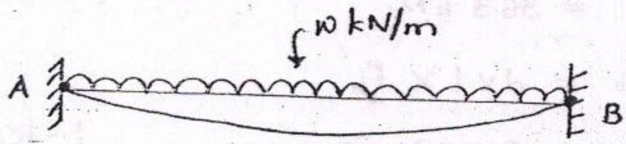
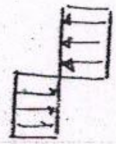
Wednesday
30th October 2013

Limit state method of beam design :

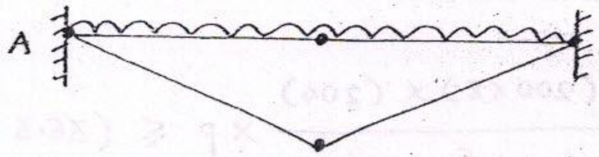
(i) Types of sections :

ⓐ Plastic section :





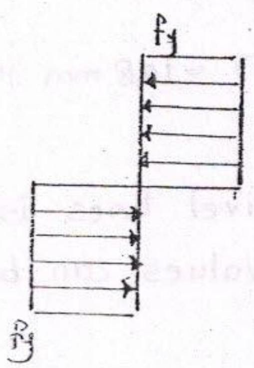
First two plastic hinges are formed at $w_1 = \frac{12 M_p}{l^2}$ when loaded further,



Third hinge is formed at $w_2 = \frac{16 M_p}{l^2}$ and plastic collapse mechanism is obtained.

When the load is increased from $\frac{12 M_p}{l^2}$ to $\frac{16 M_p}{l^2}$, the first two plastic hinges at A and B rotate till plastic collapse mechanism is formed.

The section has capacity to develop plastic moment i.e. plastic hinge and has sufficient ductility so that it can rotate till the plastic collapse mechanism is formed. This section is called 'plastic section'. The stress distribution for plastic section is

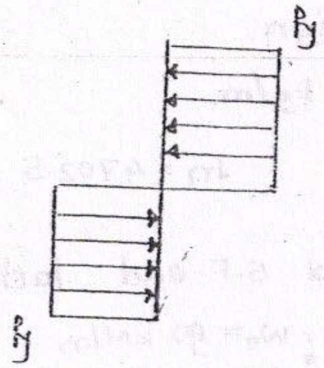


(collapse mechanism can be formed with these sections)

stress distribution (for plastic sections)

ⓑ Compact section :

It has the capacity to develope plastic moment (i.e. plastic hinge) but doesn't have capacity to rotate till plastic collapse mechanism is formed. (It means that we cannot load the beam after first plastic hinge is formed. Because, it will buckle)

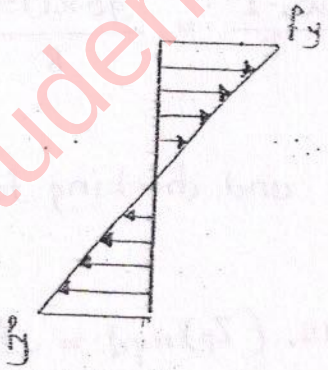


(collapse mechanism can not be develped in these sections)

stress distribution for compact section.

ⓒ Semi-compact section :

The section has capacity to develop yield moment only. It doesn't have the capacity to develope the plastic moment or plastic hinge.

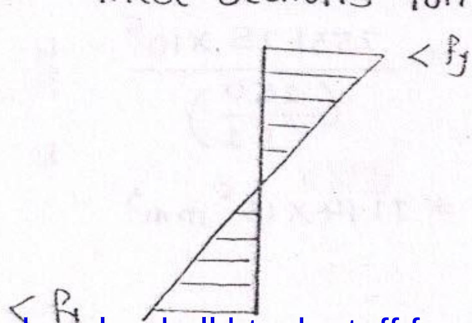


(No plastic moment or plastic hinge is formed, becouse if we load beyond f_y, component will buckle)

stress distribution for semi-compact section.

ⓓ Slender section :

These sections fail before reaching f_y.



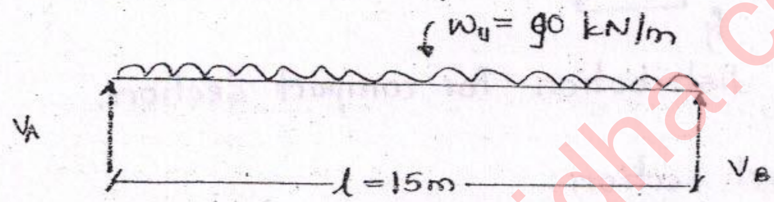
We assume that all sections are plastic sections and design the beam based on collapse mechanism.

Q. Design a suitable section for steel plate beam girder carrying UDL of 60 kN/m (inclusive of its self wt) over an effective span of 15 m. check for shear and deflection. Use Limit state method of design. *(Marks)*

Use ISWB 600 @ 145·1 kg/m.

