B. Tech.

Second SEMESTER EXAMINATION, 2008-09

Engineering Physics-II

Note : This question paper contains three sections.

Section : A

Q. 1. Attempt all parts. All parts carry equal marks.

(a) The wave nature of material particles was first proposed by

Ans. de-Borglie

(b) In Compton Effect, a photon scattered at right angle to the incident direction, the Compton shift will be

Ans. 0.242 A°

Pick the correct choice from following :

(c) The quantized energy of a particle of mass m in a one dimensional box of length L is given by

(i) $\frac{n^2 h^2}{2mL^2}$ (ii) $\frac{n^2 h^2}{8mL^2}$

(iii) $\frac{n^2 \pi^2 h^2}{2mL^2}$ (iv) $\frac{n^2 h^2}{8\pi^2 mL^2}$

Ans. (ii) $\frac{n^2 h^2}{8mL^2}$

(d) In dielectrics, the polarization is

(i) Linear function of applied electric field

(ii) Square function of applied electric field

(iii) Exponential function of applied electric field

(iv) None of the above

Ans. (i) Linear function of applied electric field

(e) The Clausius-Mossotti relation is

(i) $\frac{\varepsilon_r - 1}{\varepsilon_r + 1} = \frac{N\alpha}{3\varepsilon_0}$ (ii) $\frac{\varepsilon_r + 1}{\varepsilon_r - 2} = \frac{N\alpha}{3\varepsilon_0}$

(iii) $\frac{\varepsilon_r + 1}{\varepsilon_r - 1} = \frac{3N\alpha}{\varepsilon_0}$ (iv) $\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha}{3\varepsilon_0}$

Ans. (iv) $\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha}{3\varepsilon_0}$

(f) Ultrasonic wave can be detected by

(i) a telephone

(ii) Quincle's method

(iii) Kundt's method

(iv) Hebb's method

Ans. (iii) Kundt's method

(g) Statement that displacement current between the plates of a capacitor

(i) flows when charge decreasing on the plates

(ii) flows when charge increasing on the plates

(iii) flows when charge remain constant on the plates

(iv) flows when no charge on the plates

Ans. (i) flows when charge decreasing on the plates

Choose the correct on :

(i) Statements (I) and (II) are correct

(ii) Statement (II) is correct

(iii) Statements (III) and (IV) correct

(iv) Statement (IV) is correct

Ans. (i)Statements (I) and (II) are correct

(h) The ratio of electric field E and magnetic field H has the dimension of

(i) Power (ii) Resistance

(iii) Inductance (iv) Capacitance

Ans. (ii) Resistance

(i) Hard superconductors observe

(i) breakdown of Silsbee's rule

(ii) incomplete Meissner effect

(iii) high critical field and transition temperature

(iv) all the above

Ans. (iv) all the above

(j) The Chemical Vapours Deposition is a techniques in nanotechnology for

(i) determination of the size of nanoparticles

(ii) identification of nanoparticles

(iii) characterization of nanoparticles

(iv) synthesis of carbon nanotubes.

Ans. (iv)synthesis of carbon nanotubes.

Section : B

Q.2 Attempt any three parts. All parts carry equal marks :

(a) The kinetic energy of an electron is 4.55×10^{-25}. Calculate velocity, momentum and the wavelength of the electron.

Ans. Given K.E. $= 4.55 \times 10^{-25}$ J

mass of electron $m_e = 9.1 \times 10^{-31}$ kg

we know that KE $= \frac{1}{2} mv^2$

$$\therefore \quad v = \sqrt{\frac{2kE}{m}} = \sqrt{\frac{2 \times 4{\cdot}55 \times 10^{-25}}{9{\cdot}1 \times 10^{-31}}}$$

$$\therefore \quad p = mv = 9.1 \times 10^{-31} \times 10^3$$

$$= 9.1 \times 10^{-28} \ kg\text{-}m/s$$

$$\lambda = \frac{h}{mv} = \frac{6.62 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^3} = 7.28 \times 10^{-7} \ m$$

$$= 7280 \times 10^{-10} m = 7280 Å$$

(b) Calculate the uncertainty in the position of a dust particle with mass equal to 1 mg if uncertainty in its velocity is 5.5×10^{-20} m/s.

Ans. Given m = 1kg, $= 10^{-3} g = 10^{-6} kg$

uncertainty in velocity, $\Delta v = 5.5 \times 10^{-20} \ m/s$

we know that $\Delta x . \Delta p = \frac{h}{4\pi}$

or $\quad \Delta x \times m \Delta v = \frac{h}{4\pi}$

or $\quad \Delta x = \frac{h}{4\pi m \Delta v}$

$$= \frac{6.6 \times 10^{-34}}{4 \times 3.14 \times 10^{-6} \times 5.5 \times 10^{-20}}$$

$$= 9.58 \times 10^{-10} \text{m}$$

(c) The dielectric constant of helium at 0°C and 1 atmospheric pressure is 1.000074. Find the dipole moment induced in helium atom when the gas is in an electric field of intensity 100 volt/m. Number of atoms per unit volume of helium gas are 2.68×10^{27}.

Ans. Given $\varepsilon_r = 1.000074$

E = 100 volt/m

$N = 2.68 \times 10^{27}$

also we know that $\varepsilon_0 = 8.85 \times 10^{-12}$

we know that dipole moment,

$$p = \frac{\varepsilon_0 (\varepsilon_r - 1) E}{N}$$

$$= \frac{8.85 \times 10^{-12} \times (1.000074 - 1) \times 100}{2.68 \times 10^{-27}}$$

$$= 24.44 \times 10^{-42} \text{ C-m}$$

(d) The permeability, permittivity and conductivity of aluminium are $\mu_r = 1, \varepsilon_r = 1$ and $\sigma = 3.54 \times 10^7$ mho/m. Find the skin depth if the wave enter in aluminium with frequency of 71.56 MHz.

