# Design of Steel Structure – II

7th Sem.

CE-401-F

# PLASTIC ANALYSIS – It is defined as the analysis in which the criterion for the design of structures is the Ultimate Load. We can define it as the analysis inelastic material is studied beyond the elastic Limit (Which can be observed in stress-strain Diagram). Plastic Analysis derives from a simple mode failure in which plastic hinges form.

Actually the ultimate load is found from the strength of steel In plastic range. This method of analysis is quite rapid and has rational approach for analysis of structure. It control the economy regarding to weight of steel since the sections required by this method are smaller than those required by the method of elastic Analysis. Plastic Analysis has its application In the design of indeterminate structures.

Basics of Plastic Analysis –

Plastic Analysis is usually Based on the idealization of stress strain curve as perfectly plastic. In this analysis It is assumed that width thickness ratio of plate elements is small so the local buckling doesn't occur. Broadly speaking the section will be declared as perfectly plastic. Keeping in mind these assumptions, it can be said that section will reach its plastic moment capacity and after that will be subjected to considerable moment at applied ~~load~~ Moments.

Principles of Plastic Analysis –

There are 3 conditions for Plastic Analysis –

→ a) Mechanism Condition – When the ultimate Load is reached collapse, mechanism usually formed.

→ b) Equilibrium Condition –

$$\Sigma F_x = \Sigma F_Y = \Sigma F_z = 0$$

→ c) Plastic Moment Condition – The B.M. at any section in the structure should not be more than the full Plastic Moment of the section.

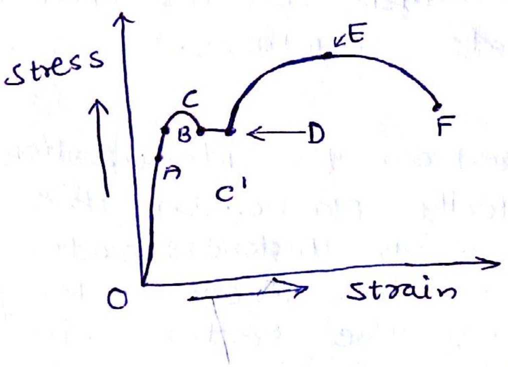
# Plastic Moment – If we consider the case of s.s. Beam, When the load is gradually applied on it, B.M. & Stresses increases. As the load is increased, the stresses in fibers of beam reach to yield stress. At this stage the moment which has converted the stresses into the yield stress is said to be as Plastic Moment. It is denoted by Mp.

$$\text{Shape Factor} = \frac{\text{Plastic Moment}}{\text{Yield Moment}} = \frac{M_P}{M_y}$$

Shape Factor usually depend upon the shape of the cross-section.

# Mechanisms — When the Load is applied on the body which is elastic, it will show resistance against deformation, such a body is called to be as structure. On the other hand if no resistance is shown against the body then it is known as mechanisms.

Idealized Stress-strain curve For Mild Steel—




Hooke's law

Stress ∝ Strain

Point

A = Limit of proportionality beyond which linear variation ceases.

B = Elastic Limit or the maximum stress upto the specimen regain its original length on Removal of the applied load.

C = Upper Yield Point at which there is a definite increase in strain without any further ↑ in stress.

The stress drops abruptly from upper yield stress to lower yield stress and strain ↑ at constant stress upto the point D.

E represents the point of max. stress corresponding to the ultimate load, beyond which an ↑ of strain occurs with a ↓ of stress until the specimen breaks at F.

$\sigma_u$, $\sigma_L$ = upper & lower yield stress

E = Young's Modulus

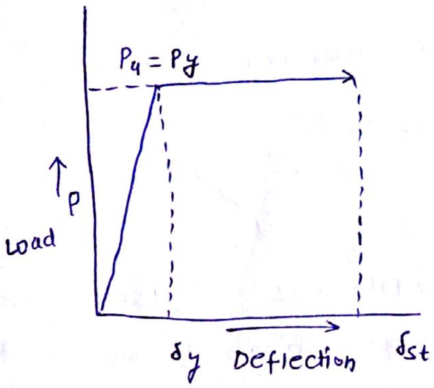
⇒ The elastic deformation occurs upto the elastic limit.