$I_{xx} = 106198 \cdot 5 \text{ cm}^4$ $\quad$ $I_{YY} = 4702 \cdot 5 \text{ cm}^4$.

Analysis (finding factored S.F. and factored B.M.)



Factored UDL $= 1 \cdot 5 \times 60 = 90$ kN/m

Factored S.F. $= V_u = \dfrac{w_u \cdot l}{2} = \dfrac{90 \times 15}{2} = 675$ kN

Factored B.M. $= M_u = \dfrac{w_u \cdot l^2}{8} = \dfrac{90 \times (15)^2}{8} = 2531 \cdot 25$ kNm

Design:

(fixing size of beam and checking for primary criteria)

ⓐ Size of beam:

$$\text{Plastic modulus, } (Z_p)_{reqd} = \frac{M_u}{\sigma_{bc}}$$

where, $f_y = 250$ MPa (assume)

$$\sigma_{bc} = \frac{f_y}{1 \cdot 1}$$

$$(Z_p)_{reqd} = \frac{2531 \cdot 25 \times 10^6}{\left(\frac{250}{1 \cdot 1}\right)}$$

$$= 11 \cdot 14 \times 10^6 \text{ mm}^3$$

Note:

$$(Z_p)_{beam} = S.F. \times Z \qquad \left(\because S.F. = \frac{Z_p}{Z}\right)$$

$$\because \quad Z = \frac{I_{xx}}{y} = \frac{106198.5 \times 10^4}{300}$$

$$(Z_p)_{beam} = 1.12 \times \frac{106198.5 \times 10^4}{300}$$

$(\because$ S.F. for I section $= 1.12)$

$$= 3.96 \times 10^6 \text{ mm}^3 < (Z_p)_{reqd.}$$

So cover plates are provided at top and bottom.

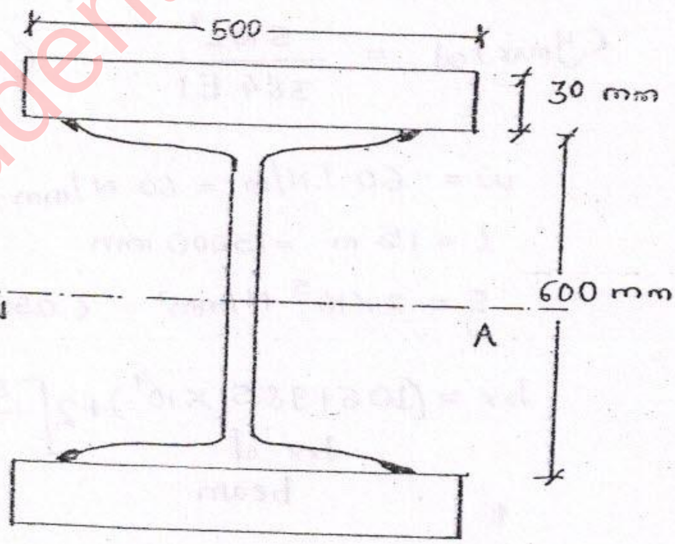
$$A_p, \text{ required on each side} = \frac{(Z_p)_{reqd} - (Z_p)_{beam}}{d}$$

$$\therefore A_p = \frac{11.14 \times 10^6 - 3.96 \times 10^6}{600}$$

$$= 11{,}966 \text{ mm}^2$$

Use cover plate of size 500 × 30 mm $(A_p = 15000 \text{ mm}^2)$

(Use welding so that A_p will not be increased by 50 %)



$$(Z_p)_{provided} = (Z_p)_{beam} + (Z_p)_{plates} \quad \left[\frac{A}{2}(\bar{y}_1 + \bar{y}_2)\right]$$

$$= 3.96 \times 10^6 + (500 \times 30)(315 + 315)$$

$$= 13.41 \times 10^6 \text{ mm}^3 > (Z_p)_{reqd.}$$

ⓑ Check for shear :

$\tau_{va,cal}$ - calculated shear stress in web.

$$\frac{V}{D \times t_w} \leq \left(\tau_{va} = \frac{f_y}{\sqrt{3} \times 1.1} \right)$$

- permissible shear stress in LSM.

$$\tau_{va,cal} = \frac{675 \times 10^3}{600 \times 10} = 112.5 \quad N/mm^2$$

(Assume thickness of web, $t_w = 10$ mm (not given)

D - depth of web (depth of beam only)

D = 600 mm (as per IS 800 : 2007)

$$\tau_{va} = \frac{250}{\sqrt{3} \times 1.1} = 131.25 \quad N/mm^2$$

$\tau_{va,cal} < \tau_{va}$. Hence safe.

