

MATHEMATICS - 2004

SECTION - A

Q. 1. If $A = \begin{pmatrix} 3 & -5 \\ -4 & 2 \end{pmatrix}$, show that $A^2 - 5A - 14I = O$.

Q. 2. Using properties of determinants, solve for x:

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

Q. 3. An urn contains 7 white, 5 black and 3 red balls. Two balls drawn at random. Find the probability that

- (i) Both the balls are red
- (ii) One ball is red, the other is black
- (iii) One ball is white

Q. 4. A fair die is tossed twice. If the number appearing on the top is less than 3, it is a success.

Q. 5. Evaluate: $\int \sin^4 x \, dx$.

Q. 6. Evaluate: $\int \frac{\sin^{-1} x}{x^2} dx$.

Q. 7. Form the differential equation corresponding to $y^2 = a(b - x^2)$ where a and b are arbitrary constants.

Q. 8. Solve the differential equation: $(1 + e^{2x})dy + e^x(1 + y^2)dx = 0$
Given the $y = 1$, when $x = 0$

Or

Solve the differential equation: $x \frac{dy}{dx} - y - 2x^3 = 0$.

Q. 9. Show that in a Boolean algebra, B:

- (i) $a \vee (a \wedge b) = a \vee b$
- (ii) $a \wedge (a \vee b) = a \wedge b$

Q. 10. Evaluate: $\lim_{x \rightarrow 0} \frac{x \tan 4x}{1 - \cos 4x}$

Q. 11. Differentiate $\sqrt{\cos x}$ w.r.t. x from first principles.

Q. 12. Differentiate $\cot^{-1}\left(\frac{1-x}{1+x}\right)$ w.r.t. x .

Q. 13. Find the equations of the tangent and the normal to the curve $x = 1 - \cos \theta, y = \theta - \sin \theta$ at $\theta = \frac{\pi}{4}$.

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Q. 14. Evaluate: $\int \frac{1}{3+2\sin x + \cos x} dx.$

Q. 15. Evaluate: $\int_0^{\pi/2} \frac{\cos x}{(1+\sin x)(2+\sin x)} dx.$

Q. 16. Using matrix method solve the following system of linear equations:

$$x + y + z = 3$$

$$2x - y + z = 2$$

$$x - 2y + 3z = 2$$

Q. 17. Show that a right circular cylinder which is open at the top, and has a given surface area, will have the greatest volume if its height is equal to the radius of its base.

Q. 18. Using integration, find the area of the circle $x^2 + y^2 = 16$ which is exterior to the parabola $y^2 = 6x$.

Or

Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $\frac{x}{a} + \frac{y}{b} = 1$.

SECTION - B

Q. 19. If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$, then show that the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal.

Or

Find x such that the four points $A(3, 2, 1), B(4, x, 5), C(4, 2, -2)$ and $D(6, 5, -1)$ are coplanar.

Q. 21. Find the Cartesian and vector equations of a line which passes through the point (1, 2, 3,) and is parallel to the line.

Q. 22. Find the Cartesian equation of the sphere which has the points A (2, -3, 4) and B (-5, 6, -7) as the end points of one of its diameters, also find its centre and radius.

Q. 23. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect. Find the point of intersection also.

Q. 24. Three forces \vec{P}, \vec{Q} and \vec{R} acting on a point are in equilibrium. If the angle between \vec{P} and \vec{Q} and be double the angle between \vec{R} and \vec{P} . prove that $PQ = Q^2 - R^2$

Q. 25. Two unlike parallel forces \vec{P} and \vec{Q} ($\vec{P} > \vec{Q}$) act at two points units apart. Show that if the direction of \vec{P} be reversed, the resultant will be displaced by a distance $\frac{2PQ}{P^2 - Q^2} \cdot x$ units.

Q. 26. A particle starting with some initial velocity and moving with uniform acceleration acquires a velocity of 20 cm/sec after moving through 10 cm from a point P to Q. and a velocity of 30 cm/sec after further moving 20 cm from Q to R in the same direction. Find

(i) its velocity at the point P.

(ii) its acceleration.

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(iii) The time it will take and the distance

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Or

From a point on the ground at a distance from the foot of a vertical wall, a ball is thrown at an angle of 45° which just clears the top of the wall and afterwards strikes the ground at a distance of y on the other side of the wall. Find the height of the wall.

SECTION - C

Q. 19. Find the banker's discount and true discount on a bill of Rs. 22,800 due 4 months hence at 4% per annum.

Q. 20. A bill of exchange drawn on January 4, 2003 at 5 months date was discounted on March 26, 2003 at 3% per annum. If the banker's discount is Rs. 1207.20, find the face value of the bill.

Q. 21. Three urns A, B and C contain 6 red and 4 white; 2 red and 6 white; and 1 red and 5 white balls respectively. An urn is chosen at random and a ball is drawn. If the ball drawn is found to be red. Find the probability that the ball was drawn from urn A.

Q. 22. The mean and variance of a binomial distribution are 4 and $\frac{4}{3}$ respectively. $P(X \geq 1)$.

Or

If the sum of the mean and variance of a binomial distribution for 5 trials be 1.8, find the distribution.

Q. 23. A, B and C are partners in a business. A, being a working partner, receives 10% of the total profit as salary. The remaining profit is distributed among them in the ratio of 2 : 3 : 4. If A gets Rs. 3,00,000 in all, find the shares of B and C.

Q. 24. Find the present value of an annuity of Rs. 1,200 payable at the end of each 6 months for 3 years, when the interest is 8% per annum, compounded semi-annually. [Use $(1.04)^6 = 1.2653$]

Q. 25. The total cost and demand function of an item are given by

$$C(x) = \frac{x^3}{3} = 7x^2 + 111x + 50$$

and $p = 100 - x$ respectively.

Write the total revenue function and the profit function. Find the number of items when the profit will be maximum. Find the maximum profit also.

Q. 26. An oil company requires 13,000, 20,000 and 15,000 barrels of high grade, medium grade and low grade oil respectively. Refinery A produces 100,300 and 200 barrels per day of high, medium and low grade oil respectively whereas the Refinery B produces 200,400 and 100 barrels per day respectively. If A costs Rs. 400 per day and B costs Rs. 300 per day to operate, how many days should each be run to minimise the cost of requirement?

Or

A firm makes items A and B and the total number of items it can make in a day is 24. It takes one hour to make an item of A and only half an hour to make an item of B. The maximum time available per day is 16 hours. The profit on an item of A is Rs. 300 and on one item of B is Rs. 160. How many items of each type should be produced to maximise the profit? Solve the problem graphically.