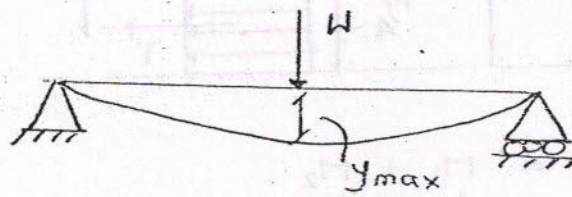
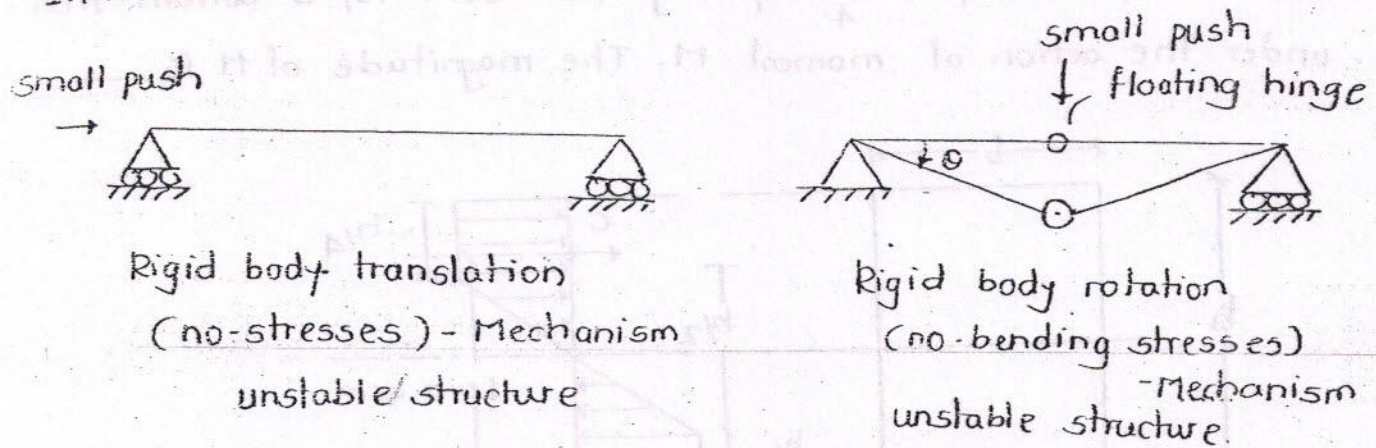


Collapse loads for standard cases:

1. Mechanism:



Bending stresses are developed
(not mechanism)

If the segments of a beam will between plastic hinges are able to move without increase in load then it is called Mechanism. (Mechanism also means rigid body translation or rigid body rotation)

At mechanism condition, (unstable condition) the beam remains straight line between the plastic hinges.

2. Static indeterminacy.

(Degree of redundancy - D_s)

Static \rightarrow using equations of statics

$$\text{i.e. } \sum X = 0, \sum Y = 0, \sum M = 0.$$

Indeterminacy \rightarrow cannot be determined.

It is the number of unknown reactions in excess of the equilibrium equations.

$$D_s = r - s$$

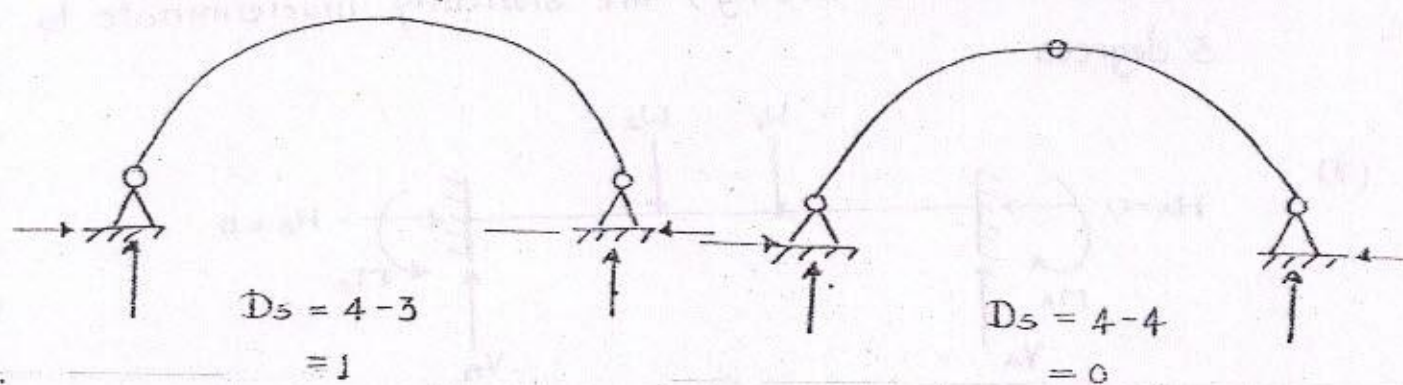
where

r - no. of unknown reactions

s - no. of eqn. of equilibrium available.

statically indeterminate

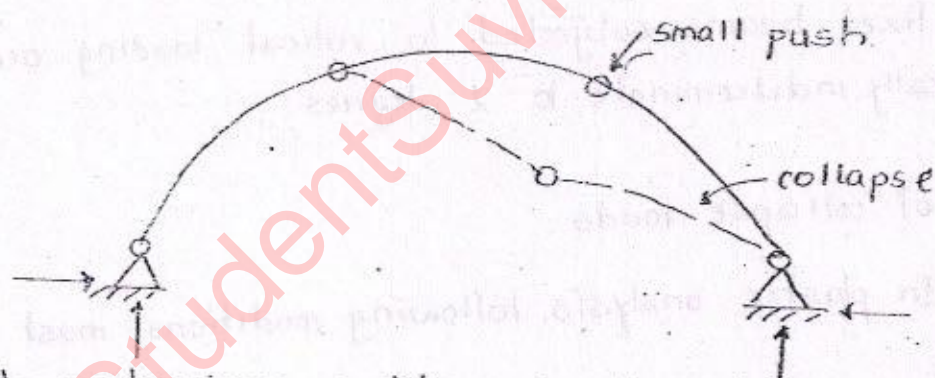
statically determinate



To make a statically indeterminate structure statically determinate, the number of plastic hinges required is equal to D_s .

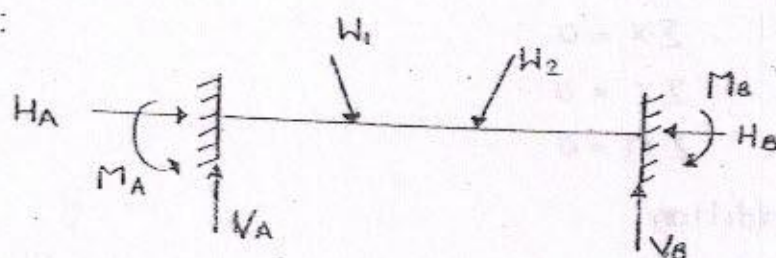
To form a mechanism in a statically indeterminate structure, the number of plastic hinges required is

$$n = D_s + 1$$



At mechanism condition structure becomes unstable.

Example:



r - no. of unknown reactions = 6

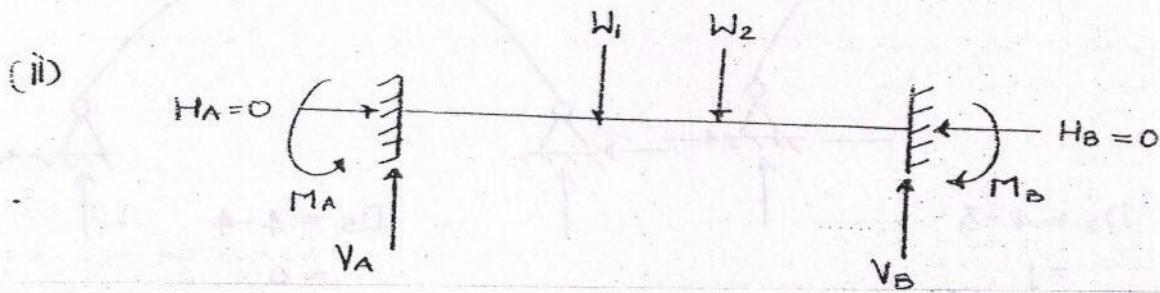
s - no. of eqⁿ of equilibrium available = 3

$$D_s = r - s$$

$$= 6 - 3$$

$$= 3$$

So, fixed beams subjected to general loading (which includes inclined loading) are statically indeterminate to 3 degrees.



$$\therefore \sum F_z = 0$$

$$\therefore H_A = H_B = 0$$

r - no. of unknown reaction = 4

S - no. of eqn^s of equilibrium = 2 ($\sum F_y = 0, \sum M = 0$)

$$D_s = r - S$$

$$= 4 - 2 = 2$$

So, fixed beams subjected to vertical loading are statically indeterminate to 2 degrees.

Calculation of collapse loads:

In plastic analysis, following conditions must be satisfied.

(i) Equilibrium conditions:

$$\sum X = 0$$

$$\sum Y = 0$$

$$\sum M = 0$$

(ii) Mechanism condition:

At collapse, sufficient no. of plastic hinges must be developed so that a part or entire structure must transform into a mechanism, leading to collapse.

(iii) Yield condition :

At collapse, bending moment at any section should not exceed the plastic moment capacity of the beam $\forall s$. (M_p)

If all the three conditions are satisfied, a unique lowest value of collapse load will be obtained.

Note:

If only (i) and (ii) conditions are satisfied or (i) and (iii) conditions are satisfied, we have two methods of analysis.

(1) Kinematic method (Upper bound theorem)

(2) Static method (Lower bound theorem)

Characteristics of Upper bound theorem :

(i) It satisfies equilibrium and mechanism conditions

(ii) It states that "the collapse load found by assuming the mechanism will always be greater than or equal to true collapse load. [$P \geq P_u$]"

where :

P - calculated collapse load.

P_u - true collapse load.