## # Scope of Plastic Analysis –

The Plastic Methods of Structural Analysis find their Principal application in – (Extra)

a) Design of Redundant Framed Mild steel structures
b) continuous or restrained Beams
c) Girders which carry load by virtue of flexural resistance.
d) For Plane Frames
e) In Truss Members (Tension & compression Members)

## # Ultimate Load – Carrying Capacity of Tension Members –




Load Deflection curve for a Tension Member

Consider a member of cross-sectional Area A, subjected to Axial tension P.

The deflection at the elastic limit is

$$\delta_y = \frac{P_y \, l}{A_s E}$$

$P_y$ = yield Load.

The Plastic flow in the member sets in as soon as the load reaches the value

$$P_y = f_y A_s$$

As soon P equal to $P_y$ the deflection ↑ instantaneously and unobstructed to $\delta_{st}$.

$\delta_{st}$ = deflection at the start of strain hardening.

Since the deformation at this stage of $\delta_{st}$ is 1·5 % of the length of member, which is too large for most structures, the usefulness of the tension members is practically finished at the start of strain hardening. Hence, for all practical Purposes the yield load $P_y$ is considered to be the ultimate load $P_u$. Hence the value of $P_u$ remains constant and is termed as the ultimate load for a tension member.

The ultimate load concept is of little use in the design of tension members, as is obvious from the curve that due to a load producing an average direct stress slightly above the elastic limit the section ruptures.

⇒ The average rupture stress in tension may be nearly twice the elastic limit ~~load~~ upon the original x-area. This reflects that there should be a large reserve capacity in a tension member. Therefore in a tension member the eccentricity of load, if any can be ignored as it is taken care of by the margin.

As per the specification of IS: 800-1984 the ultimate load carrying capacity is calculated as

$$P_{at} = 0.85\, A_s f_y$$

where $P_{at}$ = the calculated max. Load capacity

$A_s$ = effective -x- area of the member

$f_y$ = yield stress of steel.

# Ultimate Load-Carrying capacity of Compression Members –

A steel compression member of normal stiffness as used in structures buckles at an average stress somewhere near the elastic limit. Hence there may be no reserve capacity beyond the yield load in a column. If there is no buckling the ultimate load carrying capacity is the product of the -x- area and the max. stress attained before the onset of strain hardening.

$$P_u = f_y A_s$$

As per the specification of IS: 800-1984, the max. Load carrying capacity of a compression member is given by the following expression:

$$P_{ac} = 1.7\, A_s \sigma_{ac}$$

where

$P_{ac}$ = calculated max. load carrying capacity

$A_s$ = eff. -x- area of the member

$\sigma_{ac}$ = max. permissible stress in axial compression

In practice, the compression members are slender and may buckle at an average stress far below the elastic limit.

The max. Moment capacity of a beam-column member is reduced and should be checked by the following expression, depending upon the ratio of the axial force P to the yield strength of the member $P_y$.

i) if $P/P_y > 0.15$

then $$\frac{P}{P_y} + \frac{M_{pc}}{1.18\, M_p} \le 1.0$$

ii) if $\frac{P}{P_y} > 0.15$ and $> \frac{1+\beta-\lambda_0}{1+\beta+\lambda_0}$

then $\frac{P}{P_{ac}} + \frac{M_{pc}C_m}{M_o\left(1-\frac{P}{P_e}\right)} \leq 1.0$

iii) Any other case not covered by above cases:

$$\frac{M_{pc}}{M_p} \leq 1.0$$

where $P$ = axial comp. force

$P_y$ = yield strength of the axially loaded section = $A_s f_y$

$P_e$ = Euler load = $\frac{\pi^2 E A_s}{(l/r)^2}$

$M_p$ = Plastic moment capacity of the section

$M_{pc}$ = Max. plastic moment capacity of the beam column.

