

## FIRST SEMESTER EXAMINATION, 2009-2010

## ENGINEERING PHYSICS-I

Time : 2 Hours

Total Marks : 50

- Note : (i) Attempt all questions.  
(ii) Marks of each question are shown against it.

## SECTION -A

1. Attempt all parts. 10×1=10

All parts carry equal marks.

(a) Decay of  $\mu$ -meson supports.

- (i) length contraction
- (ii) time dilation
- (iii) relativity of mass
- (iv) relativity of energy.

Ans. (ii) Time dilation

(b) The kinetic energy of a particle is double of its rest mass energy. The dynamic mass of the particle in terms of its rest mass  $m_0$  is

- (i)  $m_0$
- (ii)  $2m_0$
- (iii)  $3m_0$
- (iv)  $1.5m_0$

Ans. (iii)  $3m_0$ 

(c) What happens, if monochromatic light used in Young's double slit experiment is replaced by white light?

- (i) The bright fringes become white
- (ii) The central fringe is White and all other are coloured.
- (iii) All fringes are coloured
- (iv) No fringes are observed.

Ans. (ii) The central fringe is white and all other are coloured.

(d) In the diffraction pattern due to single slit of width  $d$  with incident light of wave-length  $\lambda$  with angle of diffraction  $\theta$ , the condition for the first minimum is

- (i)  $\lambda \sin \theta = d$
- (ii)  $d \cos \theta = \lambda$
- (iii)  $d \sin \theta = \lambda$
- (iv)  $\lambda \cos \theta = d$

Ans. (iii)  $d \sin \theta = \lambda$ (e) if  $N$  be the total number of rulings on the grating,  $n$  the order of spectrum and  $\lambda$ , the wavelength of light used, then the resolving power of grating is given by

- (i)  $Nn$
- (ii)  $Nn\lambda$
- (iii)  $N\lambda/n$
- (iv)  $N/n$

Ans. (i)  $Nn$ 

(f) Polarised light can be produced by

- (i) dispersion
- (ii) scattering
- (iii) interference
- (iv) diffraction

Ans. (ii) Scattering

(g) The property of rotating the plane of polarisation is known as

- (i) optical activity
- (ii) optical rotation
- (iii) dichroism
- (iv) specific rotation

Ans. (i) Optical activity

- (h) For stimulated emission to take place in an active medium, the condition is
- the production of population inversion in energy levels
  - sufficient number of atoms in ground state
  - coherent incident radiation on the medium
  - large volume of active medium

Ans. (i) The production of population inversion in energy levels.

- (i) High information carrying capability of optical fibers is measured with their
- low losses
  - high bandwidth
  - low costs
  - high efficiency

Ans. (ii) high bandwidth

- (f) Loss of light intensity in optical fiber is due to
- scattering
  - absorption
  - refraction
  - (i) and (ii)

Ans. (iv) (i) and (ii) (scattering and absorption)

### SECTION-B

2. Attempt any three parts. All parts carry equal marks.  $5 \times 3 = 15$

- (a) The mass of a moving electron is 11 times its rest mass. Find its kinetic energy and momentum.

Ans.  $m = 11m_0$

$$K = ?, p = ?$$

We know that total energy is given as

$$E = K + m_0c^2 \quad \dots(1)$$

where  $E =$  total energy

$K =$  Kinetic energy

$$\text{or } mc^2 = K + m_0c^2 \quad (\because E = mc^2)$$

$$K = mc^2 - m_0c^2$$

$$K = 11m_0c^2 - m_0c^2$$

$$K = 10 m_0c^2$$

$$K = 10 \times (9.1 \times 10^{-31} \text{ kg}) \times (3 \times 10^8 \text{ m/sec})^2$$

$$K = 81.9 \times 10^{-14} \text{ Joule}$$

$$K = \frac{81.9 \times 10^{-14}}{1.6 \times 10^{-19}} \text{ eV}$$

$$K = 51.18 \times 10^5 \text{ eV}$$

$$K = 5.21 \times 10^6 \text{ eV}$$

$$K = 5.12 \text{ MeV}$$

Momentum  $p = mv$

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$11m_0 = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$11 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$1 - \frac{v^2}{c^2} = \frac{1}{121}$$

$$1 - \frac{1}{121} = \frac{v^2}{c^2}$$

$$\frac{120}{121} = \frac{v^2}{c^2}$$

$$v = \sqrt{\frac{120}{121}} \times c$$

$$v = 2.99 \times 10^{-21} \text{ kg m/sec}$$

- (b) In an interference pattern, the amplitude of intensity variation is found to be 5% of the average intensity. Calculate the relative intensities of interfering sources.

**Ans.** It is given that the intensity variation is 5% of the average intensity. So if the average intensity is 100, then the maximum intensity is 105 and minimum intensity is 95.

$$\frac{I_{\max}}{I_{\min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{105}{95}$$

$$\frac{a_1 + a_2}{a_1 - a_2} = 1.051$$

$$(a_1 + a_2) = 1.051(a_1 - a_2)$$

$$= 1.051a_1 - 1.051a_2$$

$$0.051a_1 = 2.051a_2$$

$$\frac{a_1}{a_2} = \frac{2.051}{0.051} = 40$$

$$a_1 = 40a_2$$

∴ Therefore, the ratio of the intensities of interfering sources is

$$\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}$$

$$\frac{I_1}{I_2} = \frac{(40a_2)^2}{a_2^2}$$

$$\boxed{I_1 : I_2 = 1600 : 1}$$

(c) A light of wavelength 6000 Å falls normally on a straight slit of width 0.10 mm. Calculate the total angular width of the central maximum and also the linear width as observed on a screen placed 1 metre away.