Characteristics of lower bound theorem :

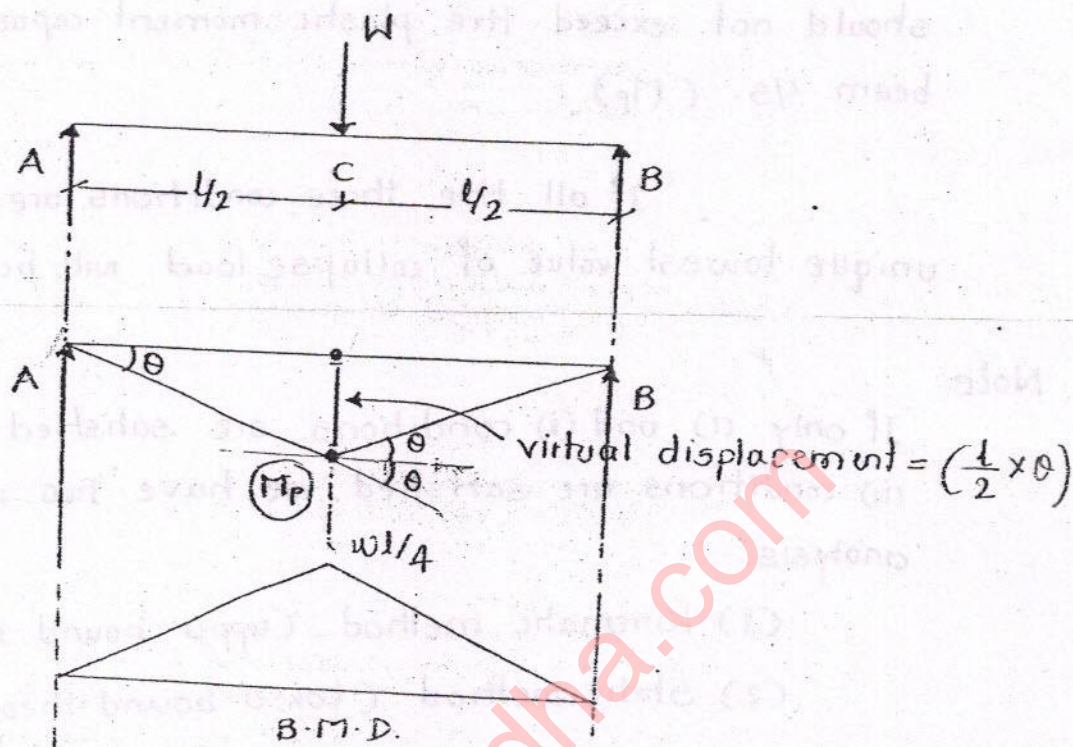
(i) It satisfies equilibrium and yield conditions

(ii) It states that "the collapse load found on the basis of collapse B.M.D. (in which B.M. at any section is less than or equal to M_p) will always be less than or equal to collapse load ($P \leq P_u$).

Kinemotic method:

1. Simply supported beam:

(a)



$$D_s = r - s$$

$$= 2 - 2 = 0 \quad (\text{statically determinate structure})$$

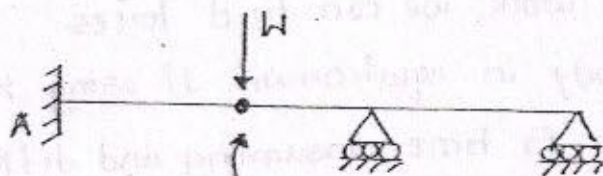
No. of plastic hinges required to form mechanism. (or to make it unstable)

$$n = D_s + 1$$

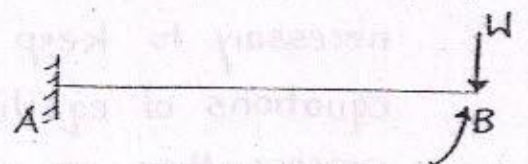
$$= 0 + 1 = 1$$

(i) Locations where plastic hinges will form:

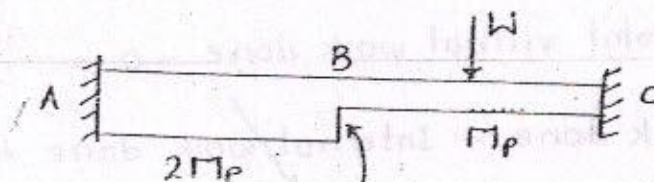
- At locations of maximum B.M.
- At rigid joints or at fixed supports. (if the angle between members do not change even after the application of loads then the joint is called rigid joint)
- Under the point loads in supported spans. (not at free ends)
- Wherever cross section changes
- Wherever the material changes.



plastic hinge can form



plastic hinge cannot form (B.M. is zero)



plastic hinge can develop



plastic hinge can form.

(ii) Principle of virtual work:

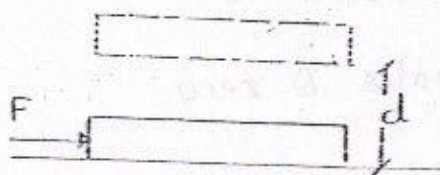
virtual - imaginary

Work = Force \times distance travelled along its line of action.

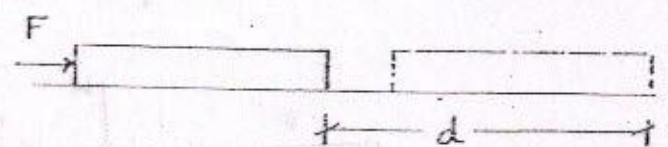
$$= F \times d \quad \text{or}$$

$$= \text{Moment} \times \text{angular displacement}$$

$$= M \times \theta$$



$$\text{Work done by } F = F \times 0 = 0$$



$$\text{Work done by } F = F \times d = Fd$$

Principle:

When the body is in equilibrium, total virtual work done by all forces is zero

Using principle of virtual work, we can find forces necessary to keep a body in equilibrium. If using the equations of equilibrium is time consuming and difficult process then we use principle of virtual work to find unknowns quickly.

(ii) Explanation:

$$U - \text{total virtual work done} = 0$$

$$\text{External work done} + \text{Internal work done} = 0$$

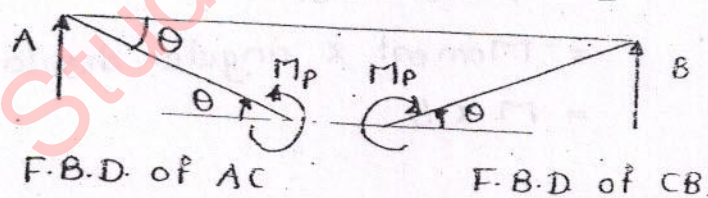
(It is always -ve because M_p and θ always act in opposite directions)

$$\therefore \text{External work done} - \text{Internal work done} = 0$$

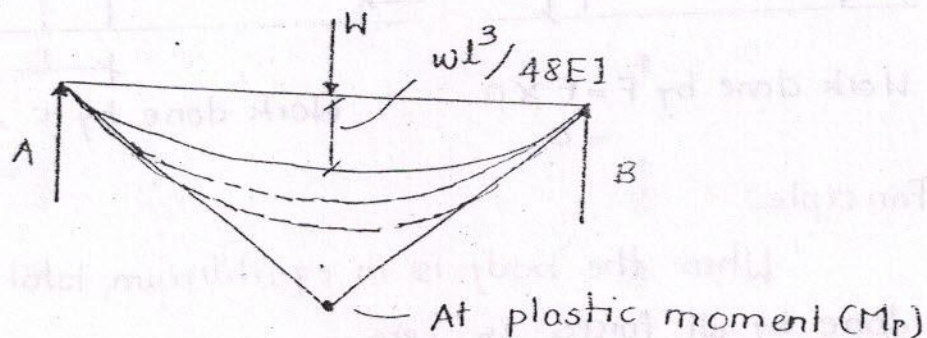
$$\text{External work done} = \text{Internal work done}$$

$$+ W \times \left(\frac{L}{2} \times \theta \right) = M_p (\theta + \theta)$$

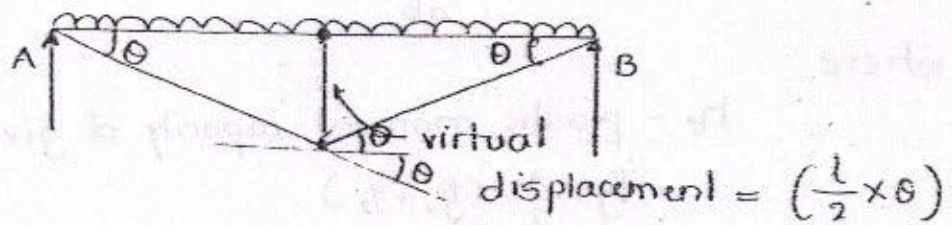
$$W = W_u = \frac{4 M_p}{L}$$



The purpose of M_p is to make θ zero



(b)

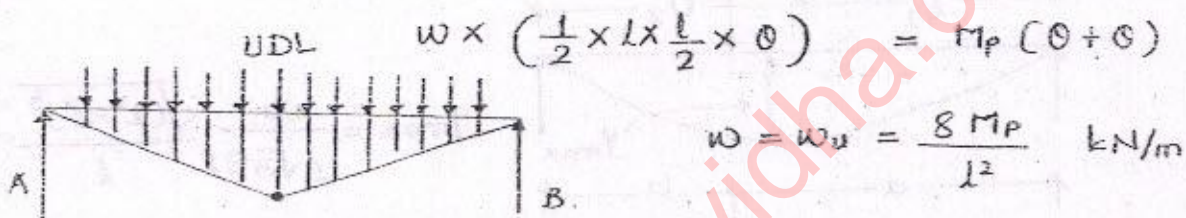


plastic hinge - requires moment M_p to rotate.

ordinary hinge - requires zero moment to rotate.

