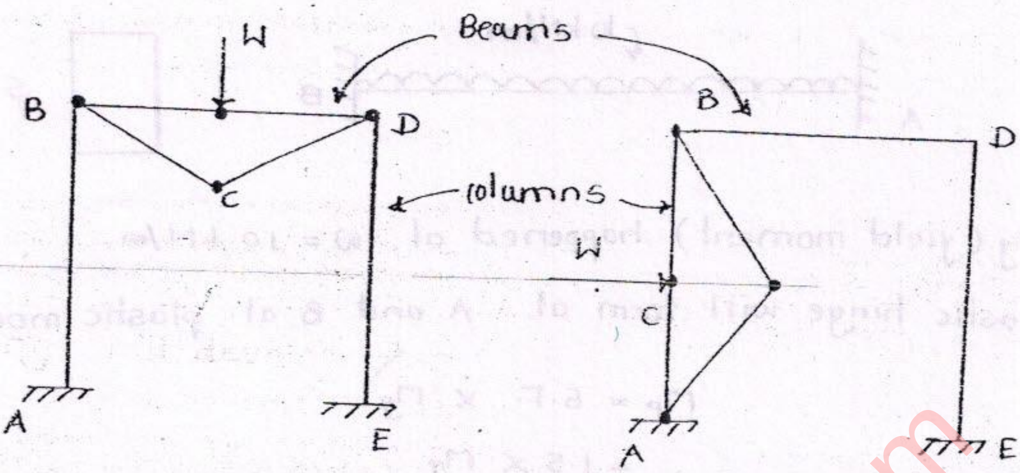
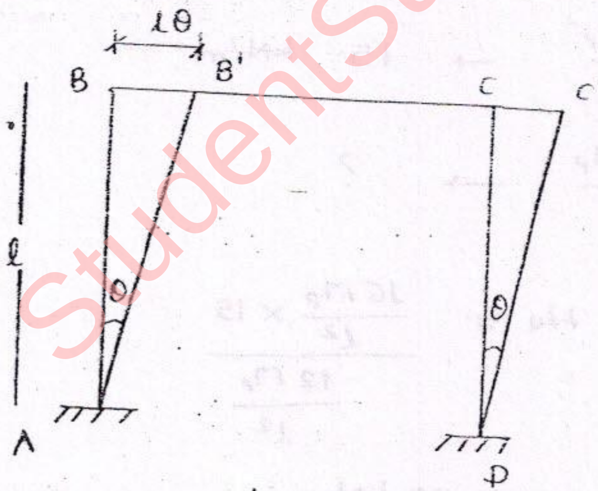
Plastic Analysis of Frames :-

Beam mechanism :



In a portal frame, if mechanism is formed in only one member i.e. either in a beam or in a column, then the mechanism is called Beam mechanism.

Sway or Panel mechanism :



$BB' = CC' = l \cdot \theta$

(axial deformations are neglected)

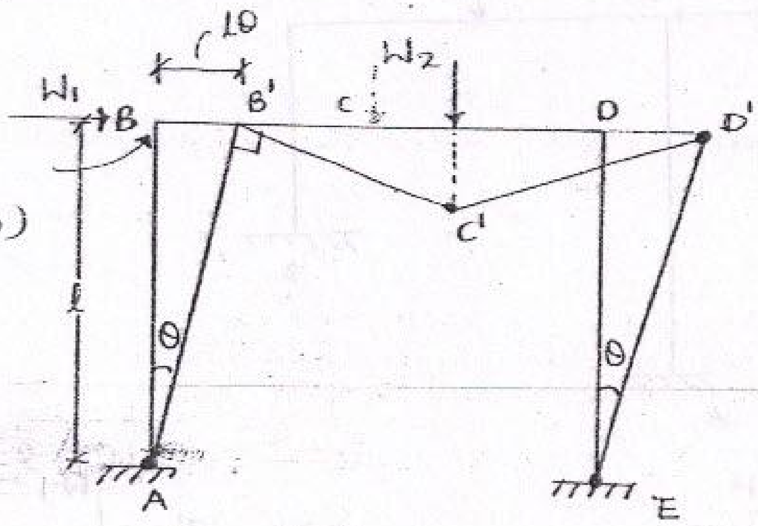
Sway mechanism in panel or Bay

If the portal frame sways, either to the left or right due to horizontal loads, then it is called a sway mechanism.

Note:

The above two mechanisms are called elementary or independent mechanisms.

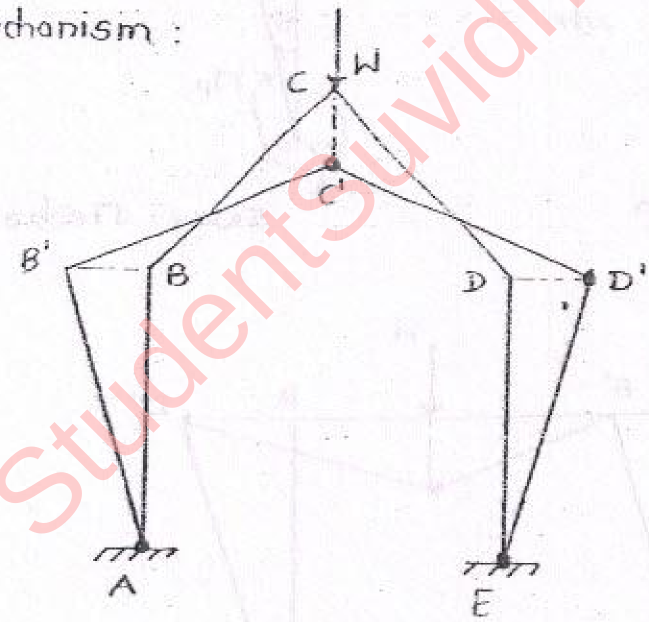
Combined Mechanism :



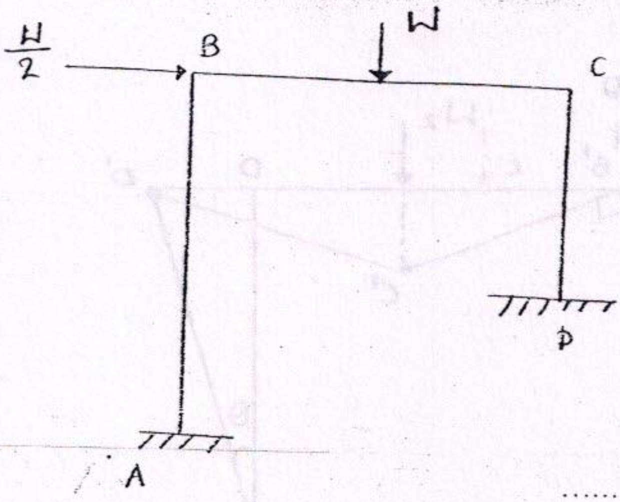
Whether, poin C comes down or not. $BB' = DD' = l \cdot \theta$.

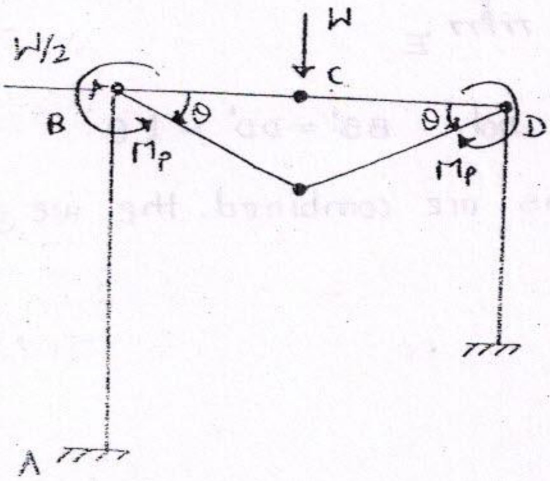
If beam and sway mechanisms are combined, the we get a combined mechanism.

Gable Mechanism :

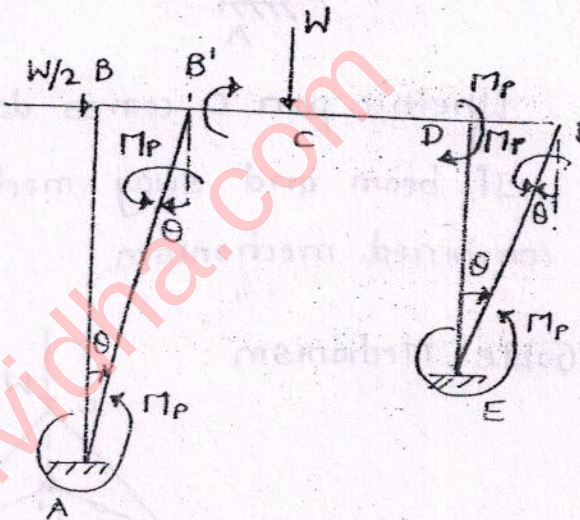


It is a variation of sway mechanism in which the column move in different directions.