Ans. Given $\mu_r = 1$

$\varepsilon_r = 1$

$\sigma = 3.54 \times 10^7 \ mho/m$

$f = 71.56$ MHz

$\mu_0 = 4\pi \times 10^{-7}$

we know that skin depth,

$$\delta = \sqrt{\frac{2}{\mu\sigma\omega}}$$

$$= \sqrt{\frac{2}{\mu_0\, u_r\, \sigma 2\pi f}}$$

$$= \sqrt{\frac{2}{4\pi\times 10^{-7}\times 3{\cdot}54\times 10^{7} \times 2\pi\times 71{\cdot}56\times 10^{3}}}$$

$$= 10\,\mu m$$

(e) A superconducting material has a critical temperature of 3.7°K in zero magnetic field and a critical field of 0.0306 Tesla at 0°K Find the critical field at 2°K.

Ans. Given : $T_c = 3.7° K$

$T = 2° K$

$H_0 = 0.0306$

$H_C = ?$

we know that,

$$H_c = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

$$= 0.0306\left[1-\left(\frac{2}{3.7}\right)^2\right]$$

$$= 0.0217 \text{ Tesla}$$

Section : C

Note : Attempt all questions. All questions carry equal marks.

Q.3 Attempt any one part of the following :

(a) Distinguish between group velocity (V_g) and phase velocity (V_p) of a wave packet and show that $V_p V_g = C^2$.

Ans. we know that phase velocity

$$v_p = \frac{\omega}{k} \quad \text{... (1)}$$

and group velocity

$$v_g = \frac{d\omega}{dk} \quad \text{.... (2)}$$

also $k = \dfrac{2\pi}{\lambda}$ & $\lambda = \dfrac{h}{mv}$

$$\therefore \quad K = \frac{2\pi mv}{h} \quad \text{....(3)}$$

Now, $E = h\nu$

or $\nu = \dfrac{E}{h}$

and $\omega = 2\pi\nu = \dfrac{2\pi E}{h} = \dfrac{2\pi mc^2}{h}$ $[\because E = mc^2]$

Thus using eqn. (3) & eqn. (4) eqn. (1) becomes,

$$v_p = \frac{2\pi mc^2}{h}\times\frac{h}{2\pi mv} = \frac{c^2}{v} = \frac{c^2}{v_g}$$

$[\because v = v_g]$

$\therefore v_p v_g = c^2$ Hence proved

Difference between group velocity & phase velocity

Phase velocity is the velocity of individual wave while group velocity is the velocity of wave packet.

(b) Derive time dependent Schrodinger wave equation.

Ans. The time independent Schrodinger equation is given by

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0 \quad \text{...(1)}$$

Time dependent Schrodinger equation may be obtained by eliminating E from equation (1)

The wave function is

$$\psi(r,t) = \psi_0(r)e^{-L\omega t} \quad \text{...(2)}$$

$$\therefore \quad \frac{\partial\psi}{\partial t} = -L\omega\psi_0(r)e^{-L\omega t}$$

$$= -i(2\pi\nu)\psi_0(r)e^{-L\omega t}$$

$$= -2\pi i\nu\,\psi(r) = -2\pi i\frac{E}{\hbar}\psi$$

$$= -\frac{LE}{\hbar}\psi\times\frac{i}{L} = \frac{E\psi}{2\hbar}$$

or $$E\psi = 2\hbar\frac{\partial\psi}{\partial t} \quad \text{...(3)}$$

Substituting this in equation (1)

$$\nabla^2\psi + \frac{2m}{\hbar^2}\left[2\hbar\frac{\partial\psi}{\partial t} - V\dot{\psi}\right]=0$$

or $$\nabla^2\psi = -\frac{2m}{\hbar^2}\left[2\hbar\frac{\partial\psi}{\nabla t} - V\psi\right]$$

or $$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = 2\hbar\frac{\partial\psi}{\partial t} \quad \ldots(4)$$

or $$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = 2\hbar\frac{\partial\psi}{\partial f}$$

This is the time dependent Schrodinger equation

or $$H\psi = E\psi$$

where H is the Hamiltonean

Q. 4. Attempt any one part of the following :

(a) Describe Bragg's spectrometer and explain how it is used to study the structure of crystals.

Ans. In 1924, locus de-Broglie thought that the matter must possess dual character like light. The dual character of matter means that it is Prestent in the form of a particle as well as it in wave like characters this is known as duality of matter. According to him a moving particle is surrounded by a wave whose wavelength depends upon the mass of the particle and its velocity. Such type of waves associated with the matter particles are known as "matter waves".

According to the de-Broglie-hypothesis, every fast moving small particle has a wave packet associated with it the wavelength of such waves depends upon the momentum of the particle. As we know

$$E = h\nu \text{ and } E = mc^2$$

$$\Rightarrow \quad h\nu = mc^2$$

$$\frac{hc}{\lambda} = mc^r$$

$$\lambda = \frac{h}{mc}$$

for material particles

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

Now we know

$$\text{kinetic energy (K.E.)} = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2v^2}{m}$$

$$E = \frac{p^2}{2m}$$

$$\Rightarrow \quad p = \sqrt{2mE}$$

so de-Broglie wavelength

$$\lambda = \frac{h}{\sqrt{2mE}}$$

(b) What do you mean by polarization of a substance ? Write different machanism of polarization in a dielectric.

Ans. Polarization of Dielectric Materials : Polarization occurs due to different atomic mechanism. If the specimen is kept in a dc electric field, polarization is because of following types of processes :

(i) Electronic polarization

(ii) Ionic polarization

(iii) Orientation polarization

and (iv) space change polarization.

(i) Electric Polarization : The displacement of the positively changed nucleus and the negative electrons of an atom in reverse directions, on applying an electric field, it results in electronic polarization. On applying a field, the electron cloud around the nucleus readily shifts towards, the positive end of the applied field. Since the nucleus and the centre of the electron cloud are separated by a definite distance, dipole moment is created within each atom. The extent of this shift is directly proportional to the field strength. As the dipole moment is the product of the change and the shift distance, dipole moment is directly proportional to the field strength. Thus

induced dipole moment,

$$\mu \propto E$$

or $$\mu = \alpha_e E \quad \ldots(1)$$

where α_e is known as electronic polariziability. It is independent of temperature.