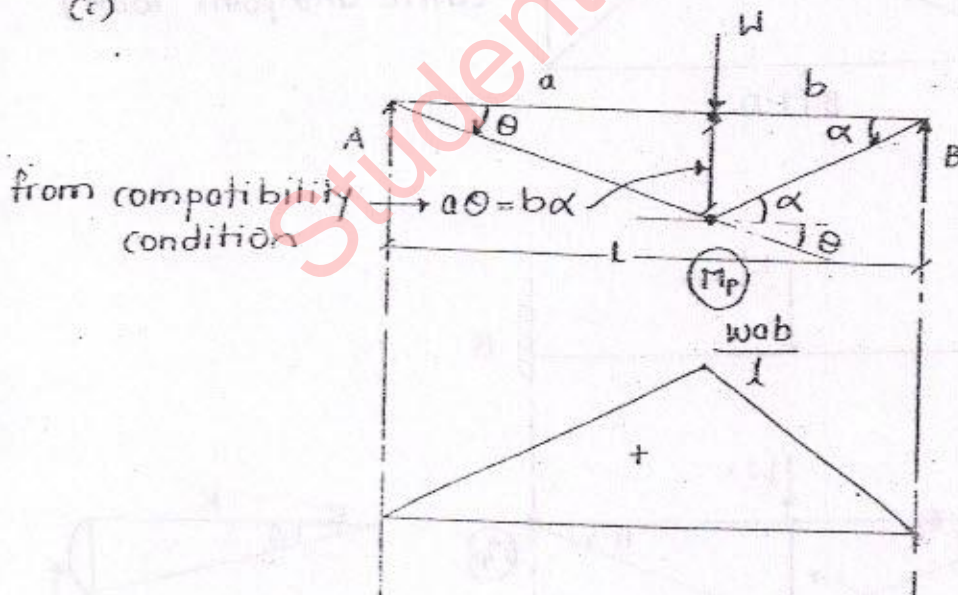
External work done = Internal work done

Intensity of UDL \times area of the displacement diag. = $M_p (\theta + \theta)$



Friday
4th October 2013

(c)



External work done = Internal work done

$$W \cdot (a\theta) = M_p (\theta + \alpha)$$

$$W \cdot (a\theta) = M_p \left(\frac{b\theta + a\theta}{b} \right) \therefore \alpha = \frac{a}{b} \theta$$

$$w = w_d = \frac{M_p \cdot l}{ab}$$

where

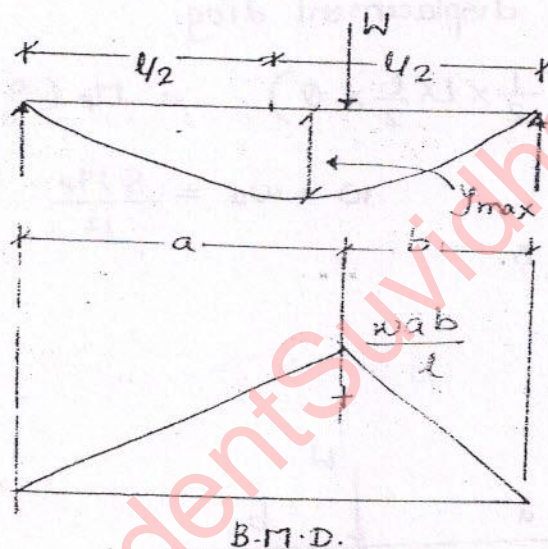
$$M_p - \text{plastic moment capacity of given beam} \text{ c/s}$$

$$= f_y \cdot \frac{A}{2} (y_1 + y_2)$$

(for a given beam M_p is constant)

Note:

At the location of max. B.M., max. deflection need not happen. Location of max. B.M. and max. deflection are independent.

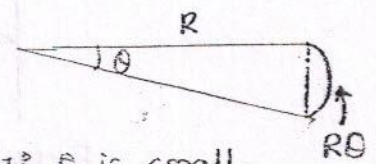
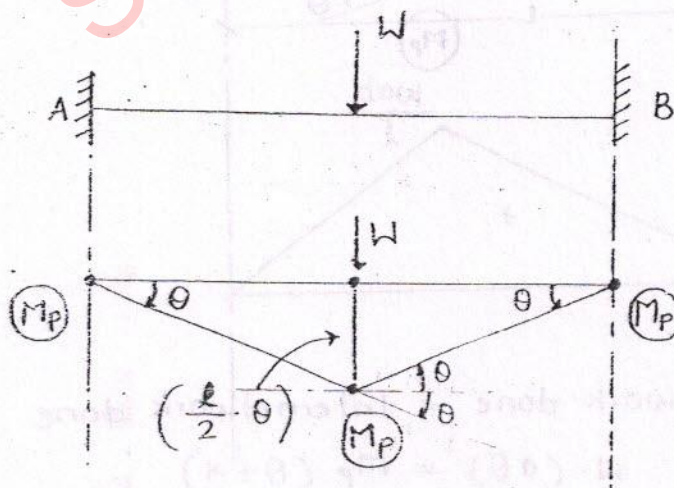


$$y_{\max} = \frac{wb}{9\sqrt{3}EI} \frac{\sqrt{(l^2 - b^2)^3}}{l}$$

(max. deflection occurs somewhere in between centre and point load)

2. Fixed beam:

(a)



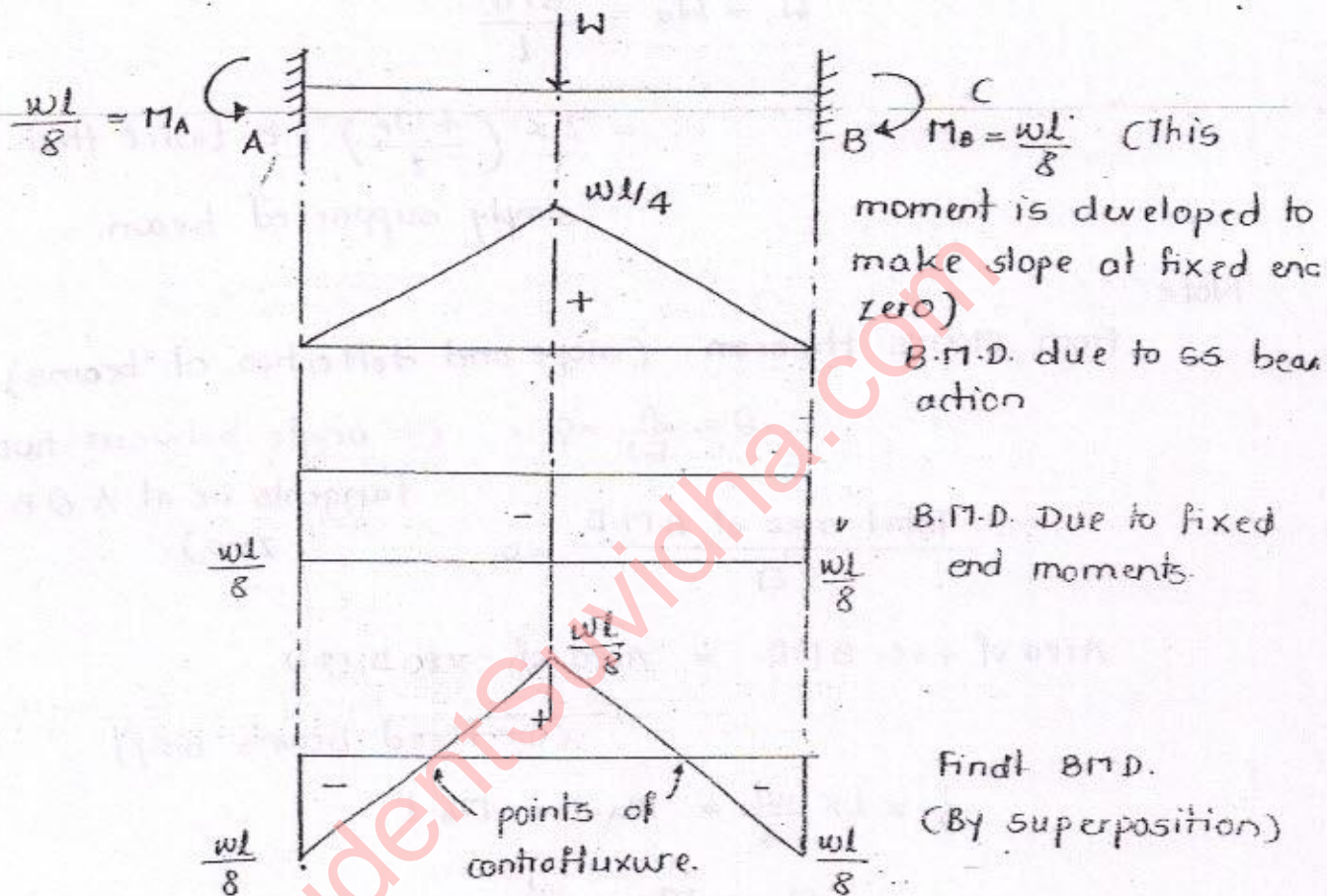
If θ is small, $R\theta$ can be taken as straight line

$$D_s = r - s$$

$$= 4 - 2 = 2$$

n - no. of plastic hinges required at collapse = $2 + 1$
 $= 3$

(i) From strength of materials.



The resultant B.M.D. due to superposition is obtained when material is linearly elastic.

$$\text{Max. +ve B.M.} = \frac{wl}{8}$$

$$\text{Max. -ve B.M.} = \frac{wl}{8}$$

(ii) sequence of plastic hinge development:

Since magnitudes of +ve B.M. and -ve B.M. are same, plastic hinges are developed at A, B & C simultaneously.

(iii) From principle of virtual work:

External work = Internal work.

$$W \left(\frac{l}{2} \times \theta \right) \quad = \quad \underbrace{M_p \cdot \theta}_{\text{At A}} + \underbrace{M_p (\theta + \theta)}_{\text{At C}} + \underbrace{M_p \cdot \theta}_{\text{At B}}$$

$$W = W_u = \frac{8M_p}{l}$$

= $2 \times \left(\frac{4M_p}{l} \right)$ i.e. twice that for simply supported beam.

