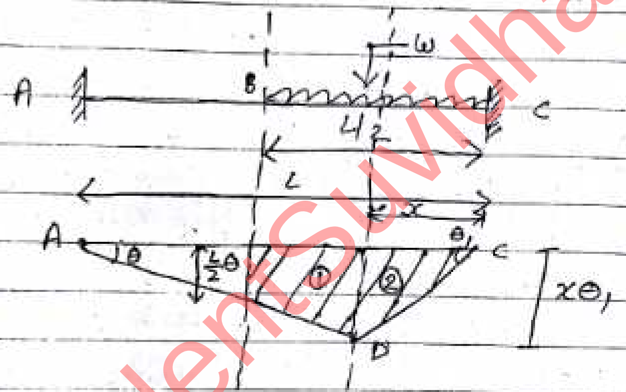
DSS

[SECTION - A]

Q1 A "beam fixed" at both ends is subjected to uniformly distributed load w on its Right Half Portion as shown. Determine the collapse load of the beam has uniform cross section.



Sol. for a fixed ended beam and loaded partly ~~or~~ for its span as shown, the possible plastic hinges will be at A, C and at point D at a distance x from support C.

From the mechanism,

$$\Delta = (L-x)\theta = x\theta_1$$

$$\therefore \left[\theta_1 = \frac{L-x}{x}\,\theta \right]$$

$\Rightarrow$ External W.D = Intensity of load $\times$ area of collapse Mechanism Diagram

$$= \frac{w_u}{L/2} \times \left[\frac{1}{2}\left(x\theta_1 + \frac{L}{2}\theta\right)\left(\frac{L}{2}-x\right) + \frac{1}{2}(x)(x\theta_1) \right]$$

As the Diagram Can be Resolved into 2 Figures i.e one Trapezoid and one Triangle

① Area of Trapezoid $= \frac{1}{2}$ (Sum of Parallel sides) $\times$ height

$$\Rightarrow \frac{1}{2}\left(x\theta_1 + \frac{L}{2}\theta\right)\cdot\left(\frac{L}{2}-x\right)$$

② Area of Triangle $= \frac{1}{2} \times B \times H$

$$\Rightarrow \frac{1}{2} \times x \times x\theta_1$$

$$\therefore \text{Ext. W.D} = \frac{2w_u}{L}\left[\frac{1}{2}\left(x\cdot\frac{L-x}{x} + \frac{L}{2}\right)\left(\frac{L}{2}-x\right)\theta + \frac{x^2}{2}\cdot\frac{L-x}{x}\,\theta\right]$$

$$= \frac{w_u}{L}\left[\left(L-x+\frac{L}{2}\right)\left(\frac{L}{2}-x\right) + x(L-x)\right]\theta$$

$$= \frac{W_u}{L}\left[\frac{3}{4}L^2 - \frac{3}{2}Lx - \frac{Lx}{2} + x^2 + Lx - x^2\right]\theta$$