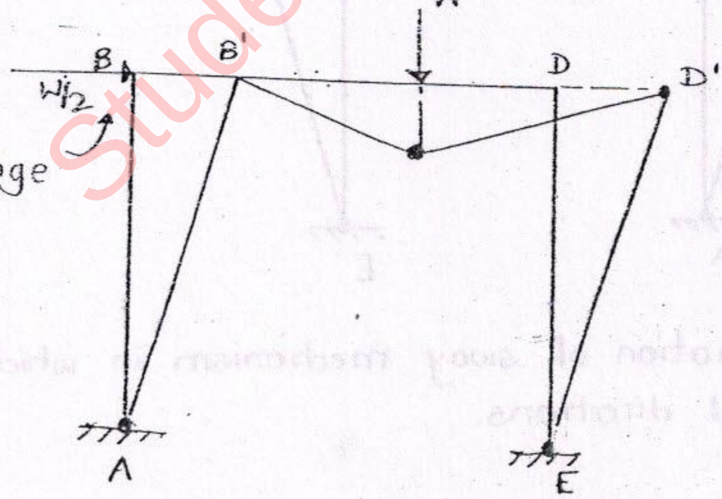


Beam mechanism



Sway Mechanism

No plastic hinge at B



Combined Mechanism

(i) In beam mechanism, at point B, moment M_p is anticlockwise while in sway mechanism at point B in beam M_p is clockwise. So the combined mechanism, at point B in the beam net moment is very less. So plastic hinge will not be developed at the joint B, in combined mechanism.

(ii) In beam mechanism, at joint D, moment is clockwise. while in sway mechanism at joint D moment is also clockwise. So in the combined mechanism, the net moment at D is more. So plastic hinge will develop at D.

(iii) Degree of statical indeterminancy, for a given beam/frame

$$D_s = r - 3$$
$$= 6 - 3 = 3$$

∴ No. of plastic hinges requied at collapse $= 3 + 1 = 4$.

∴ No. of possible hinges locations for formation of plastic hinge are $= 5$ (A, B, C, D and E)

So we must eliminate plastic hinge formation at B, Because net moment at B is very less.

(iv) No. of independent mechanisms in a portal frame is

$$k = N - D_s$$

where,

N - no. of possible locations of plastic hinges

D_s - degree of statically indeterminancy.

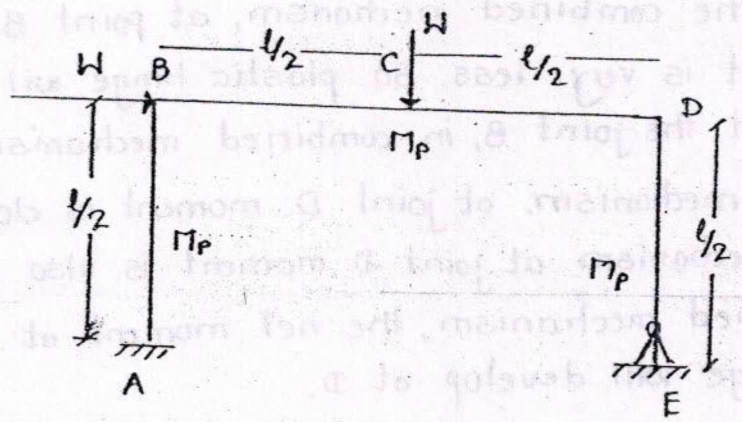
For the above frame,

$$k = 5 - 3$$
$$= 2 \text{ independant mechanisms.}$$

Sway and combined mechanisms are possible only when the frame is subjected to horizontal loads.

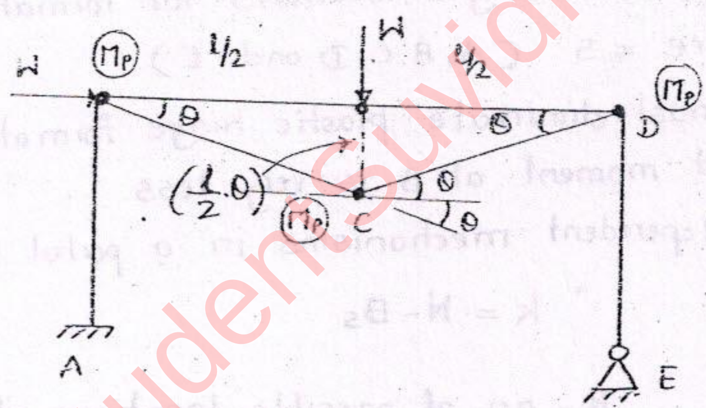
Q. Find the collapse load for portal frame shown in fig. and draw plastic moment diagram / collapsed moment diagram.

20 Marks



Since the frame is subjected to both horizontal and vertical loads, we have to check sway and combined mechanisms also.

Possibility I:



Horizontal load will not do any work because there is no movement of beam in horizontal direction in this mechanism.

External work = Internal work

$$W \cdot \left(\frac{l}{2} \cdot \theta\right) = M_P \cdot \theta + M_P(\theta + \theta) + M_P \cdot \theta$$

$$\frac{W \cdot l}{2} = 4\, M_P$$

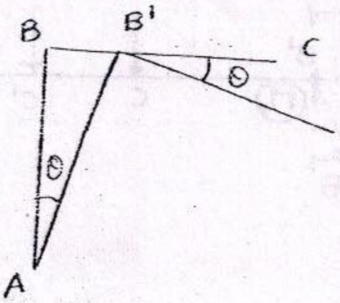
$$W = W_u = \frac{8\, M_P}{l} \quad \text{——— (i)}$$

where, M_P - plastic moment capacity of beam (it is constant)

$$M_P = f_y \cdot \frac{A}{2}(\bar{y}_1 + \bar{y}_2)$$

$$BB' = DD' = \left(\frac{L}{2}\cdot\theta\right)$$

As rotation at A is θ, rotation at B is also θ (rigid body rotation)



Here both horizontal and vertical loads do the work.

External work = Internal work

$$W\left(\frac{L}{2}\cdot\theta\right) + W\left(\frac{L}{2}\cdot\theta\right) = M_p\cdot\theta + M_p\cdot(\theta+\theta) + M_p\,(\theta+\theta)$$

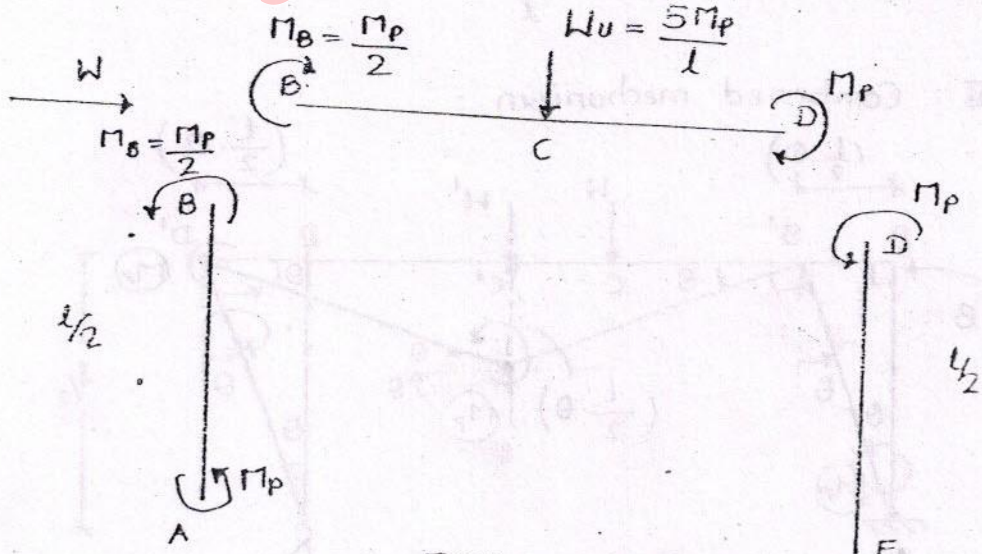
At B — At C — At A — At C — At D.

$$W = W_u = \frac{5M_p}{L} \quad \text{——— (iii)}$$

So, the correct failure mechanism is combined mechanism and true collapse load is $\frac{5M_p}{L}$.

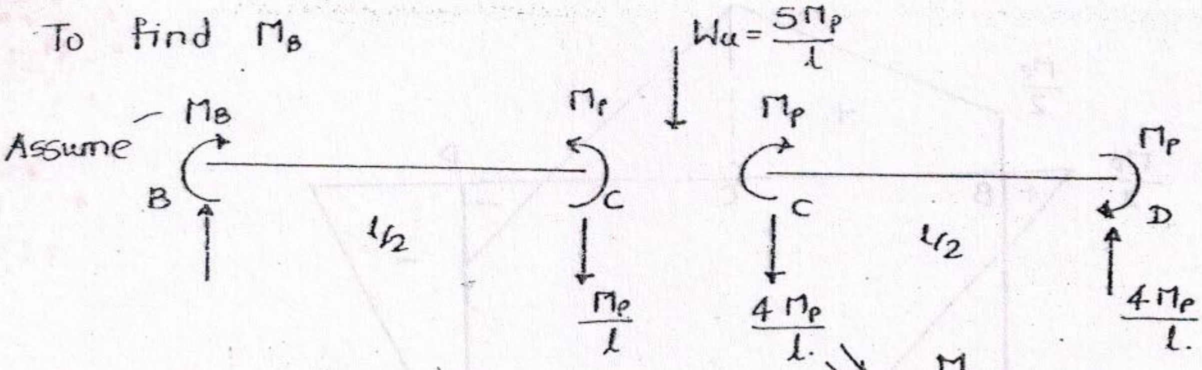
Collapse BMD / Plastic moment diagram.