Note:

From Mohr's Theorem (slope and deflection of beams)

$$\theta = \frac{\Delta}{EI} = 0$$

(∵ angle between two tangents i.e. at A & B is zero)

$$\frac{\text{Total area of B.M.D.}}{EI} = 0$$

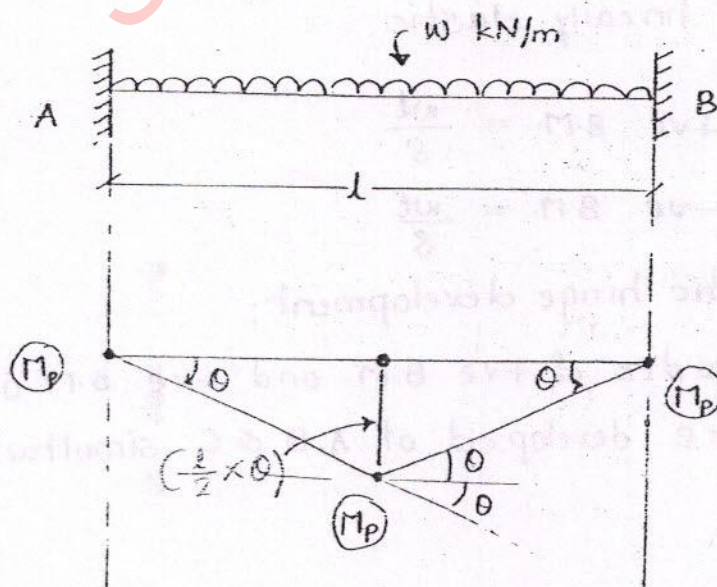
Area of +ve B.M.D. = Area of -ve B.M.D.

(for fixed beams only)

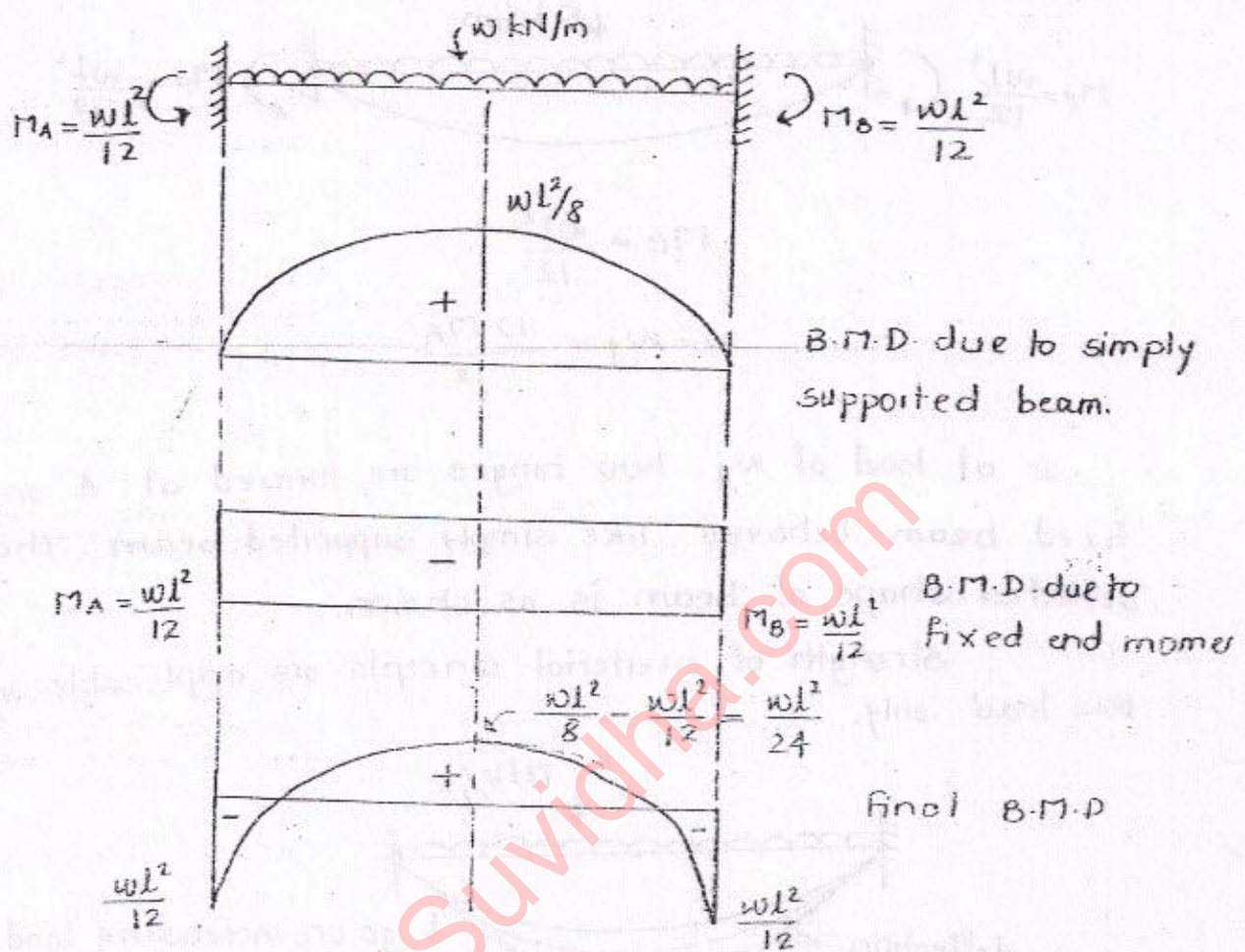
$$\frac{1}{2} \times l \times \frac{wl}{4} = M_A \cdot l = M_B \cdot l$$

$$M_A = M_B = \frac{wl}{8}$$

(b)



(i) B.M.D. from strength of materials:



$$\text{Max. +ve B.M.} = \frac{wl^2}{24}$$

$$\text{Max. -ve B.M.} = \frac{wl^2}{12}$$

i.e.

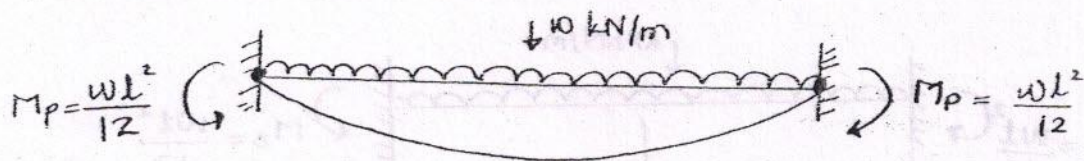
$$\text{Max. -ve B.M.} = 2 \times \text{Max. +ve B.M.}$$

(ii) Sequence of plastic hinge development.

Since -ve B.M. is more than +ve B.M. first two plastic hinges are developed at A and B simultaneously.

When loaded further, third hinge will develop at centre and then mechanism is developed.

(iii) load at which first two hinges will form:

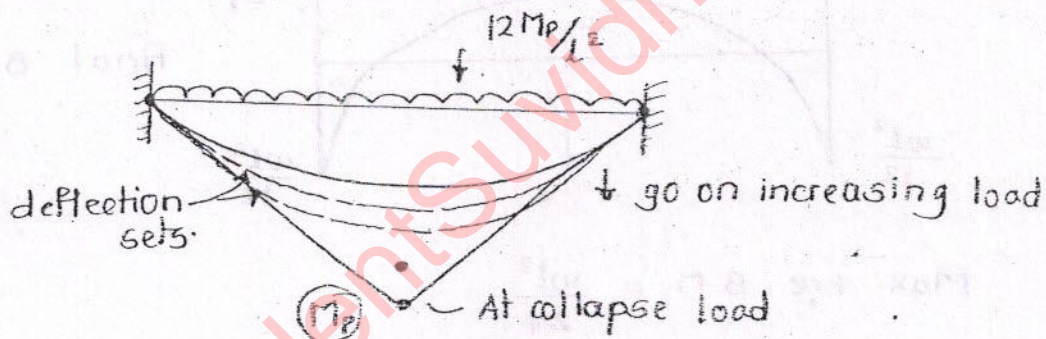


$$M_p = \frac{w l^2}{12}$$

$$w = w_y = \frac{12 M_p}{l^2}$$

so at load of w_y , two hinges are formed at A and B & fixed beam behaves like simply supported beam. The deflected shape of beam is as shown.

Strength of material concepts are applicable upto this load only.



(iv) Load at which third plastic hinge will form:

Since strength of material formula cannot be applied, use principle of virtual work.

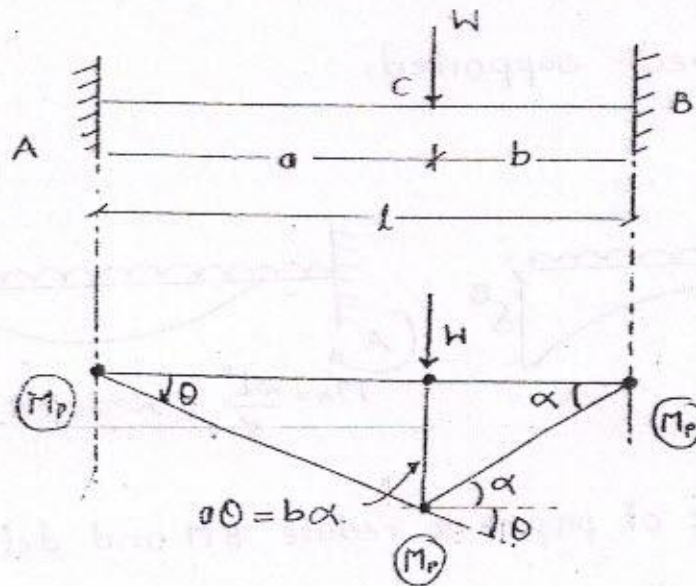
External work = Internal work.

$$w \left(\frac{1}{2} \times l \times \frac{l}{2} \times \theta \right) = M_p \theta + M_p (\theta + \theta) + M_p \theta$$

at A At C At B.

$$w_u = \frac{16 M_p}{l^2}$$

(c)



By principle of virtual work.