$M_o$ = lateral buckling strength in the absence of axial load. If the column is laterally braced, $M_o$ is equal to $M_p$

$\beta$ = ratio of end moments

$\lambda_0$ = characteristic slenderness ratio

$= \sqrt{P_y/P_e} = \frac{l}{\pi r}\sqrt{f_y/E}$

$l$ = eff. length and is taken equal to actual length. The eff. length factor in plastic analysis is taken as 1

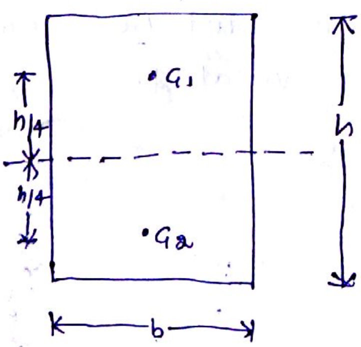
$C_m$ = coeff. defined in article 7.1.3 of I.S: 800-1984

$r$ = radius of gyration about the same axis about which the B.M. acts.

# Shape Factor -

$$f = \frac{M_p}{M_y} = \frac{f_y Z_p}{f_y Z} = \frac{Z_p}{Z}$$

$Z$ and $Z_p$ are the elastic and plastic section moduli.




Rectangular section -

the elastic modulus of section

$$Z = \frac{bh^3}{12} \Big/ \frac{h}{2} = \frac{bh^2}{6}$$

the plastic modulus of section

$$Z_P = \frac{A}{2}(\bar{Y}_1 + \bar{Y}_2) = \frac{bh}{2} \times \left(\frac{h}{4} + \frac{h}{4}\right)$$
$$= \frac{bh^2}{4}$$
$$f = \frac{Z_P}{Z} = \frac{bh^2}{4} \Big/ \frac{bh^2}{6} = 1.5$$

# Plastic collapse– When a sufficient number of plastic hinges are formed to convert a structure into a mechanism, the structure collapses. The plastic collapse of a structure depend upon its redundancy. At such a stage the deflection ↑ very fast at constant load. The collapse of a structure can be partial, complete or over complete.

If
$r$ = indeterminancy
$N$ = no. of plastic hinges
then
if

| | | |
|---|---|---|
| $N < (1+r)$ | partial collapse | (less) |
| $N = (1+r)$ | complete collapse | (equal) |
| $N > (1+r)$ | overcomplete Collapse | (more) |

→ Principal of virtual work—
It state that

If a system of forces in equilibrium is subjected to a virtual displacement, the work done by the external forces equals the work done by the Internal forces. i.e.

$$W_E = W_i$$

$W_E$ = external and $W_i$ = Internal work done

This Principle can be used to determine the collapse load. The structure is assumed to deflect through a small additional displacement after the ultimate load is reached. The work done by external forces is equated to the work absorbed by the plastic hinges and the collapse load is worked out. As the collapse eqⁿ is derived from work considerations, the B.M. diagram must be drawn to check the yield condition is not violated.

→ Theorems of PLASTIC ANALYSIS—

a) static | Lower Bound Theorem—

For a given Frame and loading if there exists any distribution of B.M. throughout the frame which is both safe and statically admissible with a set of loads P, then the value of the load P must be less than or equal to the collapse load $P_u$ ($P \le P_u$)

→ The static Theorem satisfies the equilibrium and yield conditions. Have max. FOS. represent lower limit to the ultimate Load.

b) kinematic | upper Bound Theorem—

For a given frame subjected to a set of loads P, the Value of P which is found to correspond to any assumed mechanism, must be either greater than or equal to the collapse load $P_u$ ($P \ge P_u$).

→ Represent an upper Limit to the true Ultimate Load.
Has smaller FOS.
Satisfies the equilibrium and continuity Conditions.

c) Uniqueness Theorem—

For a given frame and loading if atleast one safe and statically admissible B.M. distribution can be found, and if in this distribution the B.M. is equal to the fully plastic moment at sufficient -x- to cause failure of the frame as a mechanism due to rotation of plastic hinges, the Corresponding load will be equal to the collapse load ($P = P_u$)

→ All the 3 conditions (equilibrium, continuity, yield) are satisfy.