Since failure mechanism is combined mechanism, it means that plastic hinges are developed at A, C and D. It means that we know B.M. values at A, C and D.

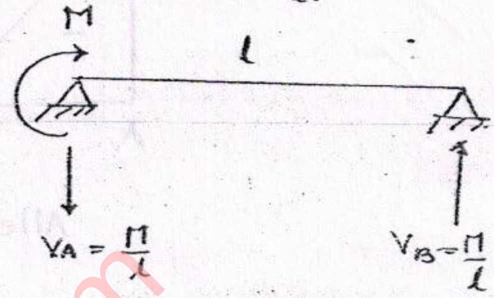


FBD.

To find M_B



From F.B.D. of BC.

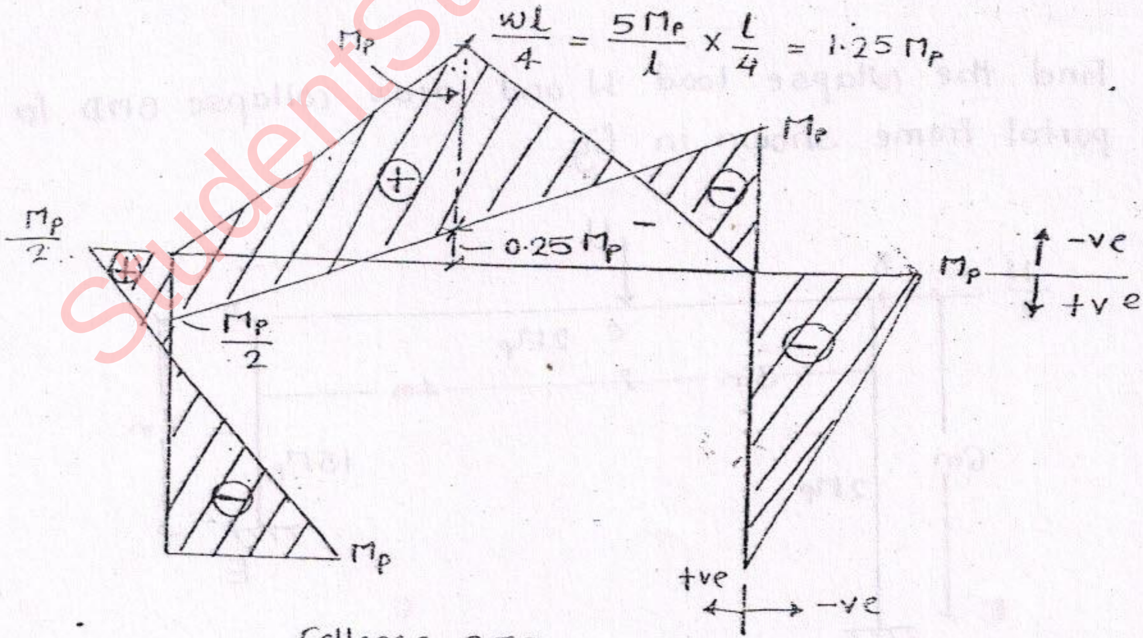


$$\Sigma M_{oB} = 0$$

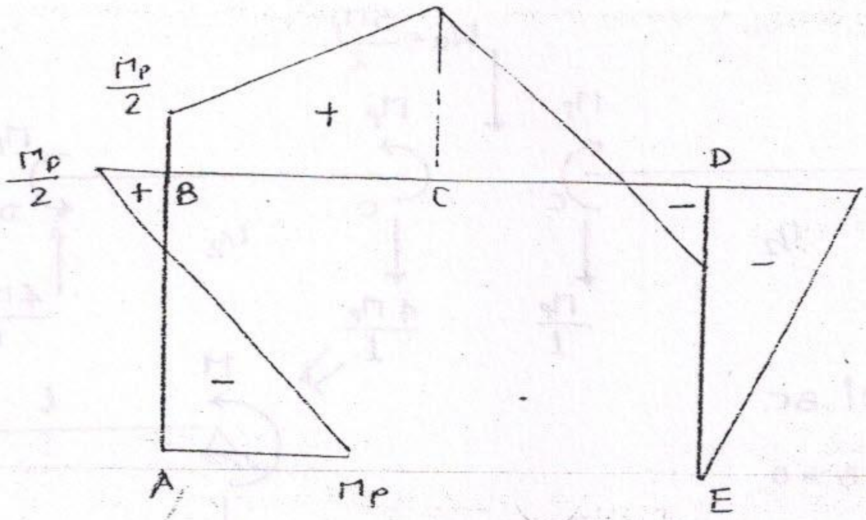
$$-M_p + \frac{M_p}{l}\cdot\frac{l}{2} + M_B = 0$$

$$\therefore \quad M_B = \frac{M_p}{2}$$

For collapse B.M.D. use deformation sign convention not the equilibrium sign convention.



Collapse BMD.

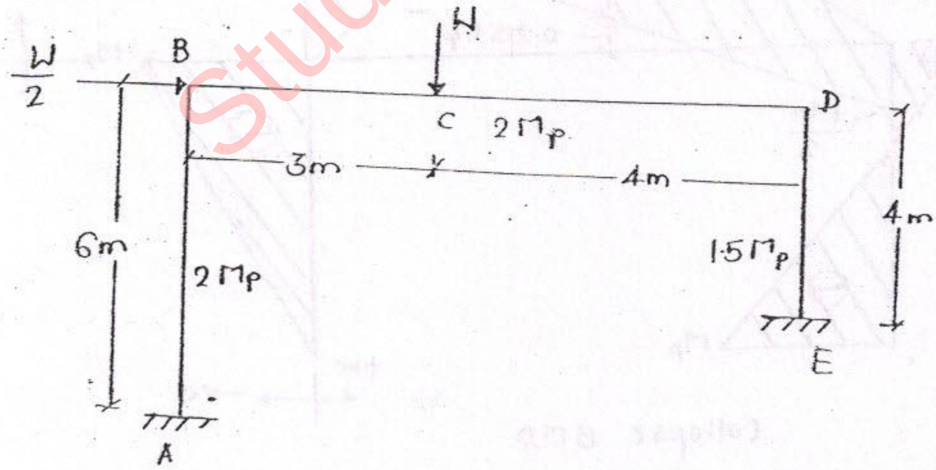


Alternate collapse B.M.D.

(i) Since there is no support at C, the applied load $\frac{5M_p}{L}$ is shared by two members CB and CD as shown in fig

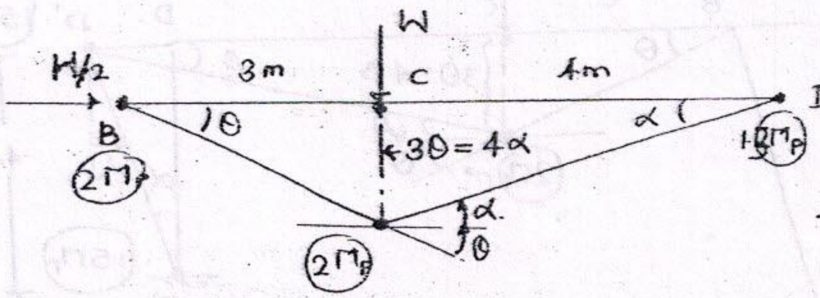
(ii) Since horizontal load W is applied exactly at joint B, the load will not bend the member AB in the F.B.D. of AB so the B.M.D. due to this load is zero.