External work done = Internal work done.

$$W \times a\theta = M_p \times \theta + M_p (\theta + \alpha) + M_p \cdot \alpha.$$

At A At C At B.

$$= M_p \cdot \theta + M_p \left(\frac{b\theta + a\theta}{b} \right) + M_p \cdot \frac{a\theta}{b}$$

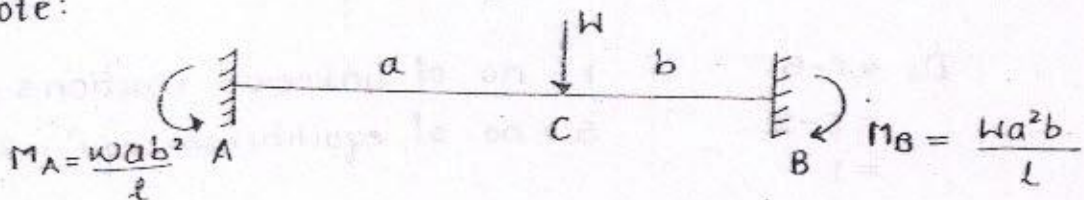
$$\therefore a\theta = b\alpha$$

$$= \frac{M_p \cdot 2l \cdot \theta}{b}$$

$$W \cdot = W_d = \frac{2M_p \cdot l}{ab}$$

i.e. twice than S.S. beam.

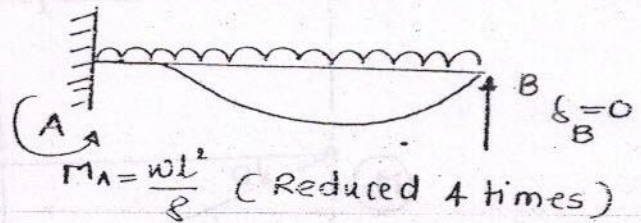
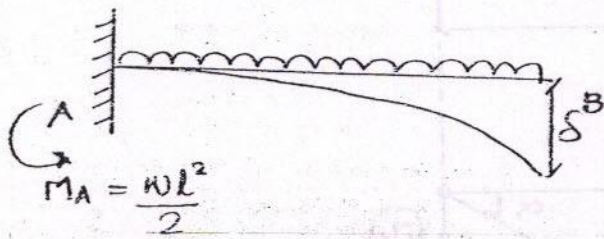
Note:



Since load is nearer to support B, first hinge is developed at B. because negative B.M. is max. at B. The formation of 2nd plastic hinge can be at A or C, depending on relative magnitudes of a, b.

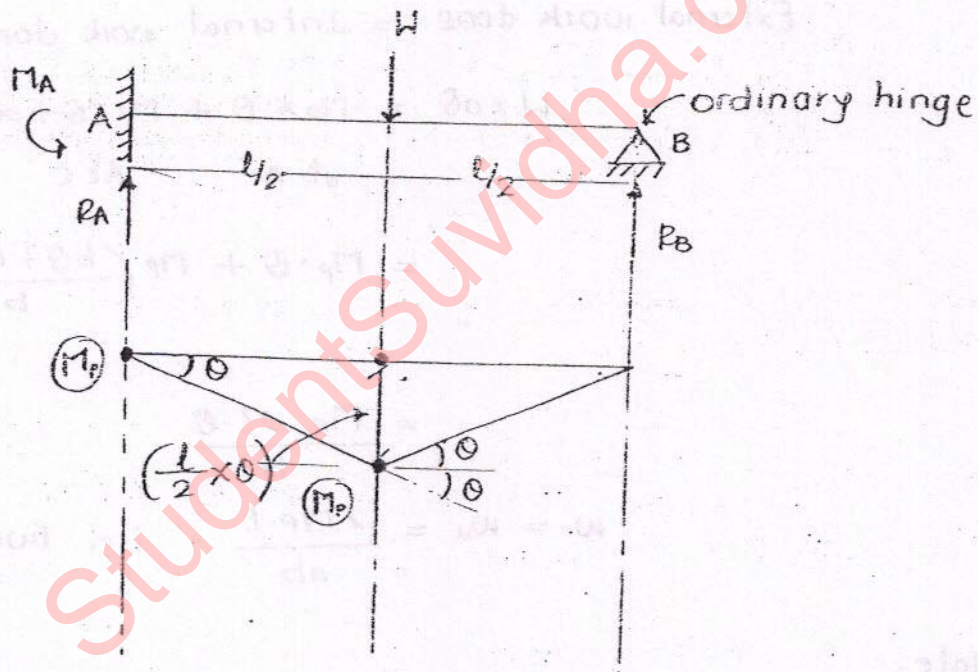
3. Propped cantilever beam:

propped - supported.



The purpose of prop is to reduce B.M. and deflection in cantilever beams.

(a)

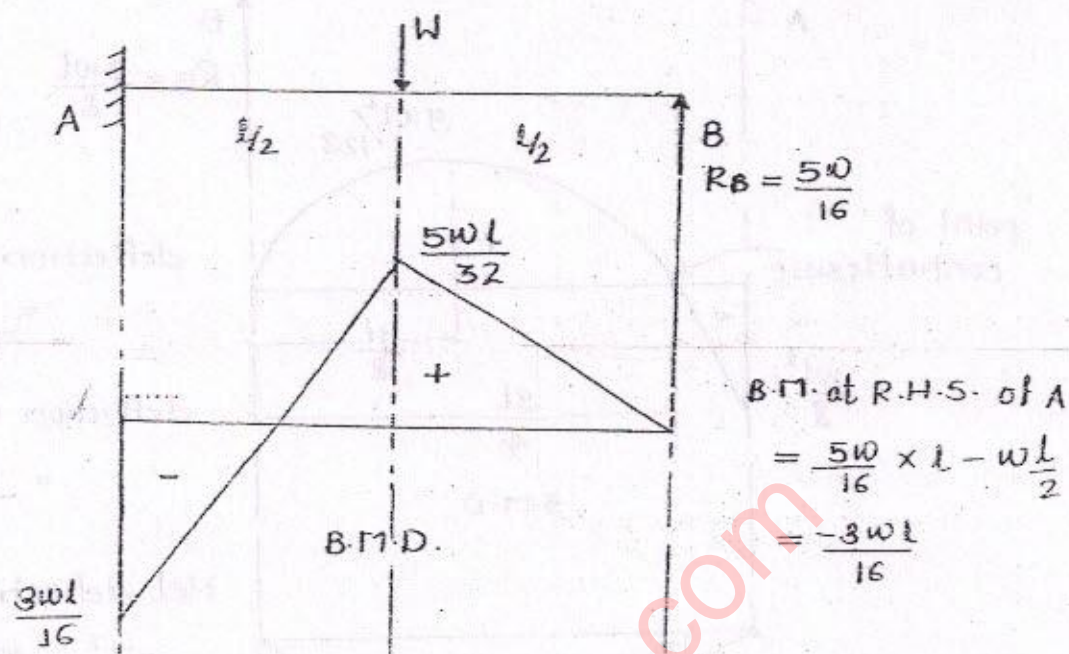


$$\begin{aligned} D_s &= r - s \\ &= 3 - 2 \\ &= 1 \end{aligned}$$

$$\begin{aligned} r &= \text{no. of unknown reactions} = 3 \\ s &= \text{no. of equilibrium eqn} = 2 \end{aligned}$$

$$\begin{aligned} \text{No. of plastic hinges required at collapse} &= D_s + 1 \\ &= 2 \end{aligned}$$

(i) From strength of materials :



Bending moment exactly under a couple is indeterminate. That is why we calculated B.M. at R.H.S. of A and not at A.

(ii) Sequence of plastic hinge formation:

Since -ve B.M. is more first plastic hinge is formed at A. When loaded further, the 2nd plastic hinge is developed and mechanism is developed.

(iii) From principle of virtual work:

External work = Internal work.

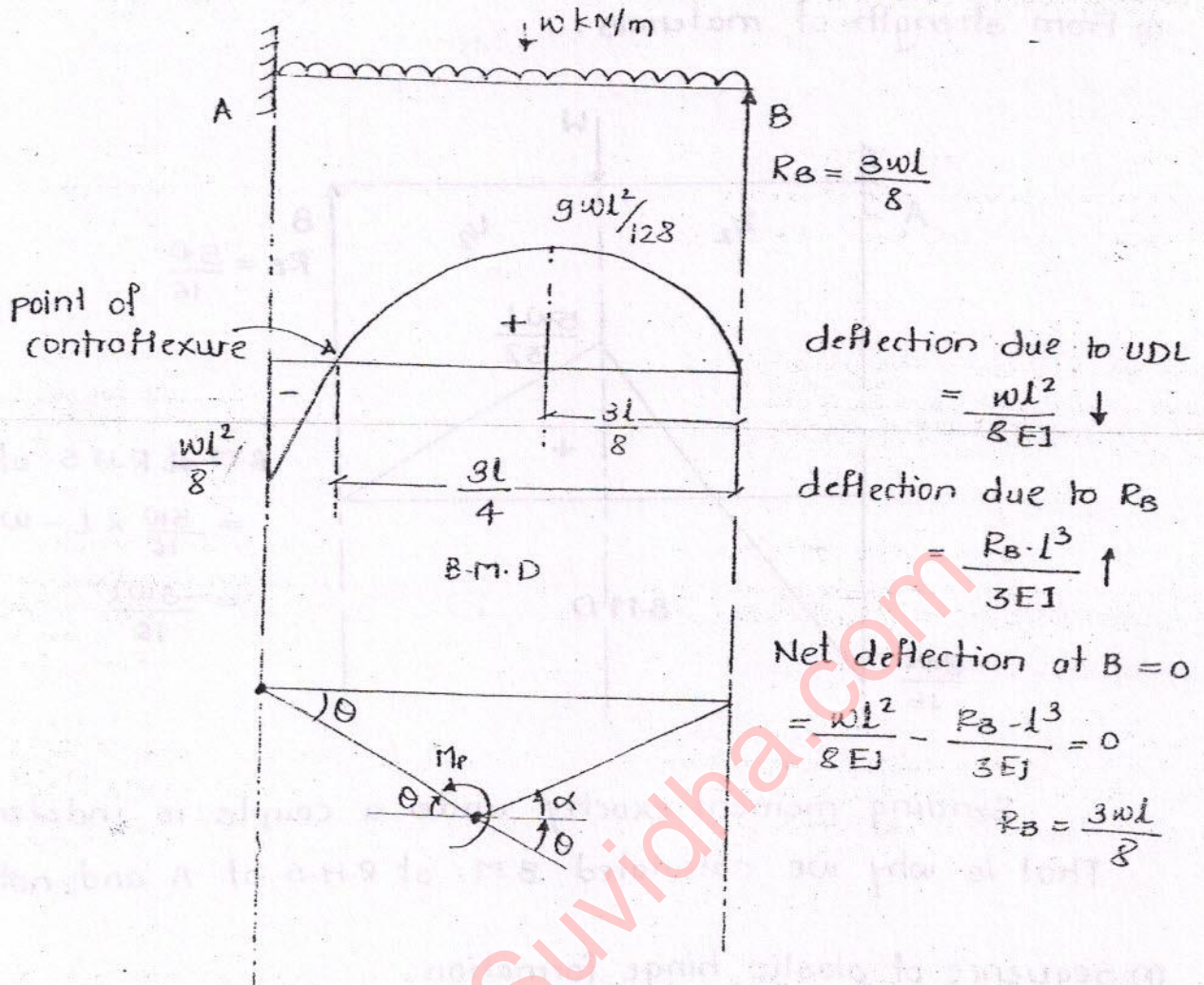
$$W \times \left(\frac{1}{2} \times \theta \right) = M_p \times \theta + M_p (\theta + \theta)$$