$$= \frac{W_u}{L}\left[\frac{3}{4}L^2 - Lx\right]\theta$$

$$\Rightarrow \boxed{W_u\left(\frac{3}{4}L - x\right)\theta}$$

Internal W.D $= M_p\theta + M_p(\theta + \theta_1) + M_p\theta_1$

$= 2M_p(\theta + \theta_1)$

$= 2M_p\left[\theta + \frac{L-x}{x}\theta\right]$

$= 2M_p\left[1 + \frac{L-x}{x}\right]\theta$

$$\Rightarrow \boxed{2M_p\left(\frac{L}{x}\right)\theta}$$

By the Principle of virtual work :-

External W.D = Internal W.D

$$W_u\left[\frac{3}{4}L - x\right]\theta = 2M_p\left(\frac{L}{x}\right)\theta$$

$$\therefore M_p = \frac{W_u}{2L}\left[\frac{3}{4}Lx - x^2\right]$$

Here M_p = plastic Moment

for the Max. value of Mp,

$$\left[\frac{dM_p}{dx} = 0 \right]$$

$$\frac{W_u}{2L}\left(\frac{3}{4}L - 2x\right) = 0$$

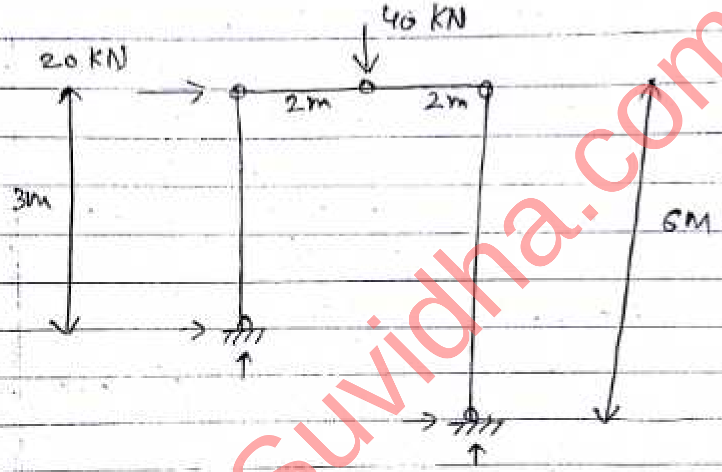
$$x = \frac{3}{8}L = \boxed{0.375L}$$

$$\therefore M_p = \frac{W_u}{2L}\left[\frac{3}{4}L(0.375L) - (0.375L)^2\right]$$

$$M_p = \frac{0.07031\, W_u}{L}$$

$$\therefore \boxed{W_u = \frac{14.22\, M_p}{L}} \quad (\text{collapse load})$$

Q2 Find out the "Collapse load of given "frame". [20]



Sol. $N = 5$ (no: of Joints) [no: of plastic hinges]

$r = 6 - 3 = 3$ (degree of indeterminancy)

[6 = no: of forces acting on span

3 = no: of spans]

No: of mechanisms $= N - r$ (n)

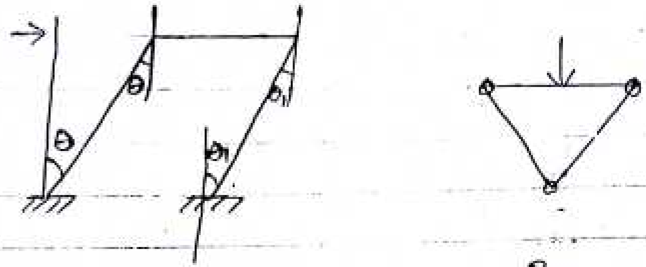
$= 5 - 3$

$= 2$

Two mechanisms will be there.

[I mechanism]:-

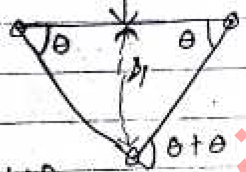
Sway & Beam



Sway   Beam

40 KN (load)

(I)



$\theta + \theta$ (Deflection)

load x Defl. $\Rightarrow$ Ext. W.D   Int. W.D

$\boxed{40 \times \Delta_1}$

$\left[\frac{\Delta_1}{2} = \theta\right]$ $(\Delta_1 = 2\theta)$

$$40 \times 2\theta = M_p\theta + M_p(\theta+\theta) + M_p\theta$$

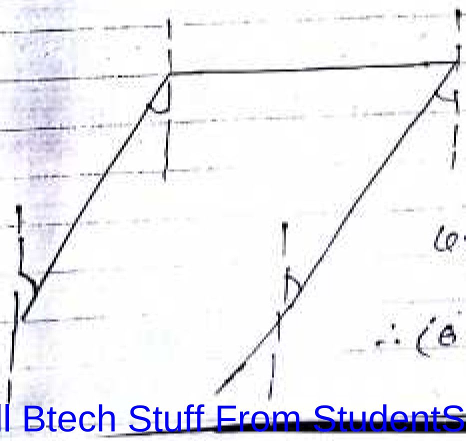
$$= M_p\theta + 2M_p\theta + M_p\theta$$

$$80\theta = 4M_p\theta$$

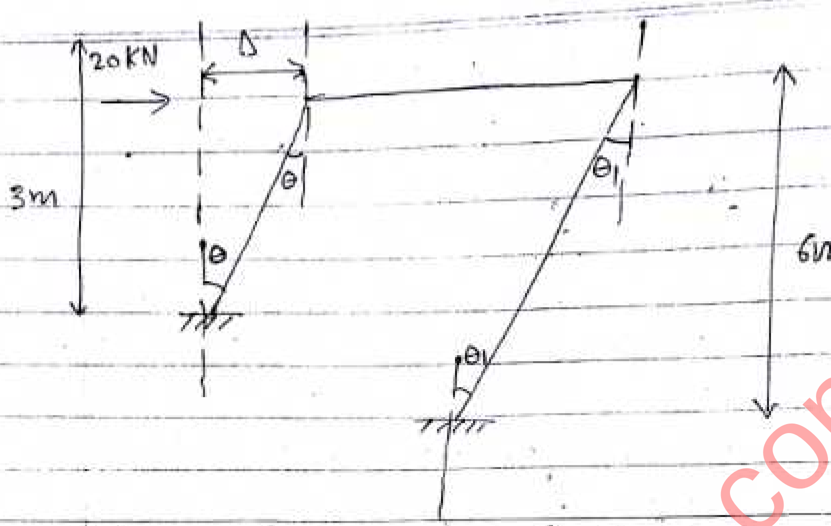
$$\therefore \boxed{M_p = 20 \text{ KN-m}}$$