Q. Find the collapse load W and draw collapse BMD for the portal frame shown in fig.



(i) To find collapse load:

Possibility I: Beam mechanism:



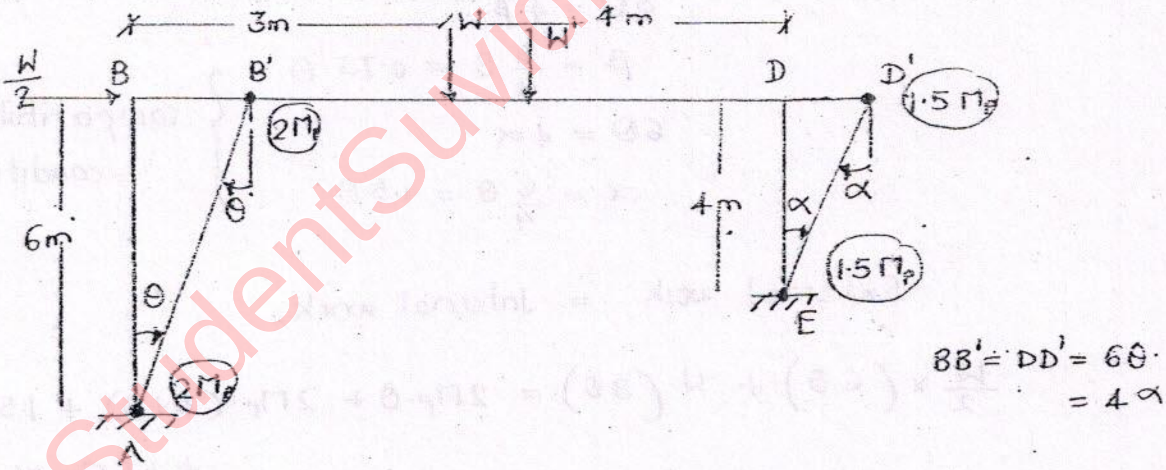
External work = Internal work

$$W(3\theta) = 2M_p \cdot \theta + 2M_p\left(\theta + \frac{3\theta}{4}\right) + 1.5M_p\left(\frac{3\theta}{4}\right)$$

at C   at B   at C   at D

$$W = W_u = 2.208\, M_p \quad \text{------- (i)}$$

Possibility II: Sway mechanism:



since lengths of AB and DE are different, angles are also different i.e. θ and α.

$$\therefore \quad 6\theta = 4\alpha$$

$$\therefore \quad \alpha = 1.5\,\theta \quad (\text{from compatibility condition})$$

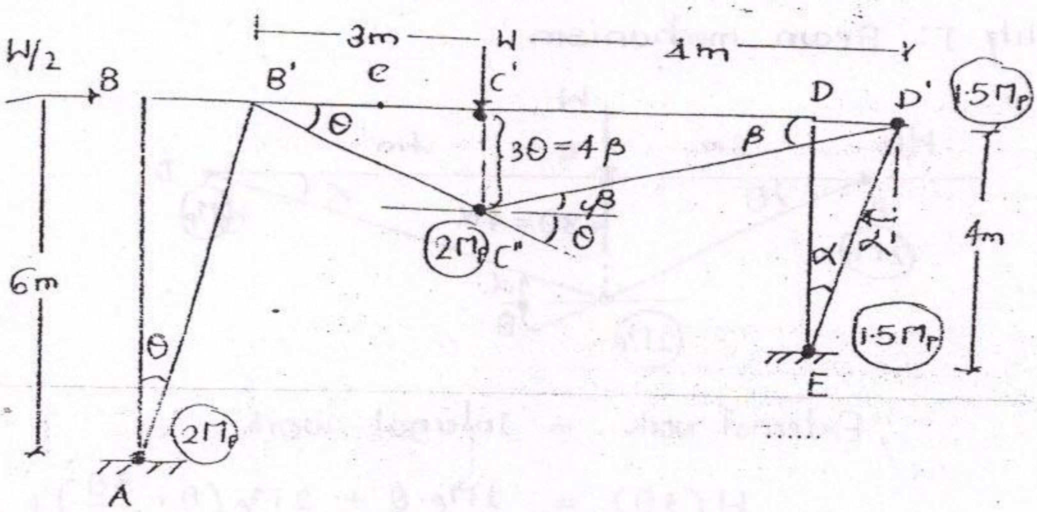
External work done = Internal work done

$$\frac{W}{2}(6\theta) = 2M_p\cdot\theta + 2M_p\cdot\theta + 1.5M_p\,\alpha + 1.5M_p$$

$$= 2M_p\,\theta + 2M_p\,\theta + 2.25M_p\,\theta + 2.25M_p$$

$$W_u = 2.83\, M_p \quad \text{------- (ii)}$$

Possibility III : Combined Mechanism.



First fix angle θ at A. ∴ $BB' = 6\theta$ ∴ $DD' = 6\theta$.

Then fix α at E

Since there is no hinge at B. slope at B is also θ ∴ $C'C'' = 3\theta$

Then fix angle β at D.

$$3\theta = 4\beta$$

$$\beta = \frac{3}{4}\theta = 0.75\,\theta$$

$$6\theta = 4\alpha$$

$$\alpha = \frac{6}{4}\theta = 1.5\theta$$

} compatibility conditions.

External work = Internal work.

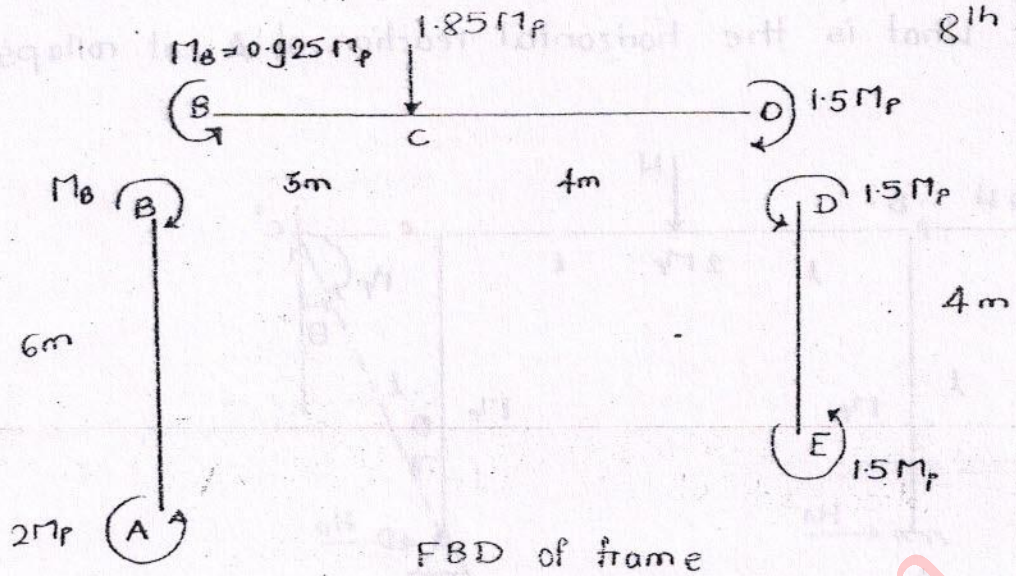
$$\frac{W}{2}\times(6\theta) + W(3\theta) = 2M_p\cdot\theta + 2M_p\cdot(\theta+\beta) + 1.5M_p(\alpha+\theta) + 1.5M_p\cdot\alpha$$

$$W = W_u = 1.85\ M_p.$$

So the failure mechanism is combined mechanism & true collapse load is $1.85\ M_p$

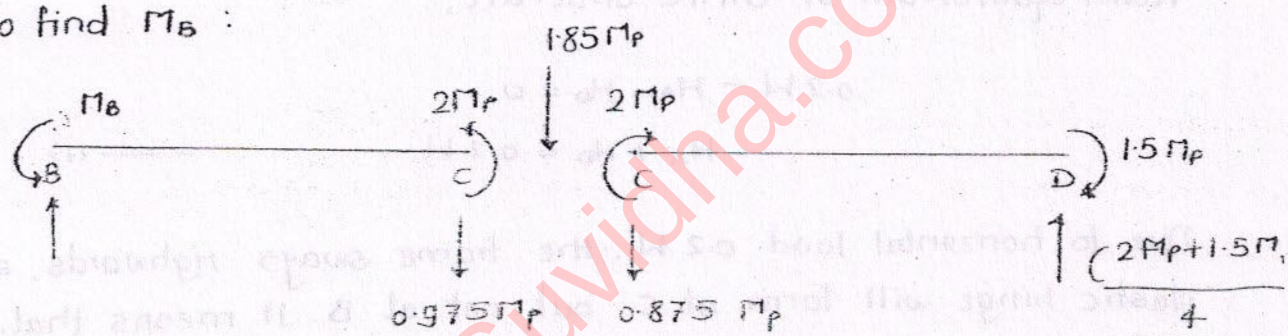
Since the correct failure mechanism is combined mechanism plastic hinges will be developed at A. C. D and E. i.e. the B.M. values at these hinges are known.