at A at C

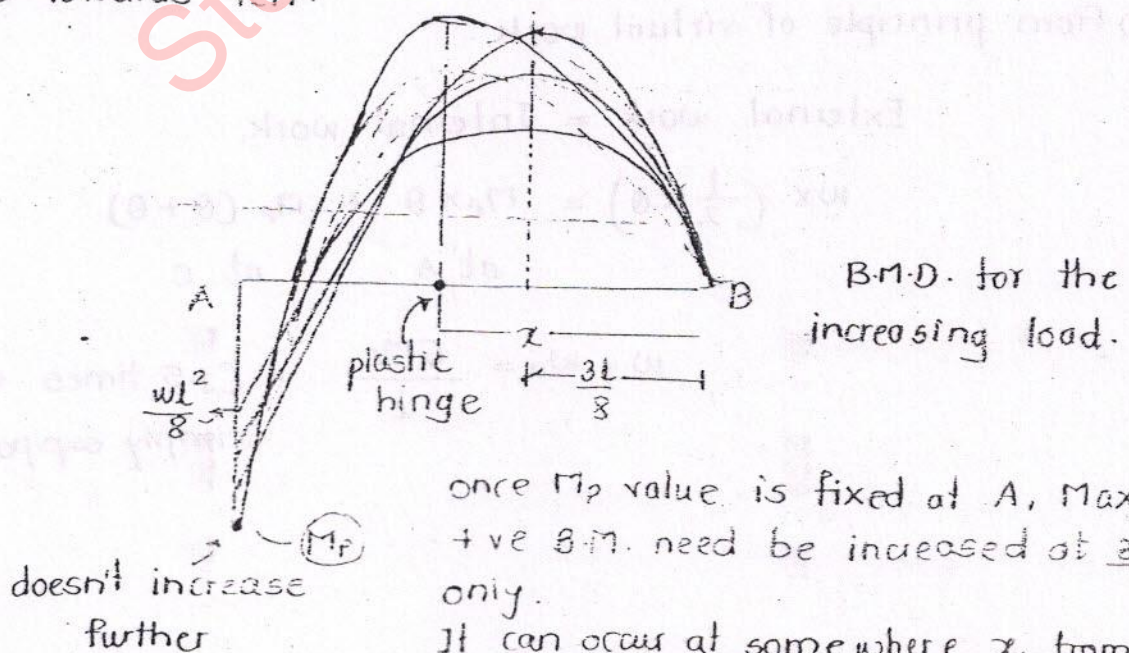
$$W = W_u = \frac{6M_p}{l}$$

(1.5 times W_u for simply supported beam)

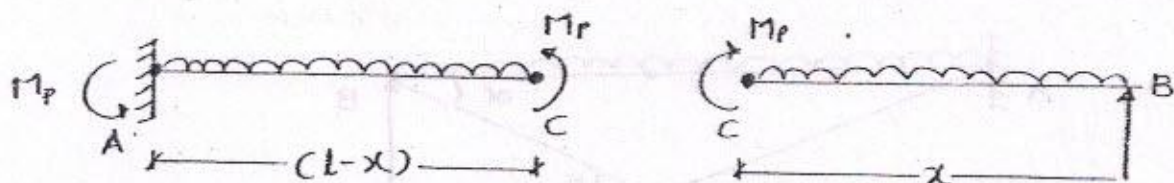
(b)



since, -ve B.M. is more than +ve B.M., first plastic hinge is developed at fixed support A. Second plastic hinge is not developed at distance of $3l/8$ from fixed end. It gets shifted towards left.



Since the location of 2nd plastic hinge is not known, use free body diagram concept, to find it quickly.



FBD of AC

FBD of CB

(No reaction at C because at the location of max. B.M. the shear force is zero.)

from F.B.D of AC,

$$\sum M_A = 0$$

$$-M_p - M_p + w(l-x) \cdot \frac{(l-x)}{2} = 0$$

$$M_p = \frac{w(l-x)^2}{4} \quad \text{--- (i)}$$

↺ +ve ↻ -ve
As external force/moment

from F.B.D. of CB,

$$\sum M_B = 0$$

$$+M_p - w x \cdot \frac{x}{2} = 0$$

$$M_p = \frac{wx^2}{2}$$

from — (i) & — (ii)

$$\frac{w(l-x)^2}{4} = \frac{wx^2}{2}$$

$$l^2 + x^2 - 2lx = 2x^2$$

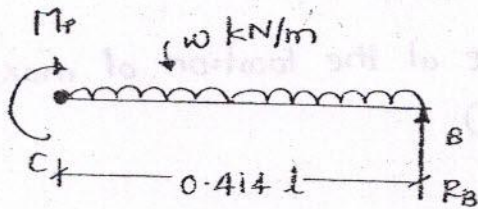
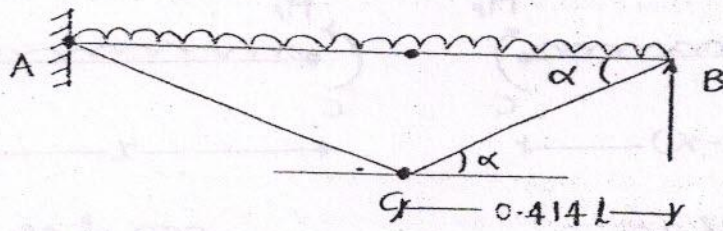
$$x = 0.414 \cdot l \quad (\text{from proped end})$$

substitute in x — (ii)

$$M_p = \frac{w \times (0.414 l)^2}{2}$$

$$\begin{aligned} \text{Collapse load, } w = w_c &= \frac{M_p \times 2}{(0.414 l)^2} \\ &= \frac{11.656 M_p}{l^2} \end{aligned}$$

- Q. A propped cantilever is subjected to collapse U.D.L. of w kN/m throughout its length. At collapse condition what is the prop reaction?



F.B.D. of CB

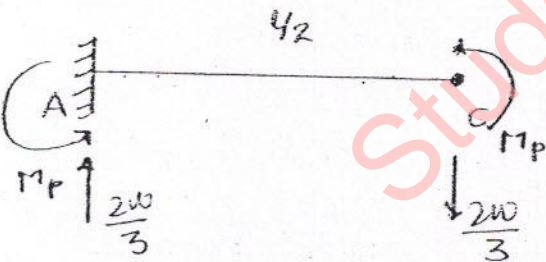
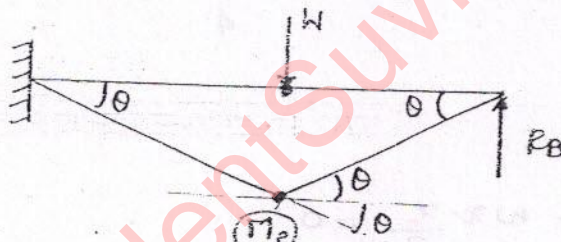
$$\sum F_y = 0$$

$$(-0.414L) \cdot w + R_B = 0$$

$$R_B = 0.414 wL$$

↑ +ve
↓ -ve

- Q. A propped cantilever is subjected to central point load w . At collapse condition, prop reaction is -



$$w_u = \frac{6M_p}{L}$$

$$W - \frac{w}{3} = \frac{2w}{3}$$

At collapse.