II mechanism :-

(II)



Here,
lengths are unequal
$\therefore$ ($\theta \neq$ same)



Since members are unequal, so,
Deflection will be unequal (θ & θ_1).

$$20 \times \Delta = M_p\theta + M_p\theta + M_p\theta_1 + M_p\theta_1$$
$$= 2M_p\theta + 2M_p\theta_1$$
$$20\Delta = 2M_p(\theta + \theta_1)$$

We have to find out Relation b/w θ & θ_1

Here, $\frac{\Delta}{3} = \theta$, $\frac{\Delta}{6} = \theta_1$

$\Delta = 3\theta$, $\Delta = 6\theta_1$

$\therefore$ $3\theta = 6\theta_1$

or, $[\theta = 2\theta_1]$

Now, $20\Delta = 2M_p(\theta + \theta_1)$

$20(3\theta) = 2M_p(2\theta_1 + \theta_1)$

$60\theta = 6M_p\theta_1$

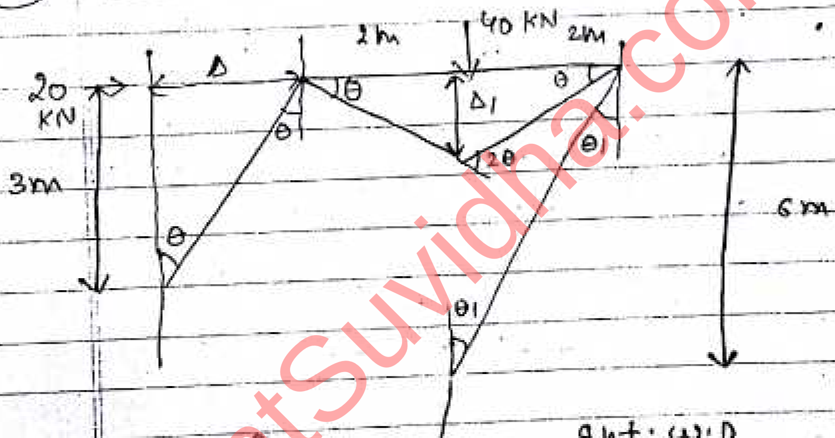
[Δ = Deflection below the load
$\Delta = 3\theta$

$$Mp = 10 \frac{\theta}{\theta_1}$$

$$Mp = 10 \times 2$$

$$\therefore \boxed{Mp = 20 \text{ KN-m}}$$

(III) "Combined mechanism" :-



$$\underbrace{20\Delta + 40\Delta_1}_{\text{Ext. W.D}} = \underbrace{Mp\theta + Mp\theta + Mp(2\theta) + Mp\theta_1 + Mp\theta_1}_{\text{Int. W.D}}$$

$$= 2Mp\theta + 2Mp\theta + 2Mp\theta_1$$

$$\boxed{20\Delta + 40\Delta_1 = 4Mp\theta + 2Mp\theta_1}$$

Here, $\frac{\Delta}{6} = \theta_1$, $\Delta = 6\theta_1$

$\frac{\Delta}{3} = \theta$, $\Delta = 3\theta$

$\frac{\Delta_1}{2} = \theta$, $\Delta_1 = 2\theta$

and $\underline{\theta = 2\theta_1}$

$$20(\Delta + 2\Delta_1) = 2(2M_p\theta + M_p\theta_1)$$
$$10(\Delta + 2\Delta_1) = 2M_p\theta + M_p\theta_1$$
$$10(3\theta + 2(2\theta)) = 2M_p(2\theta_1) + M_p\theta_1$$
$$70\theta = 5M_p\theta_1$$
$$70(2\theta_1) = 5M_p\theta_1$$
$$140\theta_1 = 5M_p\theta_1$$