Tuesday
8th October 2013



FBD of frame

To find M_B :



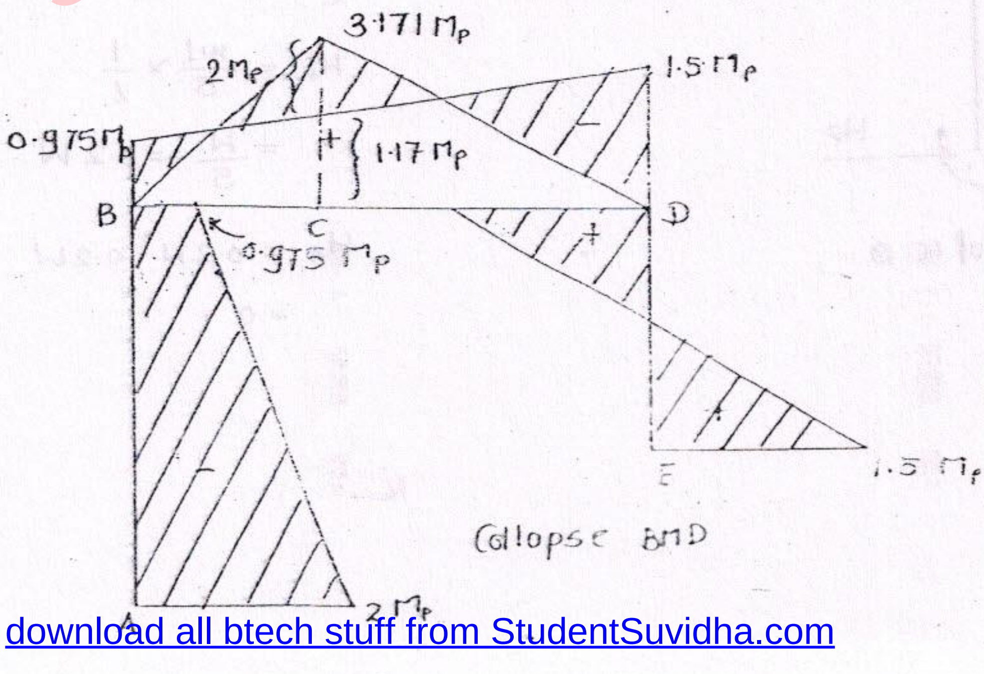
FBD of BC FBD of CD.

from FBD of BC

$\Sigma M_o\, B = 0$

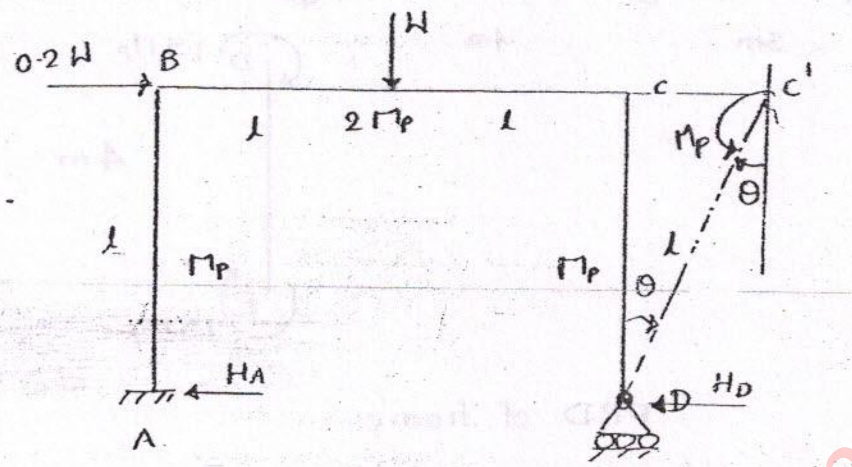
$-M_p + 0.975\, M_p \times 3 - M_B = 0$ ↻ +ve ↺ -ve

$M_B = 0.925\, M_p$



Collapse BMD

Q. Collapse moment for the frame shown in fig. is worked as $M_p = \frac{wl}{5}$. What is the horizontal reaction at A at collapse condition.

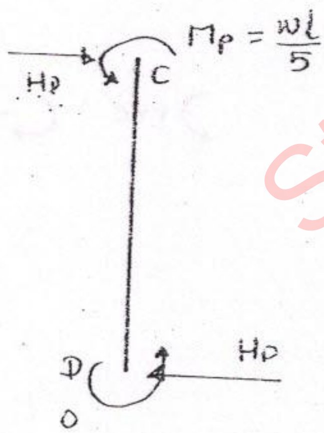


From equilibrium of entire structure,

$$0.2W - H_A - H_D = 0$$

$$H_A + H_D = 0.2W \quad \text{——— (i)}$$

Due to horizontal load 0.2 W, the frame sways righwards, so plastic hinge will form at C. but not at B. It means that, we know the B.M. value at C i.e. M_p. so draw F.B.D. of CD to find H_D.



FBD of CD.

$$\Sigma M_{\text{at}}\ D = 0$$

$$-M_p + H_D \times l = 0$$

$$\frac{M_p}{l} = H_D$$

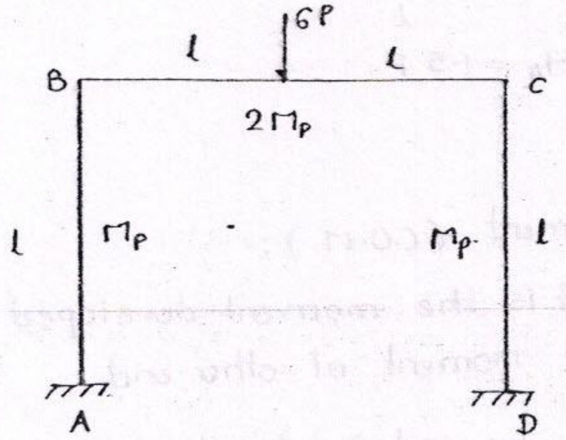
$$H_D = \frac{wl}{5} \times \frac{1}{l}$$

$$= \frac{W}{5} = 0.2W$$

$$\therefore \quad H_A = 0.2W - 0.2W$$

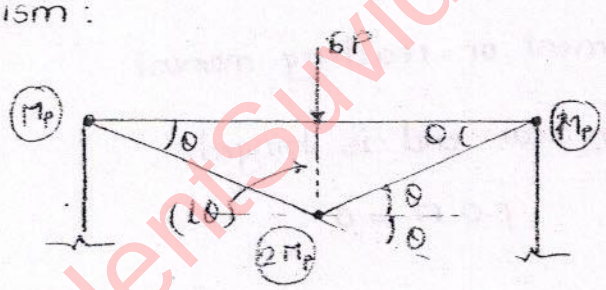
$$= 0$$

Q. At the point of collapse what is the value of horizontal thrust at point A for the loaded frame shown in fig.



Since the structure and loading are symmetrical and no horizontal loads are present, the frame will not sway in any direction. It means that sway and combined mechanisms are not possible.

Beam mechanism :

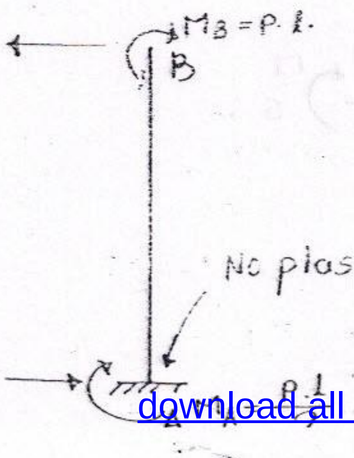


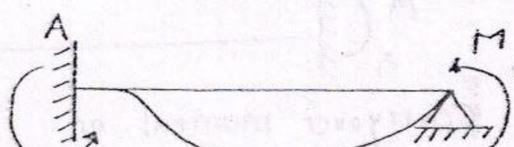
$$6P \cdot (l\theta) = M_p \cdot \theta + 2M_p(\theta+\theta) + M_p \cdot \theta$$

$$6P \cdot l = 6M_p$$

$$M_p = P \cdot l$$

Since correct failure mechanism is beam mechanism, it means that plastic hinges are developed at B, E, C only. but not at A, D





$M_A - \frac{M}{2}$ (carryover moment) or (Induced moment)

No plastic hinge from C.O.M concept

developed to make slope zero

$$H_A = \frac{1.5\, M_P}{l}$$

$$H_A = \frac{1.5\, Pl}{l}$$

$$H_A = 1.5\, P$$

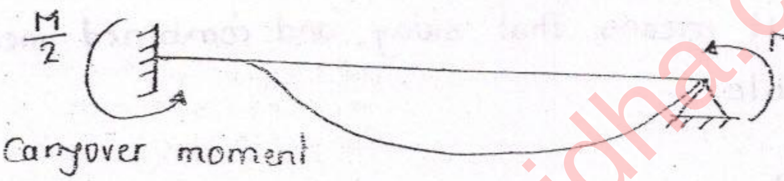
Note:

Carry over moment (C.O.M.) :-

It is the moment developed at one end due to applied moment at other end.

