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$

25. Using integration find the area of the region bounded by the parabola $y^2 = 4x$ and the circle $x^2 + y^2 = 9$.

26. Evaluate $\int_0^1 \frac{dx}{x^2 + y}$

For $M = \frac{a^2 x}{a+x}$

27. Find the equation of the plane passing through the point $(-1, -1, 2)$ and perpendicular to each of the following planes;

$$2x + 3y - z = 2 \text{ and } 5x - 4y = 6$$

OR

Find the equation of the plane passing through the points $(3, 4, 1)$ and $(0, 1, 0)$

and parallel to the line $\frac{x+3}{-2} = \frac{y-3}{-7} = \frac{z-2}{5}$

For the line $\frac{x+3}{-2} = \frac{y-3}{-7} = \frac{z-2}{5}$ find the equation of the plane passing through the point $(-1, -1, 2)$ and perpendicular to each of the following planes;

$$2x + 3y - z = 2 \text{ and } 5x - 4y = 6$$

For the line $\frac{x+3}{-2} = \frac{y-3}{-7} = \frac{z-2}{5}$ find the equation of the plane passing through the points $(3, 4, 1)$ and $(0, 1, 0)$ and parallel to the line

$$\frac{x+3}{-2} = \frac{y-3}{-7} = \frac{z-2}{5}$$

28. A factory owner purchases two types of machines. A and B. The requirements and the limitations for the machines are as follows:

Machine	Area occupied	Labour force	Daily output (in units)
A	1000 m ²	12 men	60
B	1200 m ²	8 men	40

He has maximum area of 9000 m² available. and 72 skilled labourers who can operate both types of machines. How many machines of each type should he buy to maximise the daily output?

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B	1200 crT lfl	Rixffu;	40

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29. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a scooter, a car and a truck are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver,

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