A simplified classical model of an atom is given in fig-a. In this fig-a. the nucleus of charge Z_e is surrounded from an electronic cloud of charge-Z_e distributed in a sphere of radius R. The change density ρ is

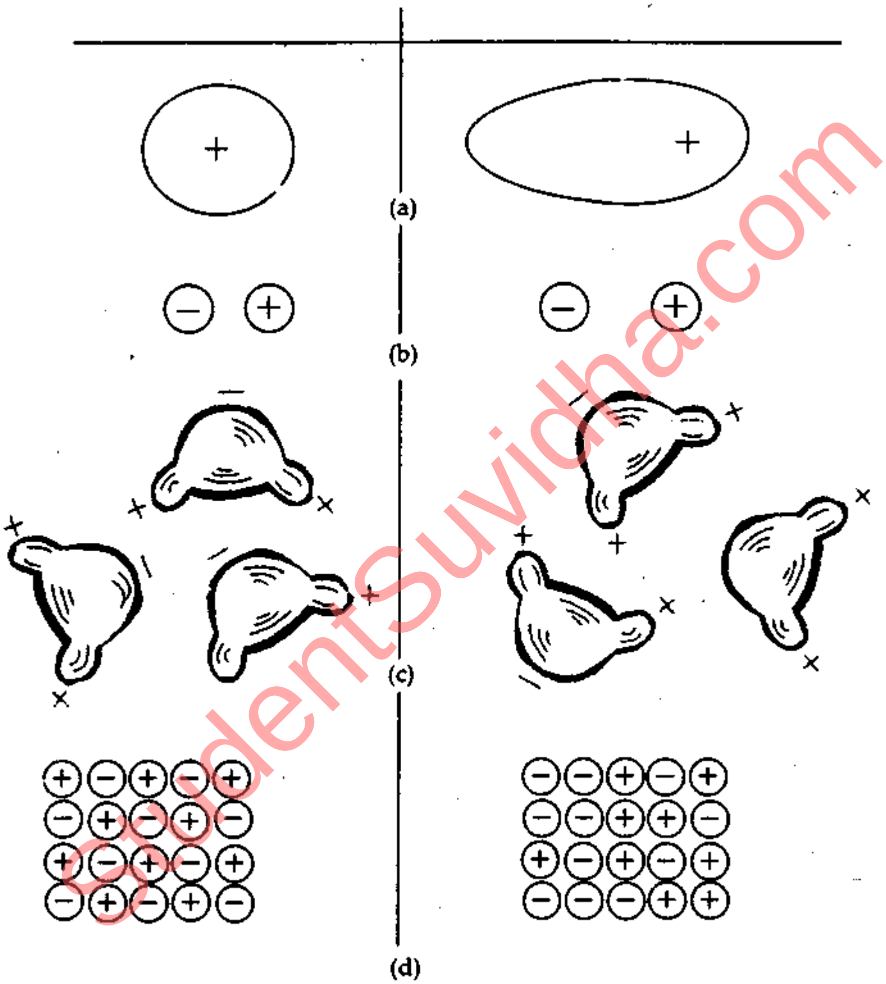

Fig. Various polarization processes : (a) electronic polarization, (b) ionic polarization (c) orientation polarization, and (d) space charge polarization

$$\rho = \frac{-Ze}{4/3\,\pi R^3} = -\frac{3}{4}\left(\frac{Ze}{\pi R^3}\right) \qquad \ldots(2)$$

If an external field having intensity E is applied, the nucleus and the electrons experience horentz forces of magnitude ZeE in reverse direction. Thus the nucleus and electron cloud are pulled apart. If they are separated, a coulob force develops the horentz force and coulomb force are equal and opposite. Thus equilibrium is reached and suppose the displacement under that condition is x.

As nucleus is much heavier than the electron cloud, it is considered that were the electron cloud is displaced if external field is applied.

$$\text{Lorentz force} = -ZeE$$

Coulomb force

$$= Ze \times \left[\frac{\text{charge enclosed in the sphere of radius } x}{4\pi\varepsilon_0 x^2}\right]$$

$$= Ze \times \left[\frac{4\pi x^3 \rho}{4\pi\varepsilon_0 x^2}\right]$$

$$= \frac{Ze}{4\pi\varepsilon_0 x^2}\left[4\pi x^3\left\{-\frac{3}{4}\left(\frac{Ze}{\pi R^3}\right)\right\}\right]$$

[Using eqn. (2)]

$$= -\frac{Z^2e^2x}{4\pi\varepsilon_0 R^3}$$

In equilibrium position

Lorentz force = Coulomb force

$$\therefore \qquad -ZeE = -\frac{Z^2e^2x}{4\pi\varepsilon_0 R^3}$$

$$\text{or} \qquad E = \frac{Ze^x}{4\pi\varepsilon_0 R^3}$$

$$\text{or} \qquad x = \frac{4\pi\varepsilon_0 R^3 E}{Ze} \qquad \ldots(3)$$

Hence the displacement of the electron cloud is directly proportional to the applied electric field E. Therefore the two electric charges $+Ze$ and $-Ze$ are separated through a distance x under the action of the applied field, thus constituting induced electric dipoles. Thus. Induced electric dipole moment,

$$\mu_e = Z_{ex} = Ze 4\pi\varepsilon_0 R^3 E/Ze$$

$$= 4\pi\varepsilon_0 R^3 E$$

$$= \alpha_e E \qquad \ldots(4)$$

where $\alpha_e = 4\pi\varepsilon_0 R^3$ is known as electronic polarizability. The dipole moment per unit volume is known as electronic polarizability. It is free from temperature.