**Ans.** The condition for central maximum is

$$e \sin \theta = \lambda$$

where  $\theta$  is angular half width.

$$\therefore \sin \theta = \frac{\lambda}{e}$$

$$\sin \theta = \frac{6 \times 10^{-5}}{0.01} = 6 \times 10^{-3}$$

∴ since  $\theta$  is very small.

$$\sin \theta \simeq \theta$$

$$\therefore \theta = 6 \times 10^{-3} \text{ radians}$$

∴ The angular width of the central maximum is =  $2\theta$

$$= 1.2 \times 10^{-2} \text{ radians}$$

Let  $y$  is the linear half width of central maximum on the screen placed at a distance  $d = 1.0$  meters.

$$Y = \theta \cdot d \quad \left( \text{angle} = \frac{\text{arc}}{\text{Radius}} \right)$$

$$Y = 6 \times 10^{-3} \times 100$$

$$\boxed{Y = 0.6 \text{ cm}}$$

Therefore, the linear width of the central maximum on the screen =  $2Y$

$$= 2 \times 0.6$$

$$= 1.2 \text{ cm.}$$

(d) (i) Calculate the specific rotation if the plane of polarisation is turned through 26.4°, traversing 20 cm length of 20 percent sugar solution.

(ii) Calculate the thickness of a halfwave plate of quartz for a wavelength of 5000 Å. Here  $\mu_e = 1.553$  and  $\mu_o = 1.544$ .

**Ans.** (i) The specific rotation is given by

$$S = \frac{100}{lc}$$

$$S = \frac{10 \times 26.4}{20 \times 0.2}$$

$$\boxed{S = 66 \text{ degree (dm)}^{-1} \text{ (gm/cm)}^{-1}}$$

(ii) For a half plate of positive crystal

$$t = \frac{\lambda}{2(\mu_e - \mu_o)}$$

$$\lambda = 5000 \text{ \AA}, \quad \mu_e = 1.553$$

$$= 5 \times 10^{-5} \text{ cms} \quad \mu_o = 1.544$$

$$t = \frac{5 \times 10^{-5}}{2(1.553 - 1.544)}$$

$$t = 2.78 \times 10^{-3} \text{ cms}$$

- (e) The core and cladding refractive indices of a step index fiber are 1.60 and 1.44 respectively. An electromagnetic wave having a wavelength of 0.8  $\mu\text{m}$  is propagating through the fiber in guided mode through the core of the fiber. Find out maximum and minimum value of phase constant.

Ans. The maximum value of phase constant

$$= \frac{2\pi}{\lambda} n_1$$

$$= \frac{2\pi}{0.8} \times 1.6$$

$$= 12.56 \text{ radian}/\mu\text{m}$$

The minimum value of phase constant

$$= \frac{2\pi}{\lambda} n_2 = \frac{2\pi}{12.56} \times 1.44 = 11.33 \text{ rad}/\mu\text{m}$$

### SECTION-C

Attempt all questions All questions carry equal marks.

3. Attempt any one part of the following  
5×5=25

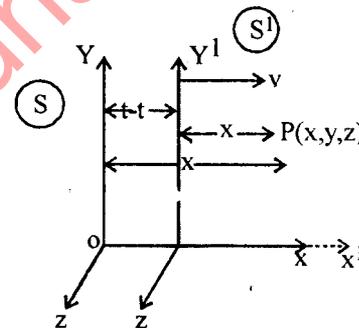
- (a) State the fundamental postulates of the special theory of relativity. Deduce the Lorentz transformation equations.

Ans. Fundamental Postulates of the Special Theory of Relativity : The special theory of relativity is based upon two basic postulates which

are as follows :

- (i) **The principle of equivalence :** This states that the laws of physics are same in all inertial frames of reference moving with a constant velocity with respect to one another.
- (ii) **The principle of constancy of speed of light:** The second postulate states that the speed of light in free space is always same in all inertial frames of reference and is equal to  $C$ , i.e. it is independent of the relative motion of the inertial frames, the source and the observer.

**Derivation of Lorentz Transformations Equations :** The equations which relate the space and time co-ordinates of two co-ordinate systems moving with a uniform velocity relative to one another are called Lorentz transformations.



Consider two observers  $O$  and  $O'$  located in two separate inertial co-ordinate systems  $S$  and  $S'$  respectively. The system  $S'$  moves with a uniform velocity  $v$  to the right along  $X$ -axis relative to  $S$ . This is equivalent to the motion of  $S$  to the left with a velocity  $v$  relative to  $S'$ .

The space and time co-ordinate of an event  $P$  described by the observer  $O$  are  $(x, y, z, t)$  and, the co-ordinate of same event as described at  $P$  by the observer  $O'$  are  $(x', y', z', t')$ .

Lorentz transformation equations which satisfy the relativity requirements according to the first postulates, must be **linear** and not quadratic or of

a higher order. A quadratic equation has two roots and a higher order equations even more. In order that an event  $(x, y, z, t)$  in the inertial system  $S$  may correspond to a single event  $(x', y', z', t')$  in the inertial system  $S'$ , and vice versa, there **must be a one to one correspondence** and the transformation equations **must be linear in space** as well as **time co-ordinates**.