# Method of Analysis—

a) Static Method—

This Consist of selecting the redundant Forces. The free & redundant B.M. diagrams are drawn for the structure. A combined B.M. diagram is drawn in such a way that a mechanism is formed. The collapse load is found by working out the equilibrium equation. It is checked that the B.M. is not more than the fully Plastic moment at any section.

b) kinematic Method— This consists of locating the possible places of plastic hinges. The possible independent and combined mechanisms are ascertained. The collapse load is found by applying the principle of virtual work. A B.M. diagram Corresponding to collapse mechanism is drawn and it is Checked that the B.M. is not more than the fully plastic moment at any section.

c) Method of Inequalities – In this the correct mechanism can be determined directly by LP techniques.

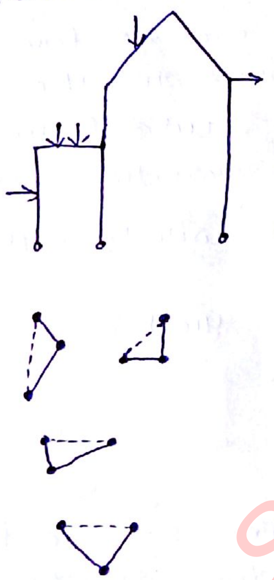
MECHANISM–

When a structure is subjected to a system of loads it is stable and hence functional until a sufficient number of Plastic hinges have been formed to render the structure unstable. As soon as the structure reaches an unstable condition, it is considered to have failed. The segments of the beam b/w the plastic hinges are able to move without an ↑ of load. This condition in a member is called Mechanism.

→ The concepts of mechanism formation in a structure due to loading beyond the elastic limit and virtual work are used in the Plastic Analysis & design of steel structures.

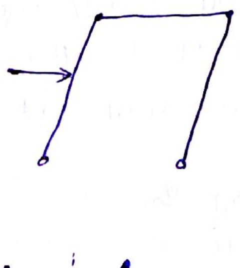
If an indeterminate structure has redundancy $r$, the insertion of $r$ plastic hinges makes it statically determine. Any further hinge converts this statically determinate structure into Mechanism. Hence for collapse the number of Plastic hinges required are $(r+1)$.

Types–

a) 

Beam Mechanisms–

All the loaded span behave as Beam Mechanism.

b) 

Sway Mechanism – It is formed due to lateral loads.

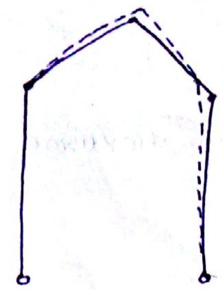
c) 

Joint Mechanism–

Formed due to action of moment.

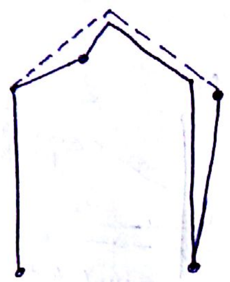
The number of members at the joint should be 3 or more.

d)



Gable Mechanism-
It is exhibited in gable frames.
Columns spread more at the top than that at the base.

e)



Composite Mechanism-
Any of the two Independent Mechanisms may be combined to form composite Mechanism. A Beam and gable Mechanism are joined to form a gable Mechanism in this figure.

Number of Independent Mechanisms -

Let

$N$ = number of possible plastic hinges
$r$ = number of redundancies
$n$ = possible independent mechanisms.

Then

$$n = N - r$$

# Design -

The important functional requirements to be satisfied during design are that the structure should be strong enough to resist external loading without collapsing, and stiff enough not to ~~enough~~ deflect unduly under that loading.

→ Plastic design methods consist of proportioning a structure under factored loads at the point of collapse.

Factored Loads = working Load x Load Factor

→ Then Max. Plastic moment $M_p$ is found.