$$\therefore \boxed{M_p = 28 \text{ KNm}}$$

We have to choose the max. value of all Mp's

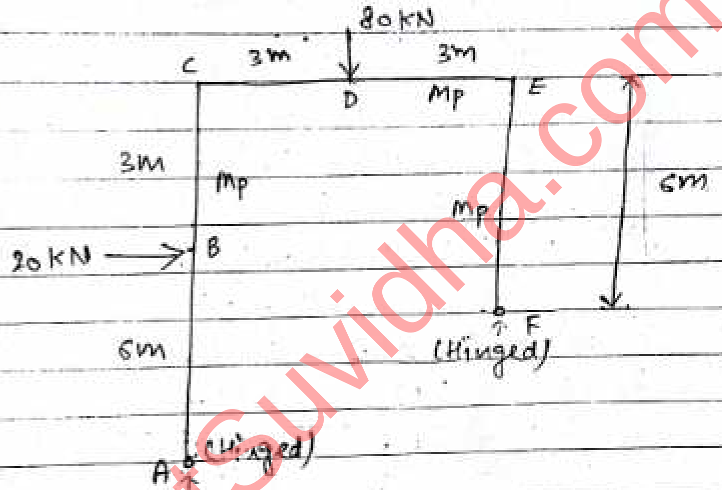
$$\therefore \boxed{M_p = 28 \text{ KNm}}$$

☆ In plastic ends, two forces will act on one end. i.e, H & V ($\rightarrow$ $\uparrow$)

☆ In Hinged ends, only one force will act on one end i.e V ($\uparrow$)

(20) MDU

Q3 Find out the collapse load for a given frame of uniform cross section under the applied uniform loads as shown.



Sol. The possible locations of plastic hinges are B, C, D, E [Correct]

No. of plastic hinges, N = 4

Degree of redundancy, $r = 4 - 3 = 1$ (9c)

$\therefore n = N - r$

$= 4 - 1$

$= \boxed{3}$ (no. of Mechanisms)

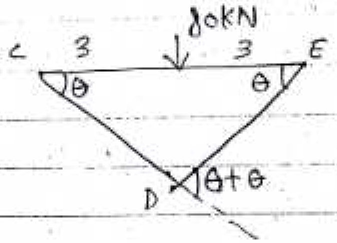
There will be 3 mechanisms :-

→ Beam mechanism, span CE

→ Beam mechanism, span AC

→ Sway mechanism.

① First mechanism :- (Span CE) :-



[Ext. W.D
$= 80\Delta$
$\frac{\Delta}{3} = \theta, \Delta = 3\theta$]

External W.D = load × Deflection
$= 80 \times 3\theta = 240\theta$

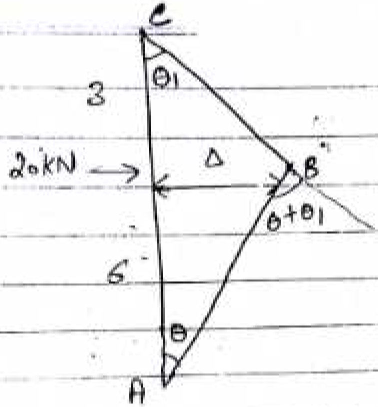
Internal W.D = $M_p\theta + M_p(2\theta) + M_p\theta$
$= 4M_p\theta$

By Principle of virtual work,
$240\theta = 4M_p\theta$
$\therefore \boxed{M_p = 60 \text{ KNm}}$

In this case, plastic hinges will be developed at three points C, E, D.

② Second mechanism :- (Span AC) :-

In this case, end A is a mechanical hinge, so, no mp will develop there
plastic hinges will develop only at B, C.



Here, $\frac{\Delta}{3} = \theta_1$, $\frac{\Delta}{6} = \theta$

$[\Delta = 3\theta_1 = 6\theta]$

& $[\theta_1 = 2\theta]$

Ext W.D = 20Δ = $20(6\theta)$ = 120

Int. W.D = $M_p\theta_1 + M_p(\theta+\theta_1)$

$= M_p(2\theta) + M_p(3\theta)$

$= 5M_p\theta$

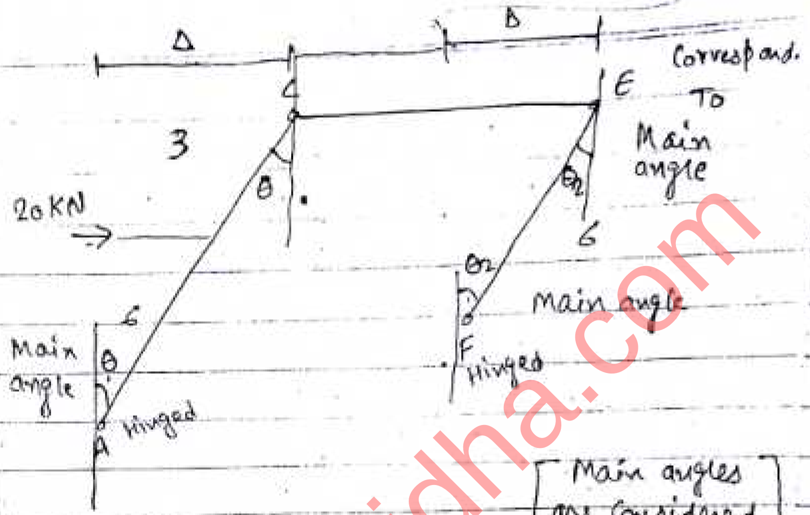
Now, $120\theta = 5M_p\theta$

$[\therefore M_p = 24 \text{ kN-m}]$

III Third mechanism :- (sway) :-

In this Case, Columns A & F are hinged, no plastic hinges will be formed.

plastic hinges will be formed at points C & E.