We shall, therefore, assume the simplest linear equations for Lorentz transformation, i.e.,

$$x = \alpha x' + \beta t'$$

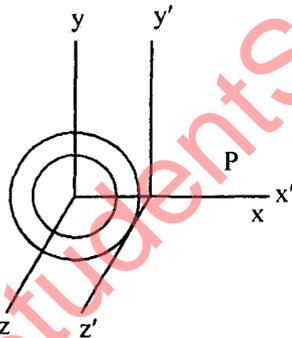
$$y = y'$$

$$z = z'$$

$$t = \gamma x' + \delta t'$$

Imagine that at the time  $t = t' = 0$  when the origin  $O'$  coincides with the origin  $O$ , a spherical pulse of light leaves the common origin of  $S$  and  $S'$ .

According to **second postulate of relativity**, the velocity of light is invariant, each observer sees a spherical wave expanding outwards with the speed  $C$  in his own system.



For observer  $O$  in the system  $S$

$$c = \frac{\sqrt{x^2 + y^2 + z^2}}{t}$$

$$x^2 + y^2 + z^2 - c^2 t^2 = 0 \quad \dots(1)$$

Similarly, for the observer  $O'$  in the system  $S'$ .

$$c = \frac{\sqrt{x'^2 + y'^2 + z'^2}}{t'}$$

$$x'^2 + y'^2 + z'^2 - c'^2 t'^2 = 0 \quad \dots(2)$$

From (1) and (2), we have

$$x^2 + y^2 + z^2 - c^2 t^2 = x'^2 + y'^2 + z'^2 - c'^2 t'^2$$

But  $y = y'$ , and  $z = z'$

$$\text{so } x^2 - c^2 t^2 = x'^2 - c'^2 t'^2$$

Substituting  $x = \alpha x' + \beta t'$  and  $t = \gamma x' + \delta t'$ , we have

$$(\alpha x' + \beta t')^2 - c^2 (\gamma x' + \delta t')^2 = x'^2 - c'^2 t'^2$$

$$\alpha^2 x'^2 + \beta^2 t'^2 + 2\alpha\beta x't' - c^2 \gamma^2 x'^2 - c^2 \delta^2 t'^2 - 2c^2 \gamma \delta x't' = x'^2 - c'^2 t'^2$$

$$x'^2 [\alpha^2 - c^2 \gamma^2] + t'^2 [\beta^2 - c^2 \delta^2] + 2x't' [\alpha\beta - c^2 \gamma \delta] = x'^2 - c'^2 t'^2$$

Comparing coefficient on both sides, we have

$$\alpha^2 - c^2 \gamma^2 = 1 \quad \dots(3)$$

$$\beta^2 - c^2 \delta^2 = -c^2 \quad \dots(4)$$

$$\alpha\beta - c^2 \gamma \delta = 0 \quad \dots(5)$$

Now when  $x' = 0$ ,  $x = vt$

Substituting these values in the equations below, we have

$$x = \alpha x' + \beta t'$$

$$\text{and } t = \gamma x' + \delta t'$$

we get

$$vt = \beta t' \quad (\text{when } x' = 0, x = vt)$$

$$t = \delta t'$$

$$v\delta t' = \beta t'$$

$$\text{or } \beta = v\delta$$

From equation (4), we have

$$(v\delta)^2 - c^2 \delta^2 = -c^2$$

$$\delta^2 [v^2 - c^2] = -c^2$$

$$\delta = \frac{c}{\sqrt{c^2 - v^2}} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \alpha$$

$$\beta = \frac{v}{\sqrt{1 - \frac{v^2}{c^2}}} = v\alpha$$

From eqn. (5), we have

$$\alpha \frac{v}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \gamma = 0$$

$$\alpha v = c^2 \gamma$$

or 
$$\gamma = \frac{\alpha v}{c^2}$$

Substituting the value of  $\gamma$  in eqn. (3), we have,

$$\alpha^2 - c^2 \frac{\alpha^2 v^2}{c^4} = 1$$

$$\alpha^2 [1 - v^2] = 1$$

$$\alpha = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \delta$$

Also 
$$\gamma = \delta \frac{\alpha v}{c^2}$$

$$\gamma = \frac{\left(\frac{v}{c^2}\right)}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\alpha v}{c^2}$$

Substituting the values of  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\delta$ , the Lorentz transformation equations will be

$$\begin{aligned} x &= \alpha x' + v\alpha t' \\ x &= \alpha(x' + vt') \\ y &= y' \\ z &= z' \end{aligned}$$

$$t = \frac{\alpha v}{c^2} x' + \alpha t'$$

$$t = \alpha \left( \frac{vx'}{c^2} + t' \right)$$

$$\begin{aligned} x &= \alpha(x' + vt') \\ y &= y' \\ z &= z' \\ t &= \alpha \left( t' + \frac{v^2}{c^2} x' \right) \end{aligned}$$

where 
$$\alpha = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

**Inverse Lorentz Transformations :** To get inverse Lorentz transformation equation, we note that  $O$  is moving with a velocity  $v$  relative to  $O'$ . So interchanging  $x, y, z, t$  with  $x', y', z', t'$  and putting  $v$  in place of  $-v$ , we get

$$\begin{aligned} x' &= \alpha(x + vt) \\ y &= y' \\ z &= z' \\ t' &= \alpha \left( t - \frac{v}{c^2} x \right) \end{aligned}$$

where 
$$\alpha = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

(b) Deduce the relativistic velocity addition theorem. Show that it is consistent with Einstein's second postulate.