Case I: When far end is fixed.

$$C.O.M. = \frac{M}{2}$$



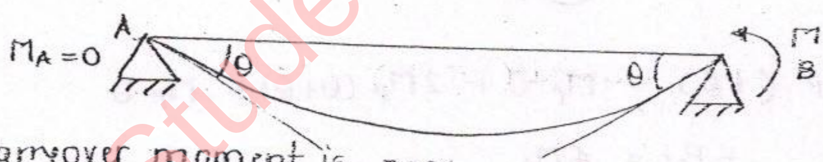
(applied moment)

Carryover moment

induced moment or reaching moment

Case II: When far end is hinged.

$$C.O.M. = 0$$

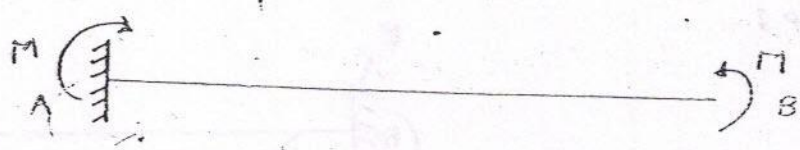


(applied moment)

(Carryover moment is zero because slope need not become zero)

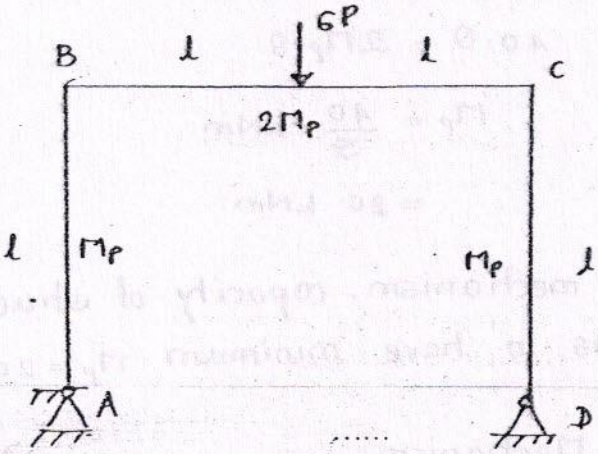
Case III: For cantilever beam.

$$C.O.M. = M$$

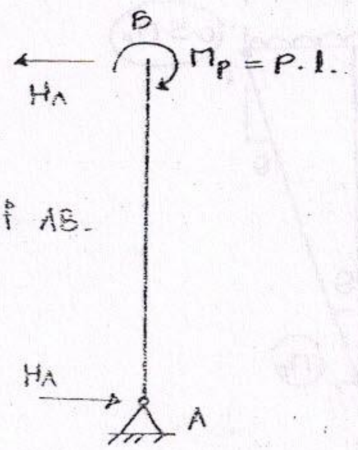


(Carryover moment or induced moment)

Q. Find H_A at collapse condition.



$M_p = P.l.$ (from beam mechanism)



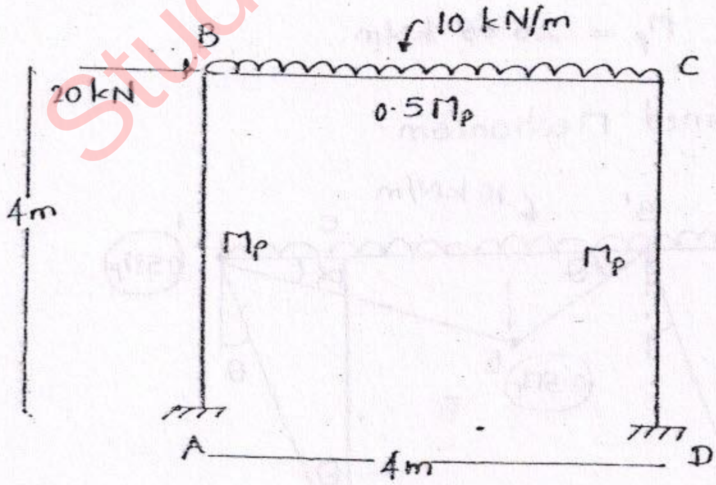
FBD. of AB.

$$H_A = \frac{P.l}{l}$$

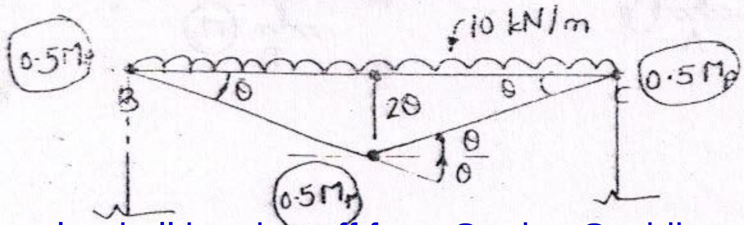
$$H_A = P$$

Q. Find M_p is $(M_p)_{Beam} = \frac{1}{2}\ (M_p)_{column}$.

10 Marks



Possibility I : Beam mechanism :



$$10\left(\frac{1}{2}\times 20\times 4\right) = 0.5\,M_p\cdot\theta + 0.5\,M_p(\theta+\theta) + 0.5\,M_p\cdot\theta$$

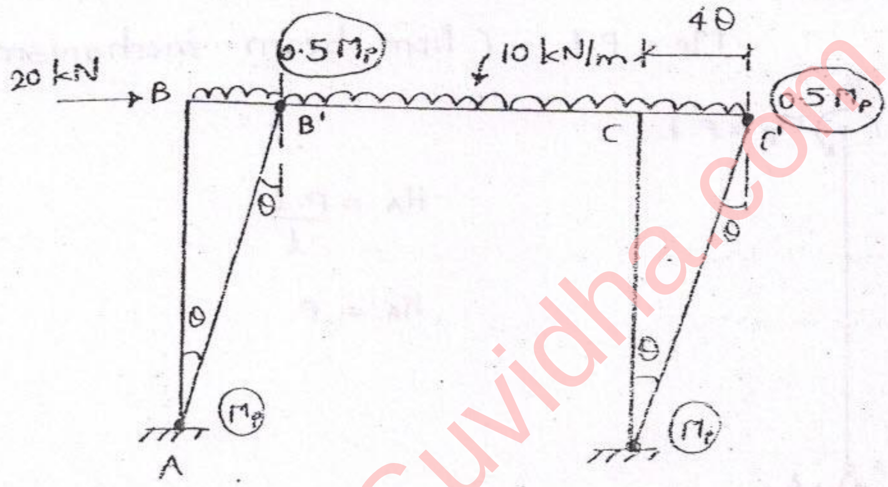
$$40\cdot\theta = 2M_p\cdot\theta$$

$$M_p = \frac{40}{2}\ \text{kNm}$$

$$= 20\ \text{kNm}$$

To prevent beam mechanism, capacity of structural member should be such as to have minimum $M_p = 20$ kNm

Possibility II : Sway Mechanism :

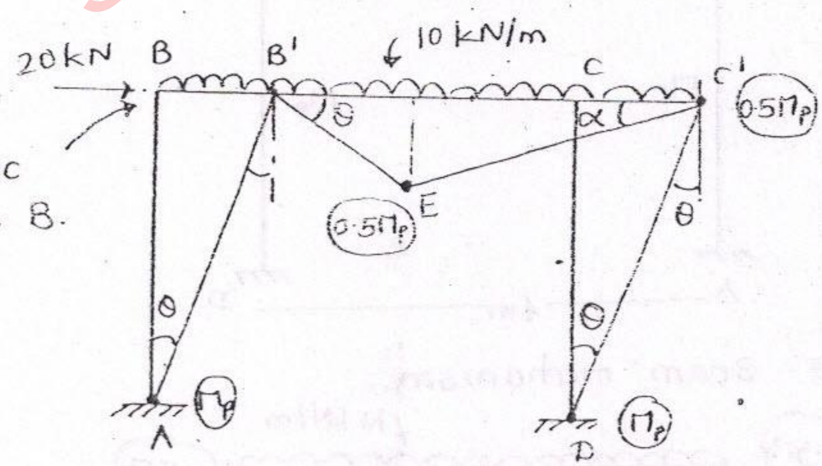


$$20\times 40 = M_p\cdot\theta + 0.5M_p\cdot\theta + 0.5M_p\cdot\theta + M_p\cdot\theta$$

$$80\cdot\theta = 3M_p\cdot\theta$$

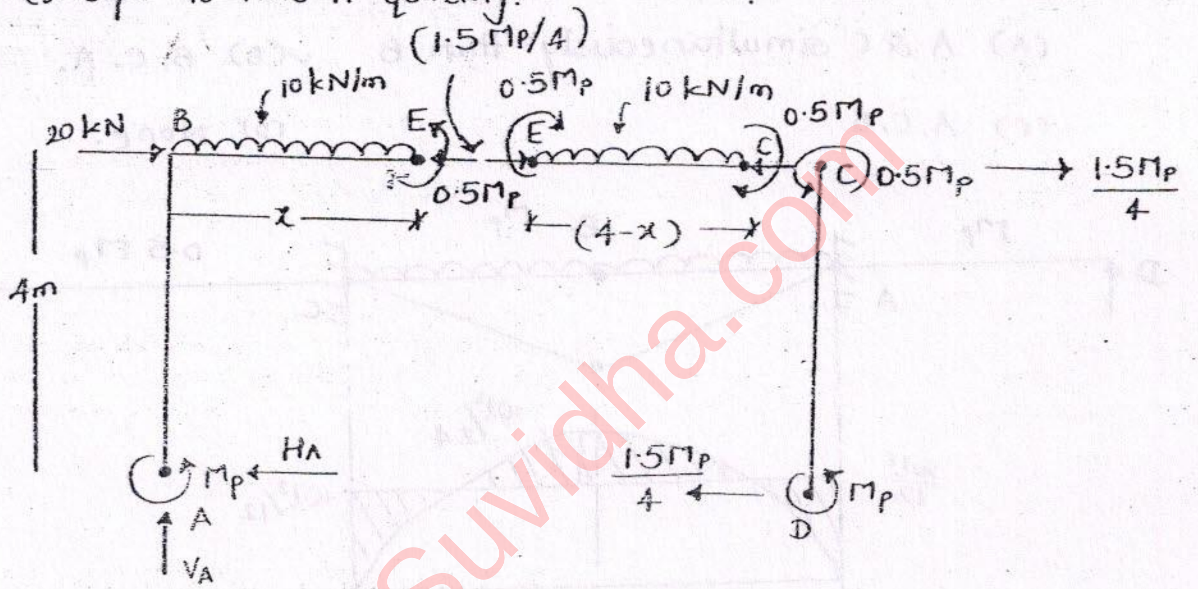
$$M_p = 26.66\ \text{kNm}.$$

Possibility III : Combined Mechanism :



(i) The location of the plastic hinge at E is not known. because the failure mechanism is not similar to proped cantilever subjected to UDL. because moment at B is not zero. So plastic hinge is not developed at a distance of 0·414 L from B.