$$P_e = N\mu_e = N\alpha_e E \qquad \ldots(5)$$

where, M = Number of atoms per metre cube. Also we know that polarization vector,

$$P_e = \varepsilon_0(\varepsilon_r - 1)E = N\alpha_e E$$

$$\text{or} \qquad (\varepsilon_r - 1) = \frac{N\alpha_e}{\varepsilon_0} \qquad \ldots(6)$$

$$\text{Thus} \qquad \alpha_e = \frac{\varepsilon_0(\varepsilon_r - 1)}{N} \qquad \ldots(7)$$

(ii) Ionic Polarization : It is due to the displacement of cations and anions in opposite directions. It occurs only in an ionic solid (see fig. b). Let an electric field is applied in the positive x-direction. The positive ions move to the right through a distance x_1 and the negative ions move to the left through a distance x_2. Taking, each unit cell has one cation and one anion, the resultant dipole moment per unit cell due to ionic displacement is given as follows :

$$\mu = e(x_1 + x_2)$$

If B_1 and B_2 are the resotring force constants of cation and anion respectively and f is the force due to applied field then,

$$F = B_1 x_1 = B_2 x_2$$

$$\text{Thus} \qquad x_1 = \frac{F}{B_1} \text{ and } x_2 = \frac{F}{B_2}$$

The restoring force constant depend upon the mass of the ion and angular frequency of the molecule in which ions are existing.

$$\text{Thus} \qquad x_1 = \frac{eE}{m\omega_0^2}$$

$$\text{and} \qquad x_2 = \frac{eE}{m\omega_0^2}$$

Where M is the mass of the negative ion.

$$\text{Thus,} \qquad x_1 + x_2 = \frac{eE}{m\omega_0^2} + \frac{eE}{M\omega_0^2}$$

$$= \frac{eE}{\omega_0^2}\left(\frac{1}{M} + \frac{1}{m}\right)$$

and $\mu = e(x_1 + x_2) = \frac{e^2 E}{\omega_0^2}\left(\frac{1}{M} + \frac{1}{m}\right)$

$$\therefore \quad \alpha_i = \frac{\mu}{E}$$

$$= \frac{e^2}{\omega_0^2}\left(\frac{1}{M} + \frac{1}{m}\right) \quad \ldots(8)$$

Therefore ionic polarizability α_i is inversely proportional to the square of the natural frequency of the ionic molecule and to its reduced mass which is equal to inverse of $\left(\frac{1}{M} + \frac{1}{m}\right)$

(iii) Orientational Polarization : In the molecule of methane (CH_4), the centre of the negative and positive charges coincide, so that there is no permanent dipole moment but in CH_3Cl molecule, the positve and negative charges do not coincide. As the electronegativity of chlorine is more than that of hydrogen, the chlorine atom pulls the bonding electrons to itself more strongly than hydrogen. Thus even in the absence of an electric field, this molecule carries a dipole moment. If an electric field is applied on these molecules, they tend to align automatically in the direction of applied field as given in fig. C. The polarization due to these alignment is known as orientation polarization which is dependent on temperature. With rise of temperature the thermal energy tends to randomize the alignment. Orientational polarization can be given as follows :

$$P_0 = N\bar{\mu} = N\mu^2 E/3kT$$

$$= N\alpha_0 E$$

Thus the orientational polarizability

$$\alpha_0 = \frac{P_0}{NE} = \frac{\mu^2}{3kT} \quad \ldots(9)$$

Hence the orientational polarizability α_0 is inversely proportional to absolute temperature of the material.

(iv) Space-Charge Polarization : The space charge polarization occurs due to the accumulation of charges at the electrodes or at the interfaces in a multiphase material as given in fig. d. The ions diffuse over appreciable distances in response to the applied field, giving rise to redistribution of charges in the dielectric medium.

The complete polarization of a material is the sum of the contribution from the different sources. Thus

$$P_{total} = P_e + P_i + P_0 + P_s \quad \ldots(10)$$

As the space-charge polarizability is very small if compared to the other types of polarizabilities, the net polarizability of a gas can be written as follows :

$$\alpha = \alpha_e + \alpha_i + \alpha_0$$

$$= 4\pi\varepsilon_0 R^3 + \frac{e^2}{\omega_0^2}\left(\frac{1}{M} + \frac{1}{m}\right) + \frac{\mu^2}{3kT}$$

Thus total polarization is given as follows :

$$P = N\alpha E = NE$$

$$\left[4\pi\varepsilon_0 R^3 + \frac{e^2}{\omega_0^2}\left(\frac{1}{M} + \frac{1}{m}\right) + \frac{\mu^2}{3kT}\right] \quad \ldots(4)$$

This equation is called largevin-Deby equation.

Q. 5. Attempt any one part of the following :

(a) Show that the magnetic suscepibility of a diamagnetic material is negative and independent of temperature.

Ans. Consider an e^- of charge' e' more in the circular orbit or radius `r' with vel `v' current through the circle is

$$i = \frac{e}{t} - \frac{e}{\frac{2\pi r}{v}} = \frac{ev}{2\pi r}$$

$\therefore$ the magnetic moment due to this current

$$M = iA = \frac{ev}{2\pi r}(\pi r^2)$$

$$M = \frac{1}{2} evr \quad \ldots(1)$$

Since magnetic field produces electric field & hence electrostatic force is experienced by the orbiting e^-

$$F = ma$$

$$F = eE \quad ...(2)$$

$$QE = \frac{v\text{(included e.m.f. forvoltage)}}{l\text{(Two point distance)}}$$