Ans. Please see Q. No. 1(c) of Ist Sem. 2007-2008.

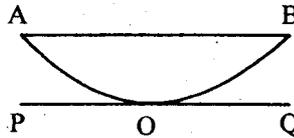
4. Attempt any one part of the following.

(a) Explain the formation of interference fringes by means of biprism using monochromatic source of light. How wavelength is measured by biprism experiment?

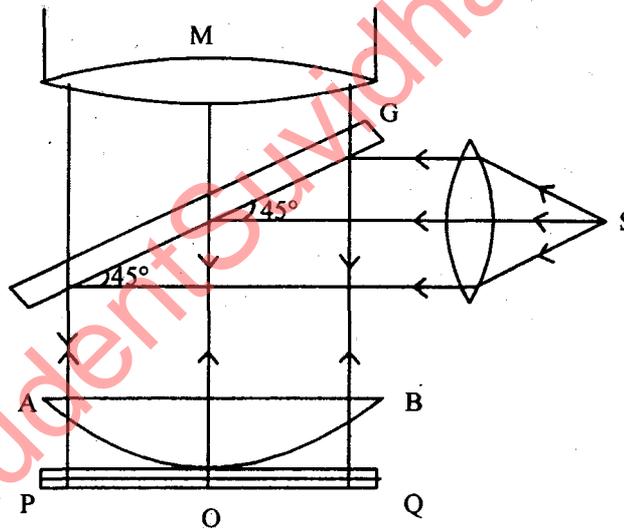
Ans. Please see Q. No. 2(a) (i) of IInd Sem. 2007-2008.

(b) What are Newton's rings? Prove that in reflected light diameters of the bright rings are proportional to the square root of odd natural number.

**Ans. Newton's Rings :** If we place a plano-convex lens of large focal length on a plane glass plate, a thin film of air is developed between the curved surface of the lens and plane glass plate. The thickness of the *air film* is zero at the point of contact  $O$  and gradually increases as we move away from the point of contact on either side. When the monochromatic light falls normally on the surface of the lens, then the light reflected from the upper surface of the film  $AOB$  interferes with the light reflected from the lower surface of the film  $POQ$ . As a result, an alternate dark and bright circular fringes (rings concentric around the point of contacts) are seen. The interference rings of equal thickness so forward were first investigated by Newton and hence called Newton's Rings.

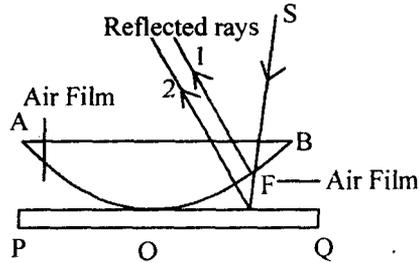


If the incident light is white, a series of concentric coloured rings is seen around the point of contact in both the reflected and transmitted system.



**Experimental Arrangement :** Newton's rings can easily be observed in the laboratory by using an apparatus as shown in figure, light from an **extended** monochromatic source  $S$  falls parallel by a lens  $L$  and then falls on a  $45^\circ$  inclined thin transparent glass plate  $G$  which partially reflects the light in the downward direction. These reflected beams fall normally on an air film formed between the convex surface of the plano-convex lens glass plate, light reflected from upper surface of glass plate lower surface of plano-convex lens interfere. These reflected beams proceed upwards and enter the observer's eye through a low power travelling microscope  $M$ . On focussing, a large number of Newton's rings,

alternately bright and dark fringes can be seen in the field of view of the microscope.



**Theory :** Circular rings are formed due to the interference of light rays reflected from the upper and lower surfaces of the air-film formed between the convex surface of a plano-convex lens, and the glass plate. The incident ray  $SF$  is divided into two **coherent rays** 1 and 2 by reflection from the upper and lower surfaces of the wedge shaped air film. These reflected rays 1 and 2 interfere and produce bright and dark circular rings around the point of contact.

The effective path difference between the interchanging rays in **reflected light** is

$$\Delta = 2\mu t \cos(r\theta) \pm \frac{\lambda}{2}$$

where

$\mu$  → Reflective index of film

$t$  → thickness of the film at the point of incidence

$r$  → angle of refraction

$\theta$  → angle of wedge

**For normal incidence**  $i = 0 = r$

and for a very small angle of wedge ( $\theta$ ).