→ Calculate $Z = Z_p / f_y$   $Z$ = Plastic Modulus
$Z_p$ = max. plastic moment
$f_y$ = yield stress

→ To keep a uniform cross-section the largest of the plastic moment values for various span is choosen and a suitable beam section is selected.

→ For economy, the section is selected for the least plastic Moment found for all the spans and cover plates are provided for the excess plastic moments in other span.

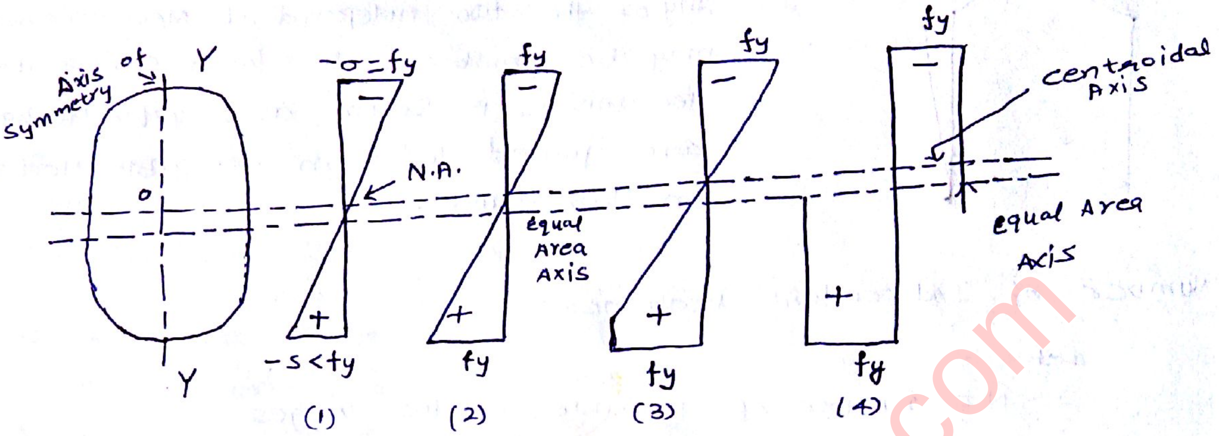
# Flexural Members –

ex –

Continuous Beam & Rigid Frames

The concepts used in the Plastic Analysis of Flexural Members are explained as follow –

Bending of Beam –




Development of Full Plasticity of a Beam section

Let us consider a beam with o as the centroid and whose -x- has an Axis of symmetry (yy). subjected to a steadily increasing B.M. M ie. from the elastic limit (stage 1) upto the plastic limit stage(4). The corresponding distribution of stresses for stages 1 to 4 are plotted in figure.

The beam is assumed to be bent by a pure couple and the effect of Axial and shear forces are neglected. The deformation and strains are assumed to be small so that stresses other than longitudinal normal stresses are neglected. It is assume that a plane sections before bending remains plane after bending and the -x- is assumed to be symmetrical w.r.t. an Axis in the plane of bending.

At

Stage 1 :- B.M. be ↑ from zero value.

longitudinal Normal stress σ and longitudinal strain ε both vary linearly. The behaviour is entirly elastic.

Stage 2 :- Moment is further ↑

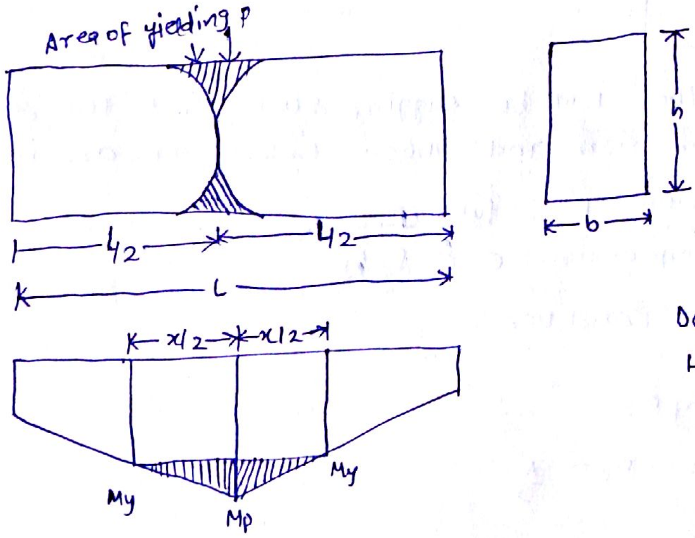
The yield spreads inwards from the upper surface of the beam.