Now by lenz law of electromagnetic Induction

$$v = \frac{-d\phi B}{dt}$$

$$\therefore \quad E = \frac{1}{l} \cdot \frac{d\phi B}{dt}$$

$$E = \frac{-1}{l}\frac{d(BA)}{dt} \qquad (\because \phi = BA)$$

$$ma = -\frac{e}{l}\frac{do}{dt}(BA) \qquad \left[\because \ E = \frac{F}{q} = \frac{ma}{e}\right]$$

$$m\frac{dv}{dt} = \frac{-e}{2\pi r}(\pi r^2)\ \frac{d(B)}{dt}$$

$$\frac{dv}{dt} = \frac{-3r}{2m}\left(\frac{dB}{dt}\right)$$

If magnetic field changes from $0 \to B$, thenvel. of orbiting e^- also changes from initial value $v_1 \to v_2$

$$i.e., \int_{v_1}^{v_2} dv = \frac{-ev}{2m}\int_0^B dB$$

$$v_2 - v_1 = \frac{-erB}{2m}$$

$$\Delta v = \frac{-erB}{2m}$$

Change in velocity of e^- produces a charge in the magnetic moment hence from equation (1)

$$\Delta m = \frac{1}{2} er\,\Delta v$$

$$\Delta m = \frac{-1}{2}\frac{e^2 r^2 B}{2m}$$

$$\Delta m = \frac{-e^2 r^2 B}{4m} \quad ...(4)$$

Since there may be many electrons orbiting in an atom & also the orbit may have any orientation w.r.t. the magnetic field, therefore we have tofind average value of Δm

$$\Delta \overline{m} = \frac{-e^2\ B\overline{r}^2}{4m} \quad ...(5)$$

$\bar{r}_x, \bar{r}_y, \bar{r}_z$ be the radi of electrons along the three axis then

$$\overline{r_0^2} = \overline{r_x^2} + \overline{r_y^2} + \overline{r_z^2}$$

where r_0, is the total radius of atom

For the atom to be a perfect (5)

$$\overline{r_x} = \overline{r_y} = \overline{r_z}$$

$$\therefore \quad \overline{r_0}^2 = 3\overline{r_x}^2$$

$$\overline{r_x}^2 = \frac{1}{3}\overline{r_0^2} = \overline{r_y^2} = \overline{r_z^2}$$

If the magnetic field produced as along z-axis $\overline{r^2} = \overline{r_x}^2 + \overline{r_y}^2 = \overline{r_z}^2$

Put this eq. in (5)

$$\Delta\overline{m} = \frac{-es\,B}{4m}\frac{2}{3}\overline{r_0^2} = -\frac{1}{6}e^2\ \frac{B\overline{r}^2}{m}$$

If I is the intensity of magnetisation of diamagnetic material then by the definition.

$$I = \frac{\Delta\overline{m}}{v}$$

$$I = \frac{-1}{6}\frac{e^2\ B\overline{r}^2}{v.m}$$

$$\& \text{ If} \qquad X_m = \frac{1}{H \text{ or } B} = -\frac{1}{6}\frac{e^2\ B\overline{r_0^2}}{mvB}$$

$$X_m = -\frac{1}{6}\frac{e^{2-}\ \overline{r_0}^2}{mv}$$

here X_m = magnetic susceptibility

If z electron orbit in the outermost orbit the X_m will be

$$x_m = \frac{-1}{6}\frac{e^2\ zr_0^2}{mv}$$

(b) What are ultrasonic waves ? Explain how they are produced using magnetostriction method.

Ans. Production of Ultrasonic Waves

The ultrasonic waves cannot be produced from our usual method of a diaphragm loudspeaker fed with alternating current. This is because of the fact that at very high frequencies the inductive effect of the loudspeaker coil is so large that practically no current passes through it. However, the

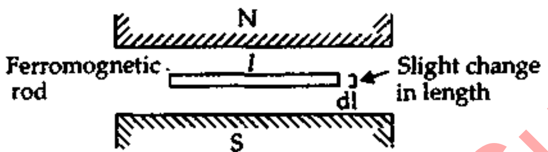

Fig. (a)

diaphragm of a loudspeaker cannot vibrate at such high frequencies. Thus other methods are used for the production of ultrasonic waves. There are two important methods namely magneto-striction and piezo electric method which are mostly used now-a-days. Magnetostriction method is used if frequencies upto 100 kHz are needed while piezo electric generators are used mostly for frequencies above that. Here we shall discuss these two methods as follows :

(a) Magnetostriction method : This method is based on the phenomenon of magnetostriction. According to this phenomenon if a rod of ferromagnetic material, such as iron or nickel, is placed in a magnetic field parallel to its length (Fig. a), a small extension or contraction occurs. This change of length is independent of the sign of the field and only depends upon the magnitude of the field and nature of the material. When the rod is placed inside a coil carrying an alternating

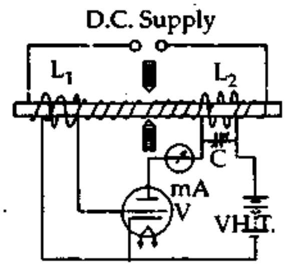

Fig. b

current, then it suffers the same change in length for each half cycle of alternating current. This results in setting up vibrations in the rod whose frequencies is twice that of alternating current. Ordinarily the amplitude of the vibrations of rod is small. While when, the frequency of the alternating current is the same as the natural frequency of the rod, then resonance occurs and the amplitude of vibrations is considerably in creased. Sound waves are now emitted from the ends of the rod. Moreover, when the applied frequency is of the order of ultrasonic frequency, the rod sends out ultrasonic waves.

An experimental arrangement due to Pierce for producing ultrasonic waves given in Fig. There is a short nickel rod which is clamped at the centre. This rod is permanently magnetised in the beginning by passing a direct current in the coil which is wrapped round the rod. There are two other coils L_1 and L_2 which are wrapped round the rod as given in the figure. The coil L_2 is connected in the plate circuit of a valve V while L_1 is connected in the grid circuit. The frequency of the oscillating platecircuit is adjusted with the help of avariable condenser C connected across the coil L_2. The direct current milliammeter reads the plate current. If the frequency of plate circuit is the same as the natural frequency of the rod, resonant vibrations are produced in the surrounding medium. The vibrations are maintained due to the coupling provided by coil L_1. If the resonant oscillations start, the milliammeter records the maximum current. The maintenanceof the oscillation of the rod is as follows :

If the plate current passing through the coil L_2 is changed, it causes a corresponding change in the magnetisation of the rod. Hence there is a change in the length of the rod. This vibration in the length cuases a variation in the magnetic flux through the grid coil L_1 which in turn changes the e.m.f. developed across L_1. This e.m.f. acts on the grid and gives an amplified current change in the plate circuit *i.e.,* in the coil L_2. In this way the plate current builds up to a large amplitude with a frequency determined by the frequency of the longitudinal vibration of the rod. Hence the vibrations of the rod are maintained. If the frequency of the circuit becomes equal to the freqeuncy of the rod, resonance occurs and the sound waves of maximum amplitude are generated. By adjusting the length of the rod and the capacity of the condenser, high frequency oscillations of different frequencies are obtained.