$\frac{\Delta}{9} = \theta$, $\Delta = 9\theta$

$\frac{\Delta}{6} = \theta_2$, $\Delta = 6\theta_2$

and $\theta_2 = 1.5\theta$

[Main angles are Considered not corresponding only For (Hinged cndt.)]

Ext. W.D = $20 \times \Delta = 20(6\theta) = 120\theta$

(Deflection below the load) is Before the load

Int W.D = $M_p\theta + M_p\theta_2$ [Two angles]

$= M_p\theta + M_p(1.5\theta)$

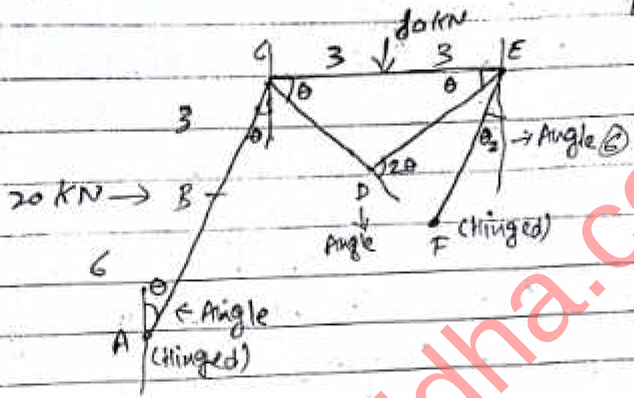
$= 2.5 M_p\theta$

Now $120\theta = 2.5 M_p\theta$

$\therefore [M_p = 48]$ kN-m

(IV) Combined mechanism :-

Sway and span CE :-



[Angles at point C & E are not considered in Int. W.D as 0° angle will be form on movement]

Ext. W.D = Σ(load × Deflection)

$= 80(3\theta) + 20(6\theta)$

$= \underline{360\,\theta}$

Int. W.D = Σ(Plastic Moment × Rotation)

$= M_p(\theta+\theta) + M_p\theta + M_p\theta_2$

$= M_p(2\theta) + M_p\theta + M_p\theta_2$

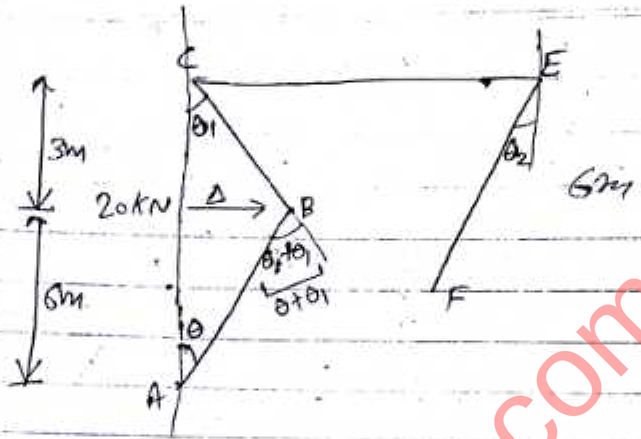
$= \underline{4.5\,M_p\theta}$

Now, $360\,\theta = 4.5\,M_p\theta$

$\therefore [M_p = \underline{80}\ \text{KN-m}]$

(V) Combined mechanism :-

Sway and span AC :-



$$Ext.W.D = \Sigma(load \times Deflection)$$
$$= 20(6\theta) + 20(6\theta)$$
$$= \underline{240\theta}$$

$$Int.W.D = \Sigma(plastic\ Moment \times Rotation)$$
$$= \underline{Mp(\theta+\theta_1)} + \underline{Mp\theta_1 + Mp\theta_2}$$
$$\left(\begin{matrix}\theta, \theta_1 \\ \theta_1, \theta_2\end{matrix}\right)$$
$$= \underline{6.5Mp\theta}$$

Now, $240\theta = 6.5Mp\theta$

$\therefore [Mp = 36.9]$ kN-m

Final Mp = largest of all

$\underline{Mp = 80\ kN\text{-}m}$