Stage 3 :- A Further ↑ in the B.M. cause the yield to spread inwards from the lower surface of the beam as well, in addition to spreading further inwards from the upper surface.

Stage 4 :- on further ↑ B.M. the whole section yields.
The two zones of yield grow in depth from the extreme fibre towards the N.A. and the corresponding B.M. is the fully plastic moment.
Any further ↑ in B.M. results only in rotation since no greater resistance moment than the fully plastic moment can possibly be developed until strain hardening occurs.

→ **Plastic Hinge**— A plastic hinge is a zone of yielding due to flexural in a structural member. Although hinges do not actually form, it can be seen that large changes of slope occur over the small length of the member at positions of max. moments.
A structure can support the computed ultimate load due to the formation of plastic hinges at certain critical sections. The members remains elastic until the moment reaches a value Mp, the max. MOR of a fully yielded -x-. Any Additional moment will cause the beam to rotate with little ↑ in stress. The rotation occurs at a constant moment Mp. The zone acts as if it was hinged except with a Constant restraining moment Mp. The plastic hinge, therefore can be defined as a yielded zone due to flexure in a structure ~~with~~ in which infinite rotation can take place at a constant restraining moment Mp of the section.
The possible places for plastic hinges in a structure with prismatic members are points of concentrated loads.

→ Hinge length— Consider a ss □ar beam subjected to gradually increasing concentrated load P, at the centre. A plastic hinge will be formed at the centre.




Development of plastic Hinge & Hinge length

$M_p = \frac{PL}{4}$

$M_y = f_y z = f_y \times \frac{bh^2}{6}$

$= f_y \times \frac{1}{6} \times 4 \times \frac{bh^2}{4}$

$= \frac{2}{3} \times f_y \times \frac{bh^2}{4}$

$= \frac{2}{3} \times f_y \times Z_p$

$= \frac{2}{3} M_p$

From B.M. Diagram

$\frac{M_p}{\frac{L}{2}} = \frac{M_y}{\frac{L}{2} - \frac{x}{2}}$

$\frac{(L-x)}{2} M_p = \frac{L}{2} M_y$

$(L-x) M_p = L M_y$

$(L-x) M_p = L \times \frac{2}{3} M_p$

$L - x = \frac{2}{3} L$

$x = \frac{L}{3}$

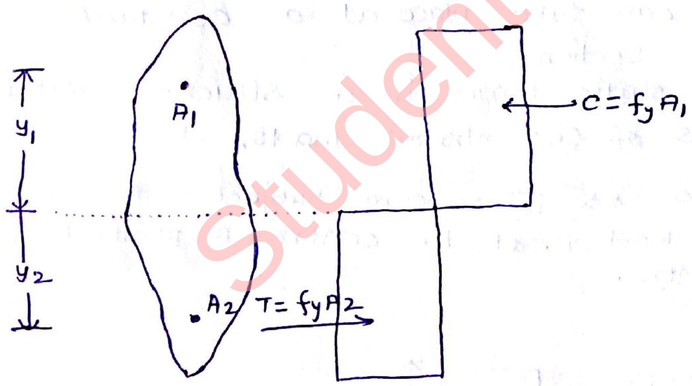
∴ The hinge length of the plasticity zone is equal to $\frac{1}{3}$rd of the span.

→ Fully Plastic Moment of a Section – It is the max. MoR of a fully yielded -x-.

The following assumptions are made to evaluate its value in addition to the assumption made earlier while discussing bending of beams.