(b) Piezo-electric method : This method is based on *Piezo-electric effect.* According to this effect if certain crystals like quartz, rochelle salt, tourmaline etc. (Fig. C) are stretched or compressed along certain axis, an electric potential difference is produced along a perpendicular axis. The converse of this effect is also true *i.e.* If an alternating potential difference is applied along the electric axis, the

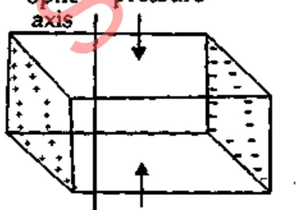

Fig. c

crystal is set into elastic vibration along the corresponding mechanical axis. When the frequency of electric oscillations coincides with the natural frequency of the crystal, the vibrations will be of large amplitude. This phenomenon is used for the production of ultrasonic waves. The alternating potential difference is obtained by a valve oscillator.

The experimetnal arrangement is given in Fig. The high frequency alternating voltage which is applied to the crystal is obtained from. *Hartley oscillatory circuit.* The Hartley circuit consists of a tuned oscillatory circuit. One end of the tuned circuit is connected to the plate of valve *V* while the other is connected to the grid. The coil L_1 is tapped near the centre and joined

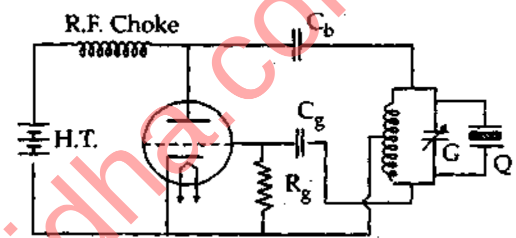

Fig. d

to the cathode. The quartz crystal *Q* is connected in parallel of variable condenser C_1.

The proper grid bias is found by means of grid leak resistor R_g and grid condenser C_g. The d.c. voltage is applied to the plate through radio frequency choke. The radio frequency choke prevents the radio frequency current to pass through high tension battery. C_b is a blocking capacitor which stops the direct current to pass through the tank circuit which bypasses the radio frequency currents. The capacity of the variable condenser C_1 is adjusted so that the frequency of the oscillating circuit is tuned with the natural frequency of the crystal. Now the quartz crystal is set into mechanical vibrations and ultrasonic waves are produced. The ultrasonic waves upto a frequency of 500 kHz with a moderate size crystal can be produced from this method. While frequency up to 15×10^7 Hz can be produced by tourmaline crystal.

The velocity of quartz along *x*-direction is given as follows :

$$\upsilon = \sqrt{(Y/\rho)}$$

For quartz we know, $Y = 79 \times 10^{10}$ N/m^2 and $\rho = 2650$ kg/m^3

$$\therefore \quad \upsilon = \sqrt{\frac{79 \times 10^{10}}{2650}} = 5450 \text{ m/s.} \qquad ...(1)$$

When t be the thickness of quartz slab in metres, then

$$\upsilon = n\lambda = n(2t)$$

where n is the frequency.

$$\therefore \quad n = \frac{\upsilon}{2t} = \frac{2725}{t} = \text{Hz}$$

When is expressed in mm, then

$$n = \frac{2725}{t/1000} = \frac{2725000}{t} \text{ Hz}$$

$$\text{or} \quad n = \frac{2725}{t} = \text{kHz}$$

So for a thickness of 1 mm of quartz crystal

$$n = 2725 \text{ kHz} = 2.725 \text{ MHz}$$

If by adjusting the variable capacitor C_1 of tank circuit, the crystal is made to vibrate at its natural frequency, then the frequency of oscillatory circuit gives the frequency of vibration of quartz crystal. Hence,

$$n = \frac{1}{2i\sqrt{L_1 C_1}}$$

Q. 6. Attempt any one part of the following :

(a) Derive and explain Poynting theorem.

Ans. Energy in electromagnetic waves (Poynting therоem & vector) : When electromagnetic wave travel from one point to another in space it transports electric & magnetic energies from point to point. The rate of this energy transfer can be expressed in terms of electric & magnetic field strength of the wave.

The rate of flow of energy per unit area in a plane electromagnetic wave can be expressed by a water $\overline{S}$ called the poynting.

$$\overline{S} = \overline{E} \times \overline{M} = \frac{1}{H_0}(\overline{E} \times \overline{B}) \text{ watt/m}^2$$

The direction of S is the direction of energy motion

$$-\int_v J \cdot E\, du = \frac{\partial}{\partial t}\int_v \frac{1}{2}(E.D + H.B)\,dv + \int_\partial (E \times M)\,ds$$

The various terms of eq.(8) may be interpreted as follows :

(1) The term $-\int J.\,Edv$ represents the rae of transfer of energy into the electromagnetic field due to the motion of charge.

(2) The term $\frac{d}{dt}\int_v \frac{1}{2}(E.D + H.B)\,dv$ $= u_e + u_m = u$ total and represents the rate of electromagnetic energy stored.

(3) The term $\int_S (E \times H)\,dS$ represents the amount of energy crossing per second through the closed surface.

The factor $E \times H = S$ is called the Poynting vector. The equation (8) thus represents the low of conservation of energy.

(b) Write down Maxwell's equations in free space and using these equations derive wave equations for both electric and magnetic fields.