$$\cos(r + \theta) = \cos \theta = 1 \quad (\theta \approx 0)$$

Thus, effective path difference

$$\Delta = 2\mu t \pm \frac{\lambda}{2}$$

For air-film  $\mu = 1$ , then

$$\Delta = 2t \pm \frac{\lambda}{2}$$

(i) For Bright Fringe :  $\Delta = n\lambda$

$$2t \pm \frac{\lambda}{2} = n\lambda$$

OR 
$$2t = (2n \pm 1) \frac{\lambda}{2} \quad \dots(1)$$

$$n = 0, 1, 2, 3, \dots \quad (+ve \text{ sign})$$

or  $n = 1, 2, 3, \dots \quad (-ve \text{ sign})$

(ii) For Destructive interference or dark fringes:

$$\Delta = (2n \pm 1) \frac{\lambda}{2}$$

$$2t \pm \frac{\lambda}{2} = (2n \pm 1) \frac{\lambda}{2}$$

$$2t = n\lambda \quad \dots(2)$$

$$n = 0, 1, 2, 3, \dots$$

**Centre Dark : (Reflected light) :** At the point of contact (or at centre)

$$t = 0$$

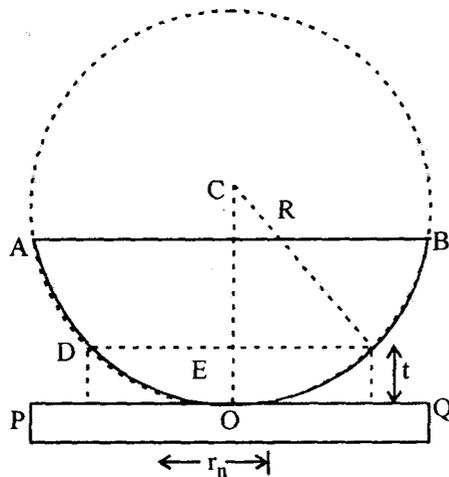
Therefore, effective path difference

$$\Delta = 2t \pm \frac{\lambda}{2}$$

$$\Delta = \pm \frac{\lambda}{2} \quad (\text{at } t = 0)$$

This is the condition for destruction interference so **centre will be dark** in case of reflected light.

**Diameters of Bright :** Let  $R$  be the radius of curved surface  $AOB$  of the lens and  $t$  the thickness of the film at any point  $F$ .



$$CF = R = CO$$

$$CE = CO - EO$$

$$CE = R - t$$

In  $\triangle CEF$ ,

$$R^2 = r_n^2 + (R - t)^2$$

$$R^2 = r_n^2 + R^2 - t^2 - 2Rt$$

( $r_n \rightarrow$  Radius of  $n$ th ring)

$$r_n^2 = 2Rt - t^2$$

$$r_n^2 = 2Rt \quad (R \gg t) \quad (t^2 \rightarrow \text{neglected})$$

$$t = \frac{r_n^2}{2R}$$

**For Bright Rings :**

$$2t \pm \frac{\lambda}{2} = n\lambda$$

$$2t = (2n + 1) \frac{\lambda}{2}$$

$$2 \frac{r_n^2}{2R} = (2n + 1) \frac{\lambda}{2}$$

$$r_n^2 = (2n + 1) \frac{\lambda R}{2}$$

In  $D_n$  is the diameter of  $n$ th bright ring, then  $D_n = 2r_n$ .

$$\left(\frac{D_n}{2}\right)^2 = (2n + 1) \frac{\lambda R}{2}$$

$$D_n^2 = \frac{4(2n + 1) \lambda R}{2}$$

$$D_n = \sqrt{2(2n + 1) \lambda R}$$

If liquid of  $RI = \mu$ , filled in place of air then,

$$2\mu t \pm \frac{\lambda}{2} = n\lambda$$

$$r_n^2 = \frac{(2n + 1) \lambda R}{2\mu}$$

$$D_n = \sqrt{\frac{2(2n + 1) \lambda R}{\mu}}$$

**5. Attempt any one part of the following.**

(a) Give the theory of plane transmission grating and show how would you use it to determine the wavelength of light.

Ans. Please see Q. No. 2(b) IInd Semester 2006-07

(b) Define the limit of resolution and resolving power Derive an expression for the resolving power of a grating.

Ans. **Resolving Power :** To see two close objects just as separate is called **resolution**. AND the **ability** or the **power** of the optical instrument to see two close objects just as separate is called **resolving power**.

**Resolution Limit :** Like eyes optical instruments also have a resolution limit.

The **smallest distance** between two close objects whose image can be seen just as separate by an

optical instrument is called resolution limit of that instrument.

Lesser the limit of resolution of the instrument, greater should be its resolving power.

$$\text{Resolving power} = \frac{1}{\text{Resolution limit}}$$

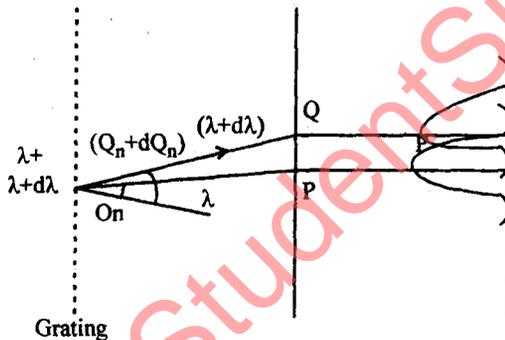
### Resolving Power of a Plane Diffraction Grating:

The resolving power of a grating is the ability to separate the spectral lines very close to each other.

“When two spectral lines in the spectrum produced by grating are first resolved, the ratio of the wavelength difference  $d\lambda$  and the mean wavelength  $\lambda$  of the spectral lines is called resolution limit of the diffraction grating and its reciprocal is called resolving power of the grating.

∴ In the position of just resolution, the resolving power of the grating is defined as

$$R = \frac{\lambda}{d\lambda}$$



Let a parallel beam of light of wavelength  $\lambda$  and  $\lambda + d\lambda$  be incident normally on the diffraction grating.