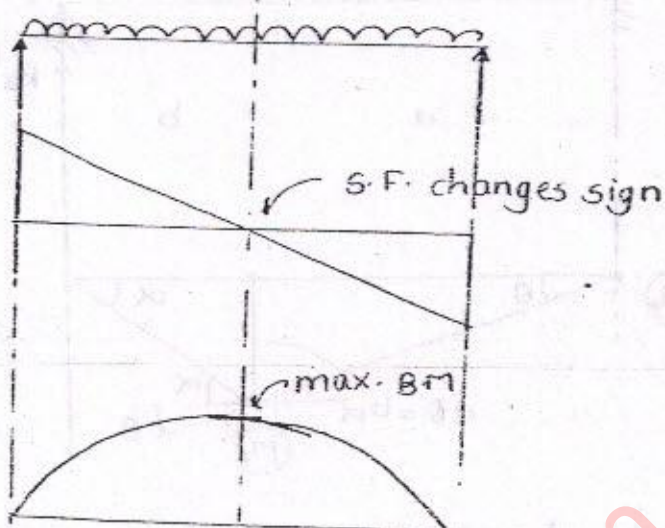
$$V_B = \frac{W}{3}$$

But in stable condition. $R_B = \frac{5w}{16}$

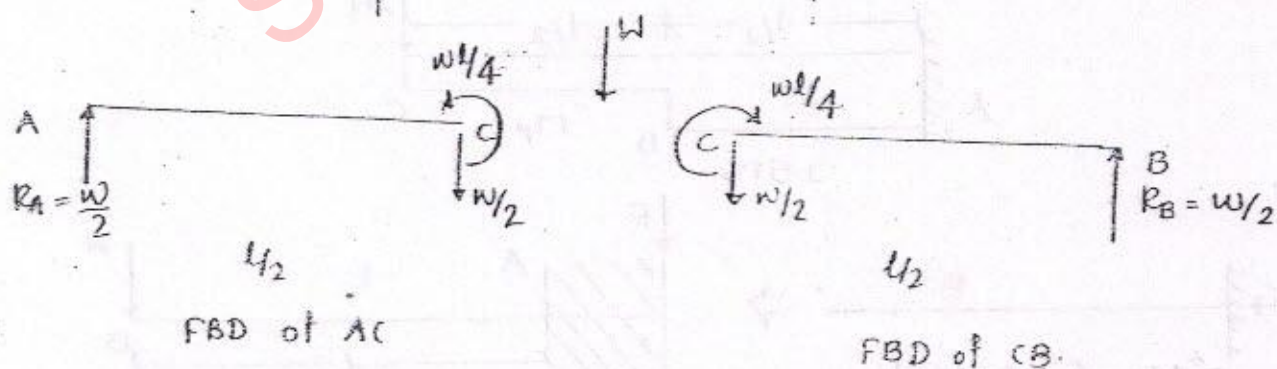
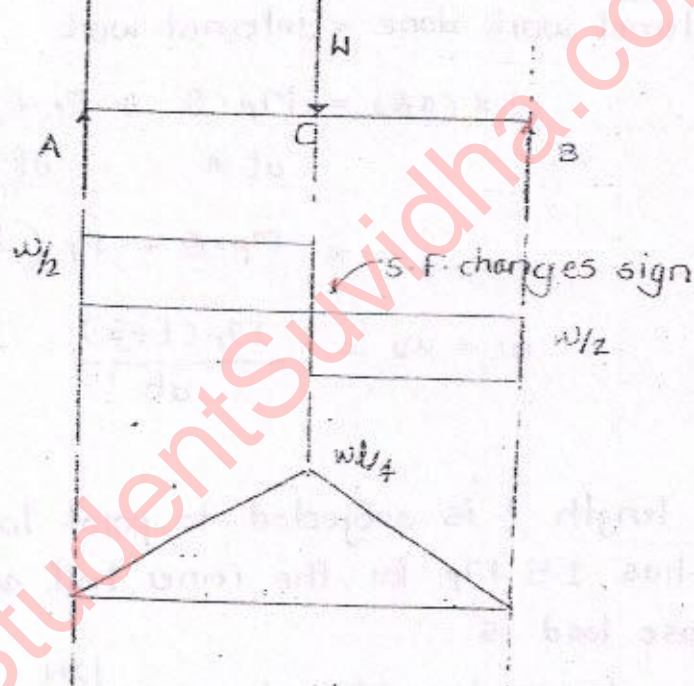
$$\begin{aligned} R_B &= \frac{w}{3} \\ R_B &= \frac{M_p}{L/2} \\ &= \frac{2M_p}{L} \\ &= \frac{2 \times \frac{wL}{6}}{L} \\ &= \frac{w}{3} \end{aligned}$$

Note:

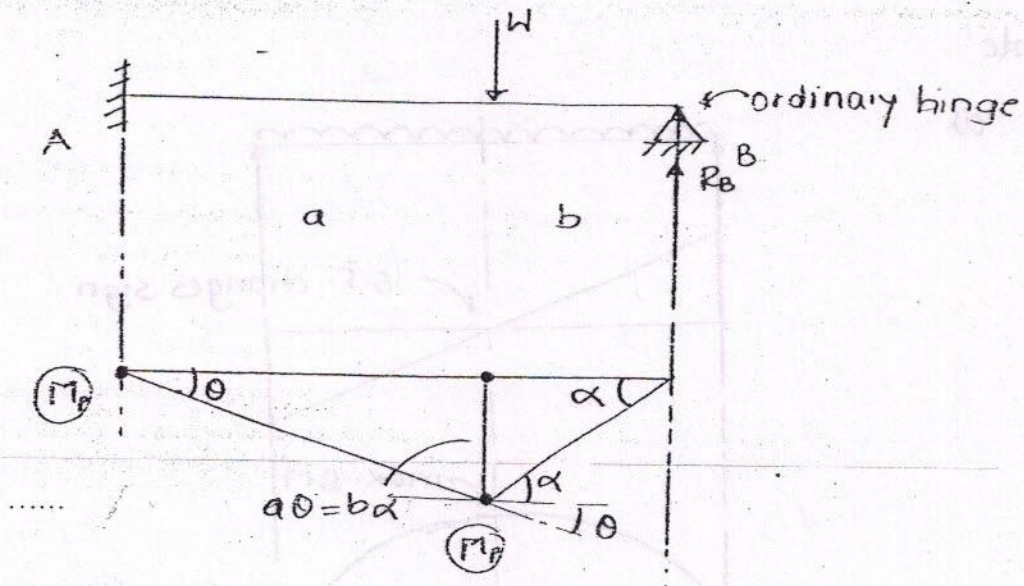
(i)



(ii)



(c)



By principle of virtual work,

External work done = internal work

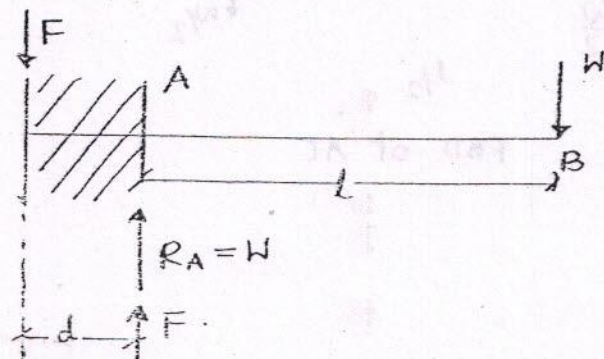
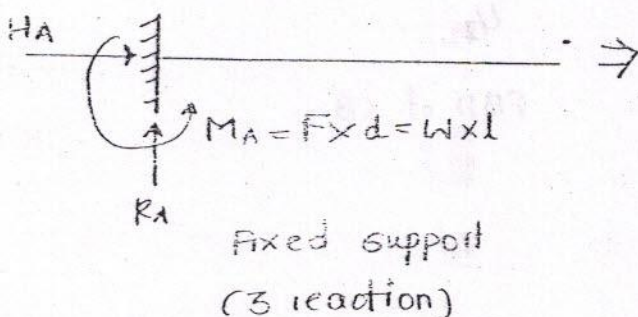
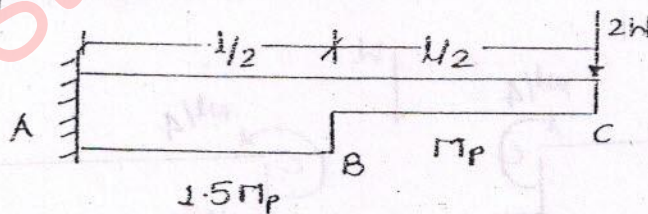
$$W \times (a\theta) = M_p \cdot \theta + M_p (\theta + \alpha)$$

at A at C.

$$= M_p \cdot \theta + M_p \left(\frac{1\theta}{b} \right)$$

$$W = W_u = \frac{M_p(L+b)}{ab}$$

Q. A cantilever of length L is subjected to point load $2W$ at free end. The beam has $1.5 M_p$ for the inner half and M_p for the outer half. Collapse load is....



M_A (shown as reaction) $\leftarrow M = F \times d$ (actual presentation)

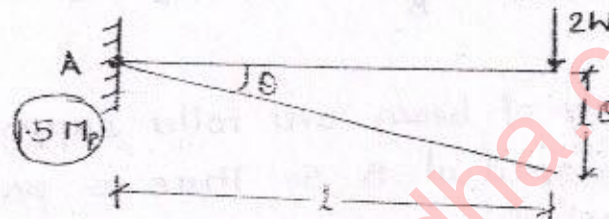
$$D_s = r - s$$

$$= 3 - 3 = 0$$

No of plastic hinges required for collapse = $D_s + 1$
 $= 1$

Plastic hinges are formed at location of max. B.M. and where the c/s changes. So we have to check possibility of plastic hinge development at A and B and least value of collapse load is true collapse load.

(i) Possibility : plastic hinge at A.



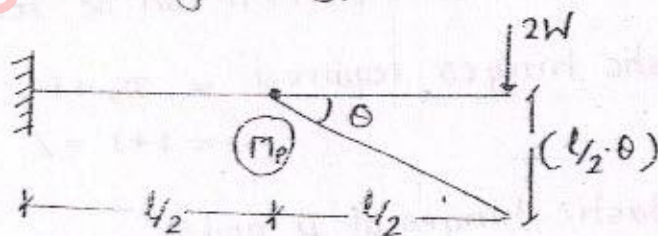
External work = internal work

$$2W \cdot l\theta = 1.5 M_p \cdot \theta$$

at A

$$W_u = W = \frac{0.75 M_p}{l}$$

(ii) Possibility : plastic hinge at B.



$$2W \times \left(\frac{l}{2} \cdot \theta\right) = M_p \cdot \theta$$

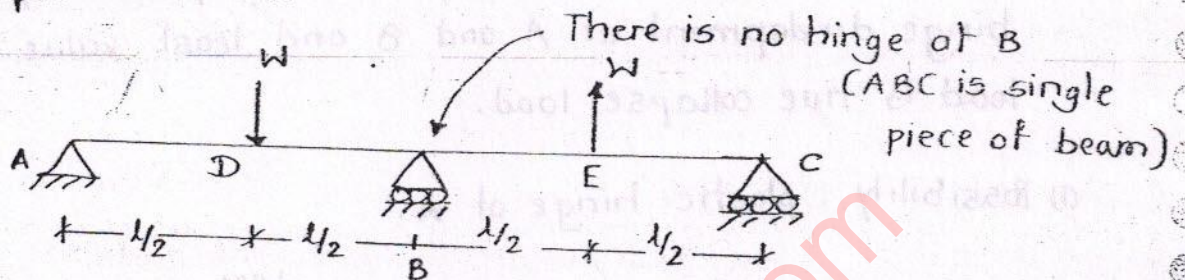
$$W = W_u = \frac{M_p}{l}$$

So the correct failure mechanism is 1st and true collapse load is $\frac{0.75 M_p}{l}$

Note:

At a joint, if two members of different capacities are meeting, always less moment capacity member is considered for calculating work done. Because plastic hinge is developed in weak member only.