(ii) Since the location of plastic hinge is not known. use F.B.D. concept to find it quickly.



From F.B.D. of ABE

$\Sigma M_o A = 0$

$$-M_p - 0.5 M_p + \left(10 \times x \cdot \frac{x}{2}\right) + 20 \times 4 - \left(\frac{1.5 M_p}{4}\right) \cdot 4 = 0$$

$$3 M_p = 80 + 5x^2 \quad \text{--- (i)}$$

From F.B.D. of EC.

$\Sigma M_o C = 0$

$$+0.5 M_p + 0.5 M_p \cdot - \left(10 \times \frac{(4-x)^2}{2}\right) = 0$$

$$M_p = (4-x)^2 \times 5 \quad \text{--- (ii)}$$

from — (i) & — (ii)

$$3 \left[5(4-x)^2\right] = 80 + 5x^2$$

$$15(4 + x^2 - 8x) = 80 + 5x^2$$

$$x = 1.52 \text{ m}$$

from —— (iii)

$$M_p = C5\,(4-x)^2$$

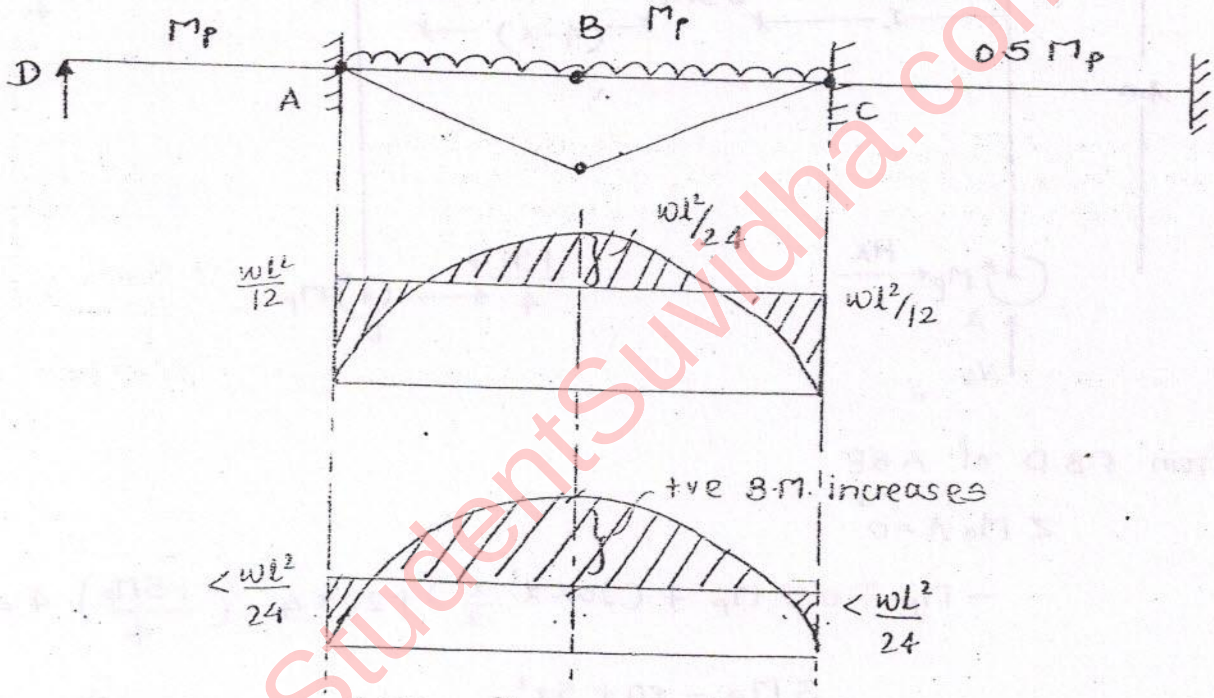
$$= 30.75 \text{ kNm}$$

So minimum M_p required for the structure is 30.75 kNm.

Q: For the continuous beam loaded as shown in fig. the sequence of plastic hinge development is...

(A) A & C simultaneously then B ✓(B) B, C, A.

(C) A, C, B (D) None.

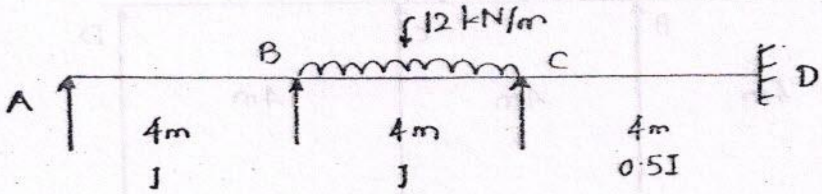


BMD after moment distribution.

(i) Since only member AC is loaded the fixed end moments at A & C will be distributed to members AD and EC so the net -ve B.M. at A and C will be less than $wl^2/24$ so +ve B.M. will be high and first plastic hinge is developed at B.

(ii) Since the the plastic moment capacity of CE is less, 2nd plastic hinge is developed at C and when loaded further 3rd plastic hinge forms at A.

Q. A continuous beam is loaded as shown in fig. find the moments at B and C.



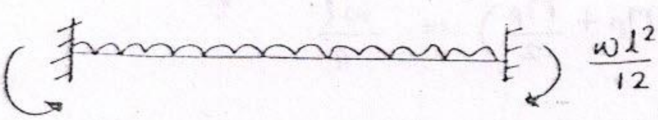
(i) Distribution factors:

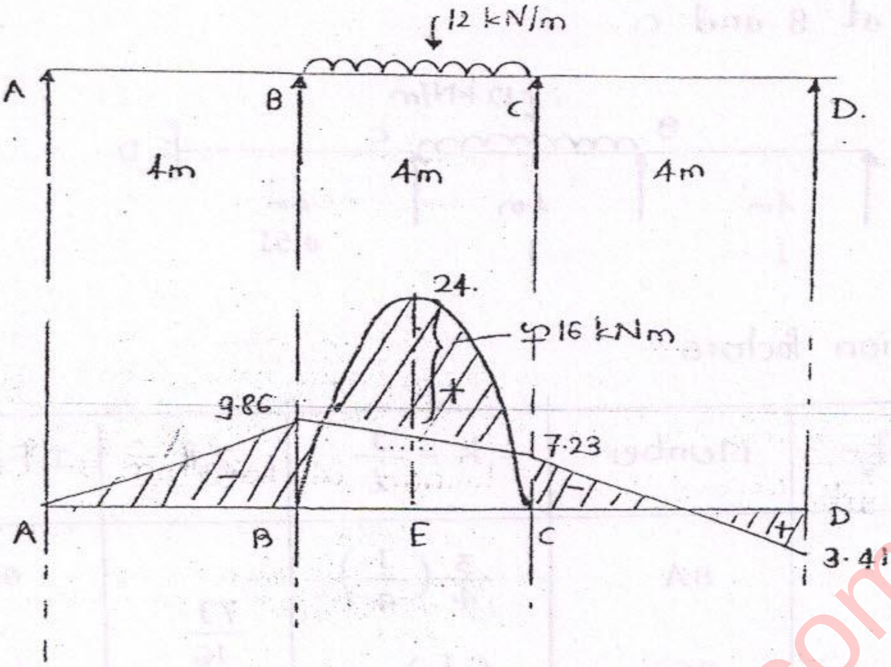
Joint	Member	$k = \frac{I}{l}$	Σk	$D.F = \frac{k}{\Sigma k}$
B	BA	$\frac{3}{4}\left(\frac{I}{4}\right)$	$\frac{7I}{16}$	0.43
	BC	$\left(\frac{I}{4}\right)$		0.57
C	CB	$\left(\frac{I}{4}\right)$	$\frac{3I}{8}$	0.67
	CD	$\frac{0.5I}{4}$		0.33

(ii) End moment distribution.