If  $n$ th principal maximum of  $\lambda$  and  $\lambda + d\lambda$  are formed in the direction  $\theta_n$  and  $\theta_n + d\theta_n$  respectively, then

For a principal maximum of wavelength  $\lambda$  at P.

$$(a + b) \sin \theta_n = n\lambda \quad \dots(1)$$

and for a principal maximum of wavelength  $(\lambda + d\lambda)$  at Q,

$$(a + b) \sin (\theta_n + d\theta_n) = n(\lambda + d\lambda) \quad \dots(2)$$

where  $n = 1, 2, 3, \dots$

and  $(a + b) \rightarrow$  Grating element.

The  $n$ th principal maximum of wavelength  $\lambda$  is formed at P. If first minimum of this wavelength is formed in the direction  $(\theta_n + d\theta_n)$  i.e. at the point Q, then according to Rayleigh's criterion the spectral lines of wavelengths  $\lambda$  will appear just resolved.

First minimum of wavelength  $\lambda$  formed in the direction  $(\theta_n + d\theta_n)$  i.e. at Q.

$$N(a + b) \sin (\theta_n + d\theta_n) = m\lambda$$

The first minimum adjacent of the  $n$ th principal maximum in the direction  $(\theta_n + d\theta_n)$  will be obtained for

$$m = nN \pm 1$$

where  $N$  is the total number of rulings on the grating.

$$(a + b) \sin (\theta_n + d\theta_n) = \frac{(nN + 1) \lambda}{N} = n\lambda + \frac{\lambda}{N} \quad \dots(3)$$

From equation (2) and (3), we get

$$n\lambda + \frac{\lambda}{N} = n(\lambda + d\lambda)$$

$$nd\lambda = \frac{\lambda}{N}$$

$$\text{Resolution limit of grating } \frac{d\lambda}{\lambda} = \frac{1}{nN}$$

and resolving power of the grating R

$$\approx R = \frac{\lambda}{d\lambda} = nN$$

= order of spectrum  $\times$  total number of ??

6. Attempt any one part of the following.

(a) Explain the phenomena of double refraction. Give the construction and theory of quarter wave plate and half wave plate.

Ans. Please see Q. No. 3(a) Ist Semester 2006-07.

(b) Discuss necessary condition to achieve laser action Describe the construction and action of ruby laser.

Ans. Necessary Conditions to Active Laser or Principle of Laser (Criteria of Laser Action) : In thermal equilibrium the population of atoms in different states is governed by Maxwell-Boltzmann law and the rate of transition of atoms by absorption a radiation is equal to the rate of transition by total emission of radiation.

$$\frac{\text{Emission rate}}{\text{Absorption rate}} = \frac{R_{nm}}{R_{mn}} = \frac{[A_{nm} + B_{nm}u(\nu)]N_n}{B_{mn}u(\nu)N_m}$$

But\*  $B_{nm} = B_{mn}$  [\* Means the probabilities of stimulated absorption and stimulated emission are same]

$$\frac{R_{nm}}{R_{mn}} = \left[ 1 + \frac{A_{nm}}{B_{nm}u(\nu)} \right] \frac{N_n}{N_m} \quad \dots(1)$$

$$\left[ \begin{array}{l} \text{Plank's law, energy density} \\ u(w) = \left( \frac{\hbar\omega^3 \mu^3}{a^2 c^3} \right) \frac{1}{e^{\frac{\hbar\omega}{kB}}} \end{array} \right.$$

For laser action two conditions must be satisfied :

- (i) The emission rate must be larger than the absorption rate
- (ii) The probability of spontaneous emission (which produces incoherent radiations) must be much smaller than the probability of stimulated radiation i.e.

$$A_{nm} \ll B_{nm}u(\nu)$$

A higher probability of stimulated emission can be achieved by the following methods :

(a) The energy density of incident radiation must be made very large i.e.

$u(\nu)$  must be very large

To increase the energy density, the emitted radiation is made to reflect again and again between two parallel mirrors in a cavity containing active medium.

(b)  $\frac{A_{nm}}{B_{nm}}$  must be made very small i.e.

$$\frac{A_{nm}}{B_{nm}} \ll 1$$

To minimize the value of  $\frac{A_{nm}}{B_{nm}}$ , metastable states of higher energy is chosen for stimulated transition

because transitions from metastable state by spontaneous emission are not allowed.

**Metastable States :** There are certain energy states whose life times are much longer (or the order of  $10^{-3}$  sec) than the life times of usual short lived ( $10^{-8}$  sec) excited states. Such long lived states are called **metastable state**.

(c)  $N_n > N_m$

$$\frac{R_{nm}}{R_{mn}} = \frac{N_n}{N_m} \left[ \text{As } \frac{A_{nm}}{B_{nm} u(\nu)} \ll 1 \right] \quad \dots(2)$$