Ans. Wave equations in free space : Consider the case of electromagnetic phenomenon in free space or more generally, in a perfect di-electric containing no charge(p = 0) and no conduction currents (J = 0). For this case the field eqautions become :

$$\nabla \cdot \vec{D} = 0 \qquad ...(a)$$

$$\nabla \cdot \vec{B} = 0 \qquad ...(b)$$

$$\nabla \times \vec{E} = -\frac{\partial B}{\partial t} \qquad ...(c)$$

$$\partial \times \vec{H} = -\frac{\partial \vec{D}}{\partial t} \qquad ...(d)$$

For free space, $\mu = \mu_0, \varepsilon = \varepsilon_0, \sigma = 0$ and $\rho = 0$

$$\vec{D} = \varepsilon_0 \vec{E} \text{ and } \vec{B} = \mu_0 \vec{H}$$

Taking curl of equaion (c) on both sides, we hve

$$\nabla \times (\nabla \times \vec{E}) = -\frac{\partial}{\partial t}(\nabla \times \vec{B})$$

$$= -\mu_0 \nabla \times \frac{\partial \vec{H}}{\partial t} \quad ...(1)$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \frac{\partial \vec{H}}{\partial t} = \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \quad ...(2)$$

$$\nabla \times (\nabla \times \vec{E}) = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\therefore \boxed{\nabla \times (\nabla \times \vec{E}) = \nabla \nabla \cdot \vec{E} - \nabla^2 \vec{E}}$$

and $\boxed{\nabla \cdot \vec{E} = \frac{1}{\varepsilon_0} \nabla \cdot \vec{D} = 0}$

$$-\nabla^2 \vec{E} = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\boxed{\nabla \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}} \quad ...(3)$$

Similarly, we can solve for magnetic field

$$\boxed{\nabla_2 \vec{H} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2}} \quad ...(4)$$

Equations (3) and (4) represent wave equations.

The waves which serves as a building block in the study of EM waves consist of electric and magnetic fields that are perpendicular to each other and to the direction of propagation and are uniform in planes perpendicular to the direction of propagation. These waves are called uniform plane waves.

Equations (3) and (4) are the wave equations for field vector $\vec{E}$ and $\vec{B}$. As we know the classical equations of wave propagating with velocity v is given by :

$$\nabla^2 \Psi = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} \quad ...(5)$$

If we compare equations (3), (4) and (5), it follows at once that the field, vectors can be propagated as waves in free space and the velocity of propagation is

$$\frac{1}{v^2} = \mu_0 \varepsilon_0 \qquad \therefore \qquad v^2 = \frac{1}{\mu_0 \varepsilon_0}$$

$$\boxed{v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}},$$

$$= \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.85 \times 10^{-12}}}$$

$$\cong 3 \times 10^8 \text{ m/sec} \equiv C$$

$$\therefore \boxed{C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}}$$

So there exist a electromagnetic wave in space and they travel in free space with velocity of light.

Plane electromagnetic waves are transverse in nature : An electro-magnetic wave, whose field vectors $\vec{E}$ and $\vec{B}$ are the functins of space coordinate and time coordinate (t), is said to be plane wave. Suppose such waves are propagating along z axis in which $\vec{E}$ and $\vec{B}$ and change only along z axis *i.e.*,

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0 \qquad \frac{\partial}{\partial z} \neq 0 \quad ...(i)$$

and $\vec{E} = \vec{E}(z, t)$ and $\vec{B} = \vec{B}(z, t)$

In vacuum Maxwell's equations are ($\rho = 0$, $J = 0$)

(a) $\nabla \cdot \vec{D} = 0$ (b) $\nabla \cdot \vec{B} = 0$

(c) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (d) $\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

From first Maxwell's equation

$\nabla \cdot \vec{D} = 0$ or $\nabla \cdot E = 0$ as $(\vec{D} = \varepsilon_0 \vec{E})$

$$\left(i\frac{\partial}{\partial x}+j\frac{\partial}{\partial y}+z\frac{\partial}{\partial z}\right)$$

$$(\hat{i}\,E_x+\hat{j}\,E_y+\hat{k}\,E_z)=0$$

$$\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z}=0$$

Since $\quad \dfrac{\partial E_x}{\partial x}=\dfrac{\partial E_y}{\partial y}=0$

$\therefore \quad \dfrac{\partial E_z}{\partial z}=0 \qquad$ or $\qquad E_z=\text{Constant}$

Similarly from equqtion (2), $B_z=\text{Constant}$

From Maxwells' third equation

$$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z}\\ E_x & E_y & E_z\end{vmatrix}=-i\frac{\partial B_x}{\partial t}-j\frac{\partial B_y}{\partial t}-k\frac{\partial B_z}{\partial t}$$

Its z component will be

$$\frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}=\frac{\partial B_z}{\partial t}$$

Since $\quad \dfrac{\partial E_y}{\partial x}-\dfrac{\partial E_x}{\partial y}=0\Rightarrow\dfrac{\partial B_z}{\partial t}=0$

$\Rightarrow B_z=\text{Constant}$

Similarly from equation (4), $E_z=\text{Constant}$

Thus, we conclude that E_z and B_z are constant with respect to space and time. But not wave is found with constant values of E_z and B_z Hence we assume them to be zero.

$$E_z=B_z=0$$

Thus, it is inferred that there are no longitudinal components in electromagnetic waves. Thus.

$$\left.\begin{aligned}\vec{E}&=i\,\vec{E}_x+j\,\vec{E}_y\\ \text{and}\quad \vec{B}&=i\,\vec{B}_x+j\,\vec{B}_y\end{aligned}\right\}\qquad\ldots(5)$$

As the wave is propagating along z axis and there is no z component of $\vec{E}$ and $\vec{B}$, both of these vectors are $\vec{E}$ and $\vec{B}$ perpendicular to the direction of propagation.Hence, Maxwell's electromagnetic waves are transvers in nature.

Q. 7. Attempt any one of the following :

(a) What are type I and II superconductors ? Explain.

Ans. Types of Superconductors

Depending on the magnetization behaviour of superconductors in an external magnetic field, superconductors can be devided into the following two categories :

(A) Type-1 super-conductor or soft superconductors.

(B) Type-2 super-conductor or hand superconductors.