1. Instability of the structure will not occur prior to the attainment of the ultimate load.
2. The connections provide full continuity such that the fully plastic moment can be transmitted.
3. The loading is proportional i.e. all the loads ↑ in a fixed proportion to one another.

Consider a fully yielded beam section. The bending stress distribution will be □ar as in figure –




Plastic Moment of a Beam Section

let the nature of the B.M. be sagging such that the fibers above N.A. are in compression and those below N.A. are in tension.

Force in tension $T = f_y A_2$

Force in compression $= C = f_y A_1$

In an equilibrium condition

$C = T$

$f_y A_1 = f_y A_2$

$A_1 = A_2 = \frac{A}{2}$

where $A_1$ and $A_2$ are the area of section above and below N.A.

A = Area of -x- of the beam.

The two forces, tensile and compressive, form a couple which resist the plastic moment.

$$M_p = f_y A_1 \bar{y}_1 + f_y A_2 \bar{y}_2$$

or $$M_p = f_y \frac{A}{2} \bar{y}_1 + f_y \frac{A}{2} \bar{y}_2$$

or $$M_p = f_y \frac{A}{2} (\bar{y}_1 + \bar{y}_2)$$

$$M_p = f_y Z_p$$

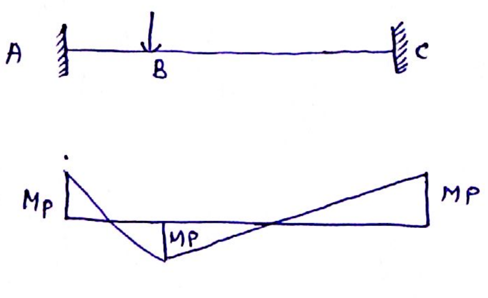
$Z_p$ = plastic modulus of section $= \frac{A}{2}(\bar{y}_1 + \bar{y}_2)$

$\bar{y}_1, \bar{y}_2$ = distance of cg of the area above and below N.A.

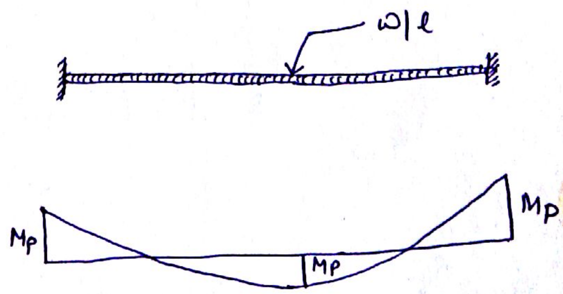
→ **Redistribution of Moment & Reserve of Strength** –

It can be explained on the basis of the action & successive formation of the plastic hinges. Under the gradually increasing load on a structure, the plastic moment is reached at a section that is most highly stressed. On further increase of load, the value of plastic moment is maintained and the section rotates. A plastic hinge is formed at the section. Other less highly stressed sections maintain an equilibrium with the increased load by a proportionate ↑ in the moment and successive formation of plastic hinges occurs at other sections, where the moment reaches the value of plastic moment. This process of formation of successive plastic hinges continues till the ultimate load is reached. Therefore, it can be said that flexural members can sustain the ultimate loads due to the redistribution of moments.

Let us take an example of a fixed ended beam with a concentrated load near one of it support. As the load P is ↑ the beam reaches its elastic limit at end A, and a plastic hinge is formed. The moments at sections B and C are less than the plastic moment.




(a)




(b)

on a further ↑ of load, the plastic section at A rotates without absorbing any more moment. The resisting moment at C increases until the plastic moment value is reached and a plastic hinge is formed at C. on still further ↑ of load, a plastic hinge is formed at B and the ultimate load is reached. ∴ it can be said that the formation of plastic hinge allow a subsequent ROM until the fully plastic moment is reached at each critical section. The ROM is the main contributing factor to the reserve of strength.

Load Factor =

$$F = P_u / P_w$$

$P_u$ = collapse Load

$P_w$ = Working load

Load Factor = FOS x Shape Factor

$$FOS = \frac{f_y}{\sigma}$$