But from Maxwell-Boltzman distribution law

$$N_n = N_o e^{-E_n/KT}$$

$$N_m = N_o e^{-E_m/KT}$$

where  $N_o \rightarrow$  number of atoms in ground state

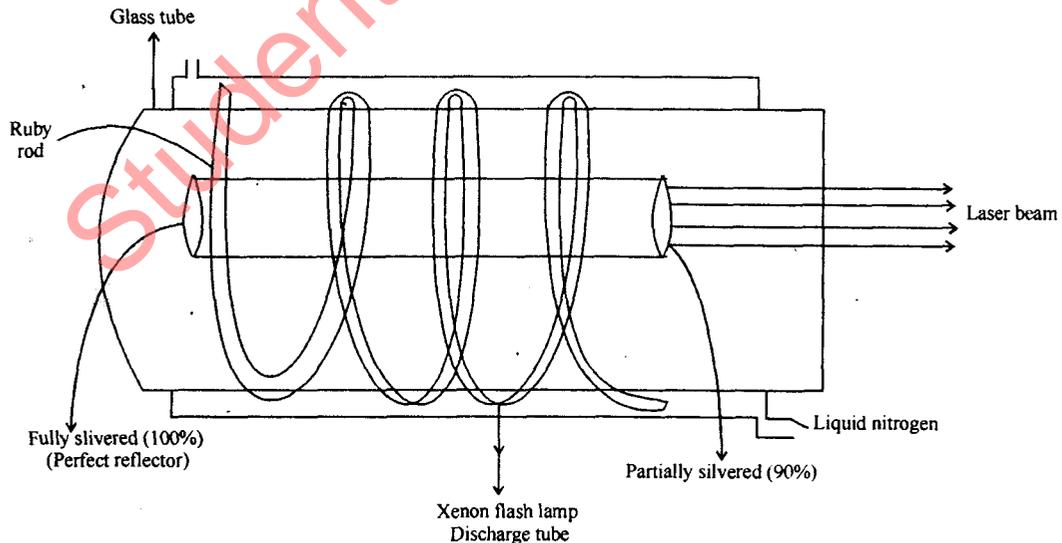
$$\frac{R_{nm}}{R_{mn}} = e^{-(E_n - E_m)/KT} \quad \dots(3)$$

**Population Inversion :** Under normal conditions of thermal equilibrium, the number of atoms in higher energy state is smaller than the number of atoms in lower energy states because

$$E_m < E_n$$

so  $N_n < N_m$

### Ruby Laser



The first laser to be operated successfully was fabricated by **Maiman in 1960**. It was **solid state laser**.

**Construction :** This laser has three main components

- (i) Active medium
- (ii) Optical resonator
- (iii) Pumping source

(i) **Active medium :** A single crystal of **pink ruby** used as an active medium. Ruby is initially a transparent crystal of aluminium oxide ( $\text{Al}_2\text{O}_3$ ) but it is doped with approximately 0.05% (by weight) of chromium oxide ( $\text{Cr}_2\text{O}_3$ ) in which  $\text{Cr}_3^+$  ion replace some of the  $\text{Al}_2^3$  ions in the crystal lattice and responsible for characteristic **pink colour** of ruby.

The Al and  $\text{O}_2$  atom are **inert**, the  $\text{Cr}_3^+$  ion are active ingredients. Ruby crystal is grown in shape of cylindrical rod with about **2 to 30 cms length** and about 0.5 to 2 cms in diameter.

(ii) **Optical Resonator :** The circular flat faces are made optical parallel and highly polished, and now ruby rod represents as an optical resonant cavity. One end of ruby rod is highly reflect (100% silvered) and the other end is partially reflection (90% silvered) to obtain an output laser beam.

The length of ruby rod is taken such that it should be equal to integral value of half wavelength.

$$L = n \left( \frac{\lambda}{2} \right)$$

where  $n$  is integer.

so that stable standing wave pattern to be formed in resonant cavity.

(iii) **Pumping Source :** The ruby crystal rod is placed along the axis of the helical xenon flash lamp tube which represents the pumping source.

The ruby rod is surrounded by helical flash lamp tube filled with xenon gas. The xenon gas discharge tube is connected to a suitable power supply through a capacitor which discharge through the xenon gas and emits the flashes.

The flash is sent through the tube and a major of the energy (flashes) emitted is used in heating the apparatus while some part of the energy (blue and green part) is absorbed by ruby which excited the  $\text{Cr}^{3+}$  ion of the rod to produce laser light.

**Working :** when light from the flash tube is made to fall upon the ruby rod, a part of this energy or flashes is absorbed by the electrons of chromium ions in the ground state and goes to excited energy bands  $E_1$  and  $E_2$ . This is called **optical pumping**.

The pumping transition to  $E_1$  and  $E_2$  bands are caused by absorption of radiation of wavelengths nearly equal to 6600 Å and 4000 Å (the blue and green part of spectrum).

The excited electrons quickly drop back to ground state but some electrons drops down to intermediate state or metastable by a very fast non-radiative transition. (i.e. the excess energy is absorbed by the lattice and does not appear in the form of electromagnetic radiation).

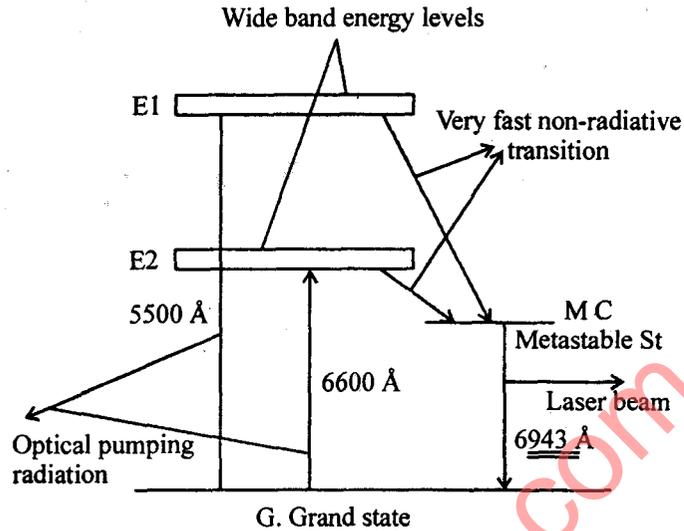
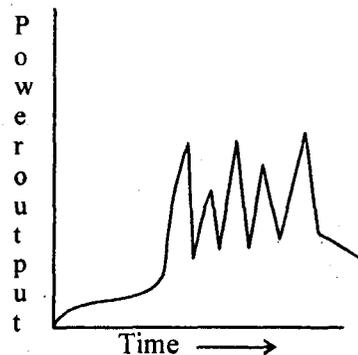