(A) Soft superconductors :

In this type of superconductor, the magnetic field is totally excluded from the

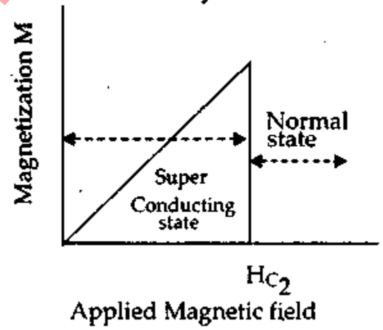

Fig. (a)

interior of the supercoductor below a certain magnetic field known as critical magnetic field (H_c). But at the critical magnetic field, the material loses superconductivity abruptly. Now the magnetic field penetrates fully within the material *i.e.,* the material is in its normal state. These type of superconductors are termeal as type-1 super conductor. This type of super conductors exhibit the Meissner effect in full *i.e.,* they are completely diamagnetic. The magnetisation curve is given in Fig. (a)

Some points are to be noted given as follows :

(a) Transition of H_c is reversible *i.e.*, when the magnetic field is reduced below H_c the maerial again aquires the property of superconductivity.

(c) Lead, tin, mercury, etc fall into type-1 category.

(b) Pure specimens of most superconducting metals are of type-1 and they have very low values of H_c (about 0·1 T).

(d) Type-1 materials are called as soft because the superconductivity is destroyed very easily at low H_c values.

(B) Hard superconductors:

This type of superconductors are characterised by the existence of two critical magnetic fields i.e., lower critical field H_{C_1} and upper critical field H_{C_2} the magnetisation curve is given in Fig. (b)

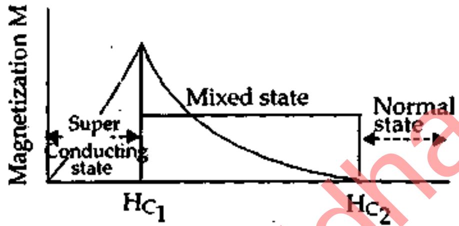

Fig. (b)

For this group of semicondutors, below H_{C_1}, the specimen is super conducting because the flux is completely excluded. At H_{C_1}, the flux starts penetrating into the speciment unit the upper critical field H_{C_2} is reached. The specimen is in the mixed state and shows superconducting electrical properties.

Hence between H_{C_1} and H_{C_2} the meissner effect is not complete. At H_{C_2} the magnetization vanishes and the specimen turns to normal conducting state. The following points are to be noted :

(a) The values of H_{C_2} may to 100 times and more than H_c values of type-1. H_{C_2} upto 50T have been found in some materials.

(b) Generally alloys or transition metals with high electrical resistivity in normal state, exhibit this type of superconductivity.

(c) This type of superconductors are used to produce magnetising coils to obtain high fields of the order of 10T or more.

(d) As the superconductivity is retained till fair large magnetic field is reached, hence the super-conductors are called hard superconductors.

(e) The magnetization vanishes gradually as the field is increased, rather than suddenly as for type-1.

(b) Describe buckyballs and their properties and uses.

Ans. **Carbon Buckyballs (C60)**

Buckyballs are another very strong cage-like structure based on its interlocked shape. It also has the unique set of properties based on its structure. The usual structure for this molecule is made of 60 carbon atoms arranged in a soccer ball-like shape that is less than one nanometer in diameter.

The C60 buckyball, also known as fullerenes, is a truncated icosahedron which is made up of 12 pentagons, each surrounded by 5 hexagons (20 hexagons). Of these, no two pentagons share a vertex.

Creation of Buckyballs

Buckyballs found in nature, naturally, and in greater amounts than expected. Buckyballs are found to exist in interstellar dust and in geological formations on Earth. Even closer to home are the buckyballs that naturally form in the wax and dirt from a burning candle, since the flame on the wick provides the sufficient conditions for such processes to occur. In laboratory, the formation of buckyballs involves transmitting a large current between two graphite electordes in an inert atmosphere.

Under these conditions, the energy from the arc is dissipated by breaking carbon from the surface. The carbon cools in the inert atmosphere and forms buckyballs.

Properties of Carbon Buckyballs

Carbon buckyballs possess some important properties given as follows :

1. Buckyballs are in fact the only known carbon allotropes which are soluble, have limited solubility, in most of the solvents.
2. Because of spherical shape, buckyballs have extremely stable configuration which is resilient to impact and deformation i.e. if it is thrown against some object it will bounce back very much similar to soccer ball.
3. Buckyballs do not bond to one another. They do however, stick together via Van der Waals forces.
4. Buckyballs have aromatic nature as electrons are free to move among other bonds in hexagon carbon rings.
5. By doping buckyballs, they can be electrically insulating, conducting, semiconducting or even superconducting.
6. The C60 buckyballs can withstand high temperatures and pressures.

Uses of Buckyballs

Carbon buckyballs have some important applications given as follows :

1. Because of their shapes, they could be used equivalently to ball bearings, and thus allow surfaces to roll over each other, making the fullerenes equivalently lubricants.
2. Buckyballs unique cage-like structure might allow it to take the place of other molecules in shuttling toxic metal substances through the human body during MRI scan.
3. Attaching metals onto the surface of fullernes offers the possibility for buckyballs to become catalysis.
4. It has been shown that fitting a potassium ion in the buckyball causes it to become superconductive.
5. The Scanning Tunneling Microscope (STM) is one of the foremost tools in microscopy which uses needle of buckyballs.

6. Buckyballs are now being considered for uses in the field of medicine, both as diagnostic tools and drug candidates.
7. Hydrogen storage as almost every carbon atom in C60 can absorb a hydrogen atom without disrupting the buckyball structure, making it more effective than metal hydrides. This could lead to applications in fuel cells.

Physical constants

Plank' constants $h = 6.6 \times 10^{-34}\ J-s$

Velocity of light in vacuum $c = 3 \times 10^{8}\ m/s$

Rest mass of electron $m = 9.1 \times 10^{-31}\ kg$