	A	B	B	C	C	D
D.F	1	0.43	0.57	0.67	0.33	1
F.E.M.	0	0	-16	+16	0	0
Balance / release		+6.88	+9.12	-10.7	-5.3	
Carry over moment	0		-5.35	+4.56		-2.65
Balance		2.35	3.02	-3.06	-1.51	
Carry over	0		-1.51	1.51		-0.75
Balance		0.65	+0.86	-1	-0.5	
Final moments	0	+9.86	-9.86	+7.23	-7.23	-3.41

$$M_{FBC} = -\frac{wl^2}{12} = \frac{-12 \times 4^2}{12} = -16 \text{ kNm}$$





B.M. at E = 16 kNm → M_P = 16 kNm

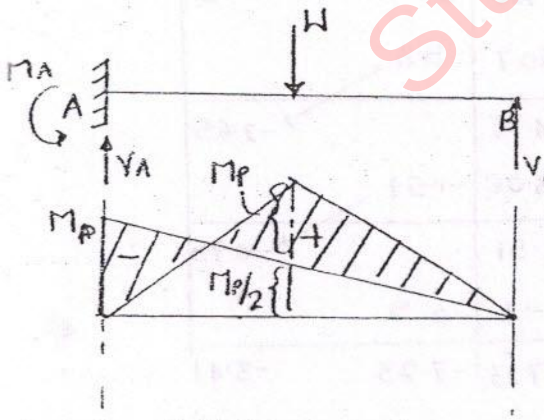
B.M. at B = 9.86 kNm → M_P = 9.86 kNm

B.M. at C = 7.23 kNm → $0.5 M_P$ = 14.46 kNm

∴ First plastic hinge will form at E, then at C and B respectively.

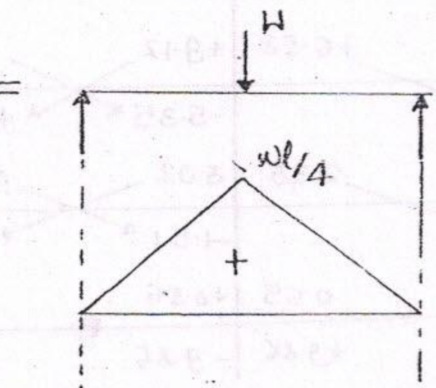
Static method of beam analysis:

It satisfies equilibrium and yield conditions.

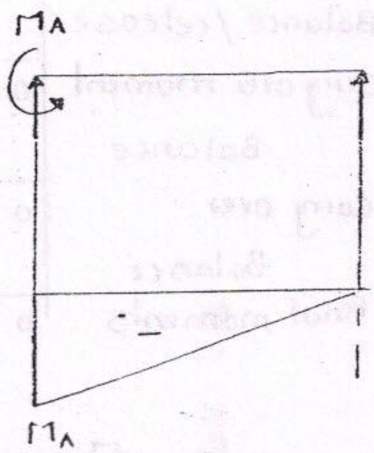


Collapse BMD.

=



free B.M.D. due to SS. beam



BMD due to redundant force

$$\left(M_P + \frac{M_P}{2}\right) = \frac{wl}{4}$$

$$W = W_u = \frac{6M_p}{l}$$

Procedure:

(i) Select the redundant force. Generally moment is taken as redundant.

(ii) Draw Free B.M.D and redundant B.M.D.

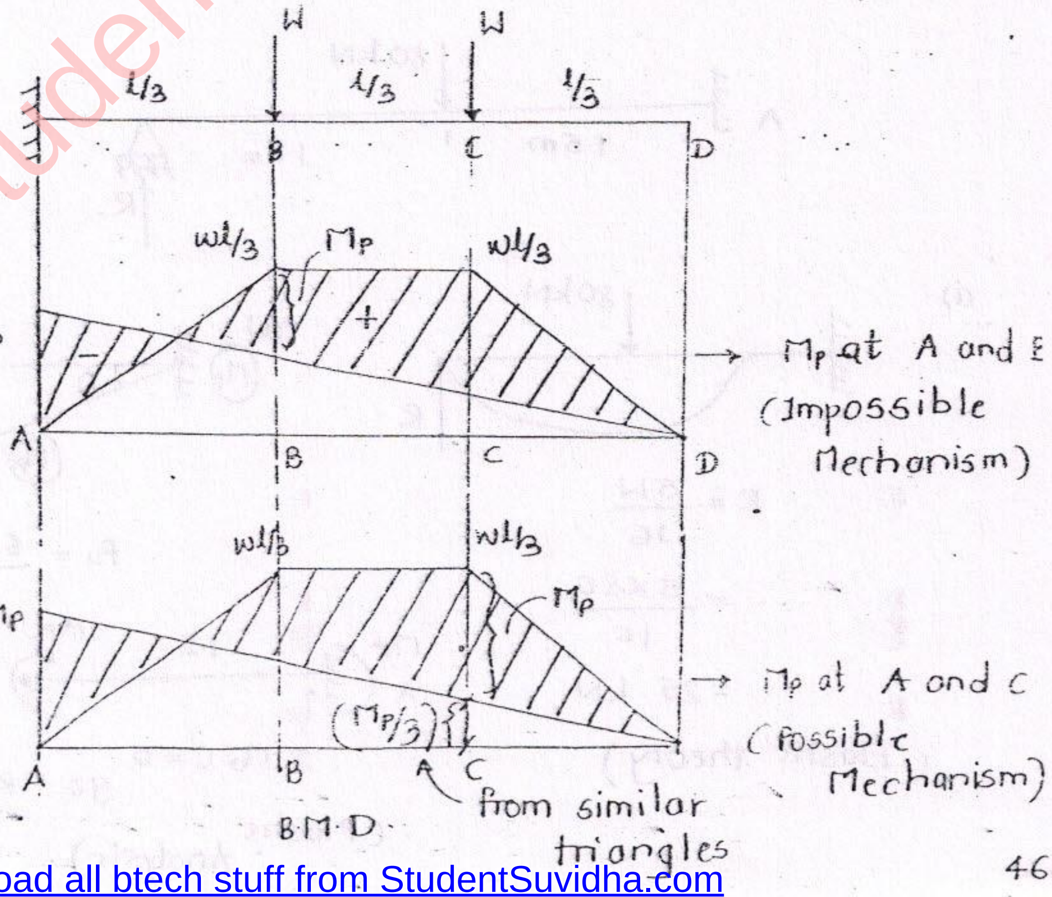
(iii) A combined BMD is drawn such that a collapse mechanism is formed. From equilibrium condition. find collapse load i.e. total free BMD ordinate in simply supported beam equals to free BMD ordinate in collapse BMD.

Free BMD ordinate in SS = Free BMD ordinate in collap

$$\frac{W \cdot l}{4} = \left(M_p + \frac{M_p}{2}\right)$$

$$W = W_u = \frac{6M_p}{l}$$

Example:



(i) If M_p is assumed at A and B then BM at C will be greater than M_p. So impossible mechanism.
(As nowhere BM should excess M_p)

(ii) If M_p is assumed at A and C then BM will not exceed anywhere. So possible mechanism.

Equating free B.M.D.

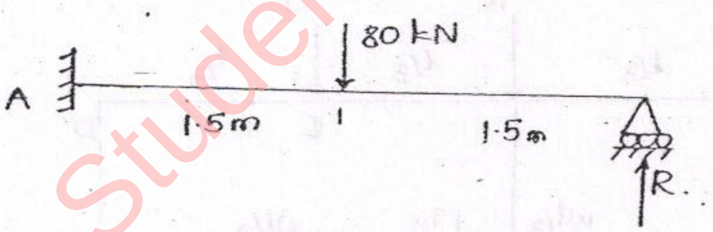
$$M_p + \frac{M_p}{3} = \frac{wl}{3}$$

$$w = W_u = \frac{4M_p}{l}$$

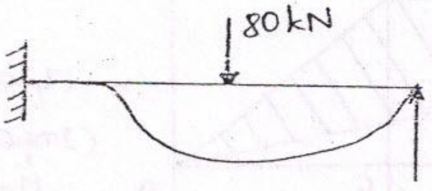
Q. The magnitude of P is increased till collapse and plastic moment carrying capacity of steel beam is 90 kNm.

(i) What is the value of R. If P is 80 kN, by elastic theory

(ii) The value of R using plastic analysis is ?



(i)



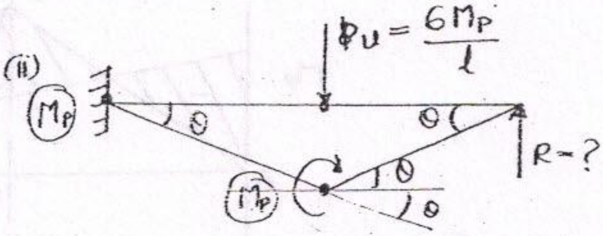
$$\therefore R = \frac{5W}{16}$$

$$= \frac{5 \times 80}{16}$$

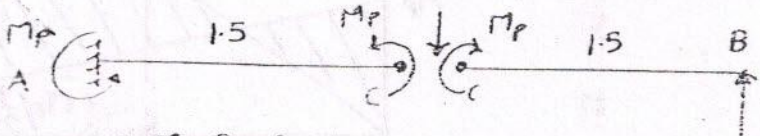
$$= 25 \text{ kN}$$

(Elastic theory)

(ii)



$$P_u = \frac{6 \times 90}{3} = 180 \text{ kN}$$



$\Sigma M_C = 0$

$$90 - R \times 1.5 = 0$$

$$R = 60 \text{ kN}$$

(Plastic Analysis)