ⓒ Check for deflection :

Deflection is calculated for service (working) loads only.

$$(y_{max})_{cal} = \frac{5 w l^4}{384 EI} \quad \left(\leq y_{max} = \frac{span}{300} \right)$$

as per LSM.

w = 60 kN/m = 60 N/mm

l = 15 m = 15000 mm

E = 2×10^5 N/mm² (assume)

$$I_{xx} = (106198.5 \times 10^4) + 2\left[\frac{500 \times 30^2}{12} + (500 \times 30) \times 315^2\right]$$

(I_{xx} of beam) (I_{koko} + A × h^2 for plates.)

$$= 4.03 \times 10^9 \ mm^4$$

$$\therefore (y_{max})_{cal} = \frac{5 \times 60 \times (15000)^4}{384 \times 2 \times 10^5 \times 4.03 \times 10^9}$$

$$= 49.07 \ mm < \left(\frac{span}{300} = \frac{15000}{300} = 50 \ mm \right)$$

Hence, safe.

Q. An ISMB 450 is used as proped cantilever beam of span 12 m. Assuming $f_y = 250$ MPa, find the factored U.D.L. 'q_u' the beam can carry inclusive of its self wt. If the load is to be applied over entire span.

20 Marks

Given. ISMB 450.

$wt/m = 72.4$ kg/m.

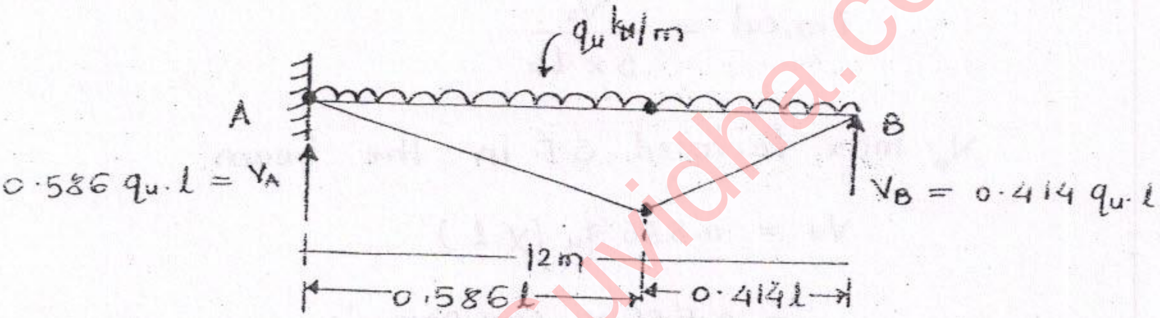
$A_g = 9227\ mm^2$

$B_f = 150$ mm

$I_{xx} = 3.039 \times 10^8\ mm^4$

$I_{YY} = 8.34 \times 10^6\ mm^4$

Use Limit state method.



$$q_u = \frac{11.656\ M_P}{l^2} \quad \text{(from plastic analysis)}$$

where,

M_P - plastic moment capacity of beam c/s

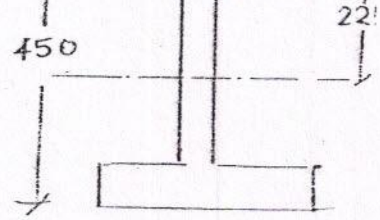
$$= \left(\frac{f_y}{1.1}\right) \cdot Z_P.$$

Z_P - plastic modulus of section

= shape factor × Z

$$Z = \frac{I_{xx}}{y} = \frac{3.039 \times 10^8}{225}$$

$$= 1.35 \times 10^6\ mm^3$$



$$Z_P = 1.12 \times 1.35 \times 10^6$$

S.F. = 1.12

$$= 1.51 \times 10^6\ mm^3$$

$$\therefore M_P = \left(\frac{250}{1.1}\right) \times 1.51 \times 10^6$$

$$= 343.8\ kNm$$

$$q_u = \frac{11.656\, M_P}{l^2}$$

$$= \frac{11.656 \times 343.8}{(12)^2}$$

$$= 27.82 \text{ kN/m}$$

Service load / working load $(w) = \frac{q_u}{1.5} = \frac{27.82}{1.5}$

$$= 18.55 \text{ kN/m}$$

Note:

If we are asked to check for shear, assume $t_w = 10$ mm.

$\tau_{va,cal}$ - calculated shear stress in web

$$\tau_{va,cal} = \frac{V_u}{D \times t_w}$$

V_u - max. factored S.F in the beam.

$$V_u = 0.586\, q_u (\times l)$$

$$= 0.586 \times (27.82 \times 12)$$

$$= 195.6 \text{ kN}$$

$$\tau_{va,cal} = \frac{195.6 \times 10^3}{(450 \times 10)}$$

$$= 43.4 \text{ N/mm}^2 \quad \left(< \tau_{va} = \frac{250}{\sqrt{3} \times 1.1} = 131.29 \right)$$

Hence, safe.