Fig. The energy levels of the  $\text{Cr}^{3+}$  ion

Since the metastable state has a long life time ( $\sim 10^{-3}$  sec), the number of atoms in this state keeps on increasing, and condition of population inversion is achieved between Metastable and ground state. Once population inversion is achieved, light amplification can take place, with in the two reflecting end mirrors of the ruby rod forming a resonant cavity.

At this stage, a triggering beam of light of wavelength  $6943 \text{ \AA}$  will initiate laser action. Usually the wavelength  $6943 \text{ \AA}$  is chosen for triggering as the probability of induced emission by this light is about 1.4 time more than at the other wavelength.

If even one photon out of the photon produced by induced emission moves parallel to the axis of the ruby crystal, it undergoes multiple reflection between the parallel mirrors. When this photon comes in contact with an excited atom in the metastable state, it stimulates this excited atom and causes to emit a fresh photon in phase with the stimulating photon. This process is repeated again and again and the photons having same phase thus multiply. This regenerative action continuous and a part of the beam comes out as a red colour laser beam from the semi-transparent end of ruby crystal.



Till the flash lamp power falls below the threshold value

**Spiking in Ruby Laser :** Ruby laser produces **high energy** pulses of coherent light of **very short duration** of the order of **microsecond**. These pulses are called spikes. These spikes are due to the pulsating flash lamp used for **optical pumping**.

**Reason :** When the pulsating flash lamp is suddenly switch on the atoms in the lower state get energy and transit to the metastable state. As soon as the energy given by flash lamp reaches a threshold value (a minimum level of energy needed to produce laser action), the population inversion build up and the laser process set in. After some time population inversion comes down, the laser action steps till the next flash occurs. The flash lamp again excites the atoms to metastable state and again laser action set on. The lasing action repeats itself.

7. **Attempt any one part of the following.**

(a) **Explain single mode and multimode fiber. What are advantages of optical fiber over copper wire?**

**Ans. Single Mode and Multimode Fibers :**

**Single Mode Fiber**

1. In a single mode fiber, the diameter of the core is very small and is of the same order as the *wavelength of light to be propagated*. It is in the range  $5\ \mu\text{m}$  to  $10\ \mu\text{m}$ . The cladding diameter is about  $125\ \mu\text{m}$ .
2. The difference in the refractive indexes of the core and the cladding materials is very small.
3. Numerical aperture of SM fiber is small.
4. SM fiber has a very high capacity of carrying information.
5. Modal dispersion of SM fiber is almost nil.
6. Material dispersion in SM fiber is low.

**Multimode Fiber**

1. In a multimode fiber the diameter of the core is large. It is in the range  $30\ \mu\text{m}$ - $100\ \mu\text{m}$ . The cladding diameter is in the range  $125\ \mu\text{m}$ .
2. The difference in the refractive index of the core and cladding material is large.
3. Numerical aperture of MM fiber is large.
4. MM fiber has low information carrying capacity.
5. Modal dispersion in MM fiber is the dominant source of dispersion.
6. Material dispersion in MM fiber is large.

**Advantage of Optical Fiber Over Copper Wire :** The advantages of optical fiber transmission over conventional copper wire system can be given as:

1. Wide bandwidth
2. Low loss
3. Electromagnetic immunity
4. Security
5. Light weight
6. Small size
7. Safety

(b) Discuss the phenomena of dispersion in optical fiber physical Constants

Planck's constant  $h = 6.63 \times 10^{-31} \text{ J}\cdot\text{s}$

Velocity of light  $c = 3 \times 10^8 \text{ m/s}$  in free space Rest mass of electron  $m_e$

$= 9.1 \times 10^{-31} \text{ kg}$ .

**Ans. Dispersion in Optical Fiber :** Dispersion is the spreading of light pulse as it travels down along the length of the fiber causing the pulses to overlap and thus making the pulses undetectable at the receiving end. Dispersion is of three types which are as follows :

1. **Modal or intermodal dispersion :** This type of dispersion we get in multimode fiber. It arises due to different velocities of different modes.
2. **Material dispersion :** It is caused due to property of material i.e. due to variation of velocity with the wavelength. We do not have absolute monochromatic light and different wavelength travel with different velocity even when all the light follow the same path. Thus, pulses reaches at the end of the fiber at slightly different times. Thus, output signal is lengthened relative to the input signal.
3. **Waveguide Dispersion :** Waveguide dispersion is important for transmission rate is the case of single mode. It occurs because guided optical energy is divided between core and cladding. Energy travels with slightly different velocity in cladding. The wavelength  $1.5 \mu\text{m}$  is the present day choice of fiber optic communication because distortion is much less at this wavelength.