Q. Find collapse load W_u :



Due to continuity of beam over roller support at B, the moment is developed at B. So, there is possibility of plastic hinge development at B.

Since the beam is not continuous at roller support C, plastic hinge will not develop at C or at A.

$$\begin{aligned} D_s &= r - S \\ &= 3 - 2 \\ &= 1 \end{aligned}$$

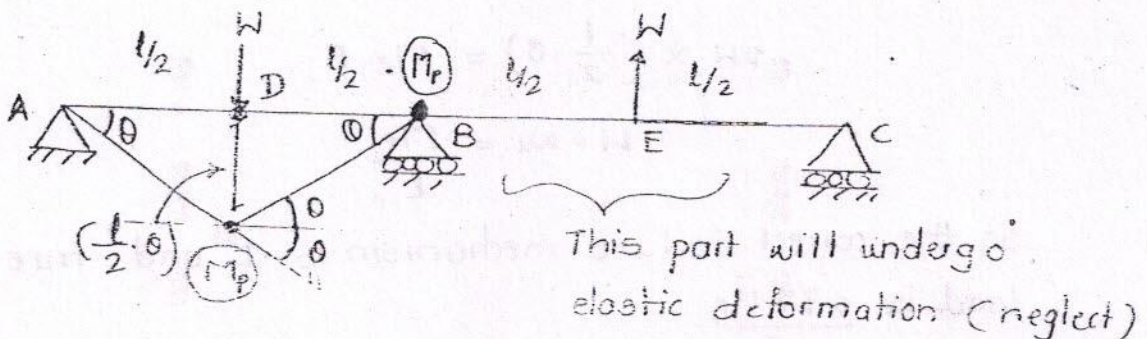
$$r = 3 \quad (R_A, R_B, R_C)$$

$$S = 2 \quad (\sum F_y = 0, \sum M = 0)$$

Here M_B is not unknown, it can be found from R_C .

$$\begin{aligned} \therefore \text{No. of plastic hinges required} &= D_s + 1 \\ &= 1 + 1 = 2. \end{aligned}$$

Possibility 1: plastic hinges at D and B



$$W \times \left(\frac{l}{2} \cdot \theta\right) = M_p (\theta + \theta) + M_p \cdot \theta$$

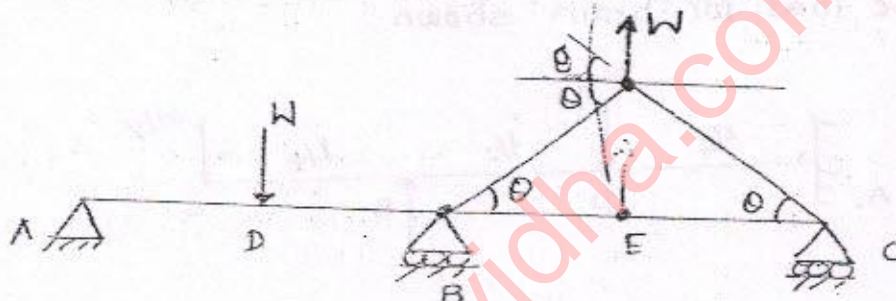
at D at B

$$W_u = \frac{6M_p}{l}$$

The failure mechanism is similar to propped cantilever subjected to central load so we can also directly write.

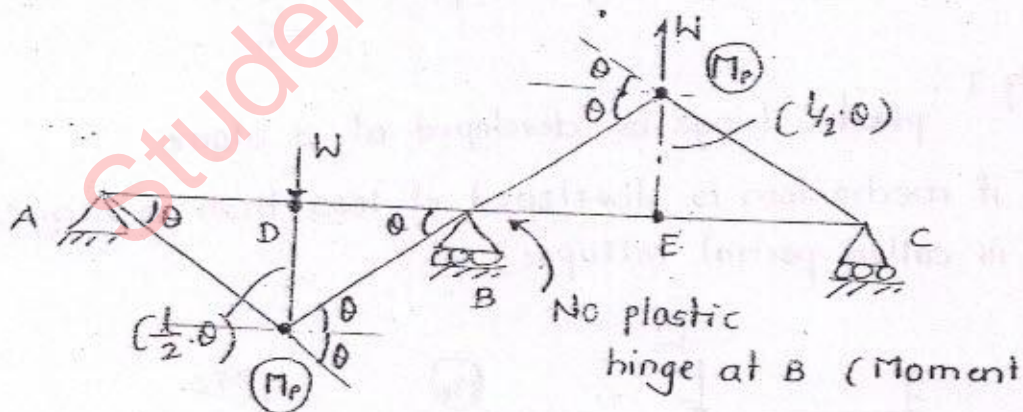
$$W_u = \frac{6M_p}{l}$$

possibility II: plastic hinges at B and E



It is similar to 1st possibility.

possibility III: plastic hinge at D and E



$$W \left(\frac{l}{2} \cdot \theta\right) + W \left(\frac{l}{2} \cdot \theta\right) = M_p (\theta + \theta) + M_p (\theta + \theta)$$

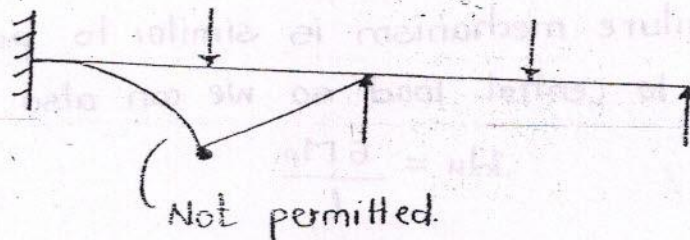
At D at E at D at E

$$W_u = \frac{4M_p}{l}$$

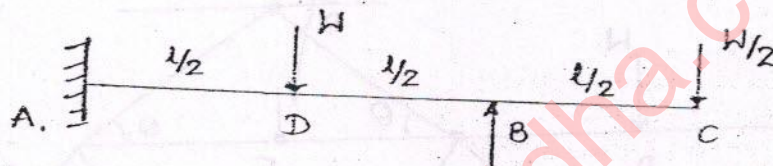
So the correct failure mechanism is IIIrd mechanism and true collapse load is $\frac{4M_p}{l}$

Note:

Between the plastic hinges, the beam behaves like a rigid body i.e. it remains as a straight line between plastic hinges. If the beam cannot maintain straight line, then that mechanism is not a possible mechanism.



Q. Find collapse load for beam shown:



$$D_s = r - s$$

$$= 4 - 3 = 1$$

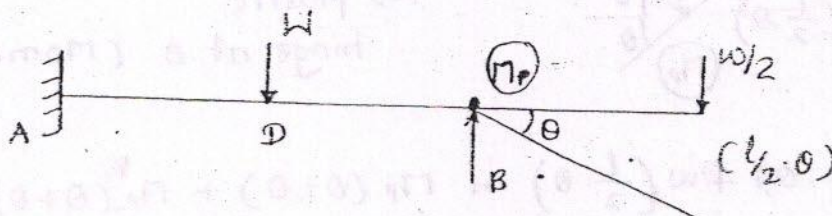
$$n - \text{no. of plastic hinges required} = 1 + 1$$

$$= 2$$

possibility I:

plastic hinge is developed at B alone

(if mechanism is developed at less than n hinges, it is called partial collapse) -

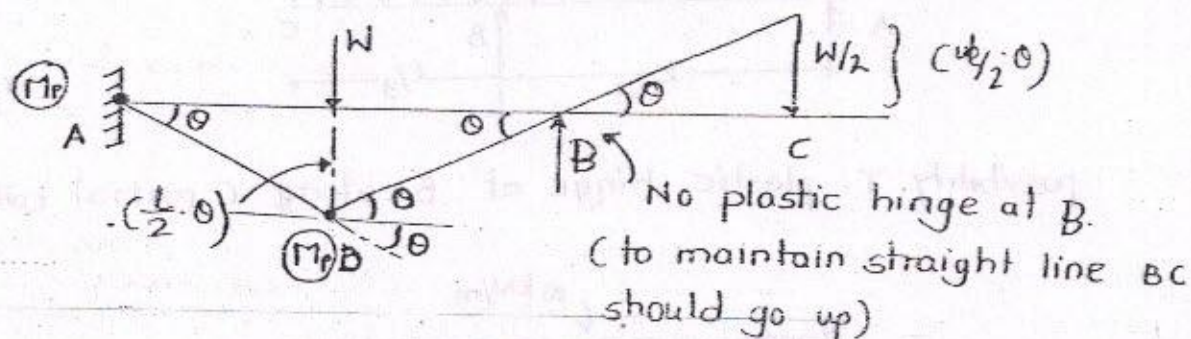


$$\frac{W}{2} \times \left(\frac{l}{2} \cdot \theta \right) = M_p \cdot \theta$$

$$W_u = \frac{4M_p}{l}$$

possibility II:

plastic hinge at A and D.



$$W \left(\frac{l}{2} \cdot \theta \right) - \frac{W}{2} \left(\frac{l}{2} \cdot \theta \right) = M_p \theta + M_p (\theta + \theta)$$

at D at C at A at B.

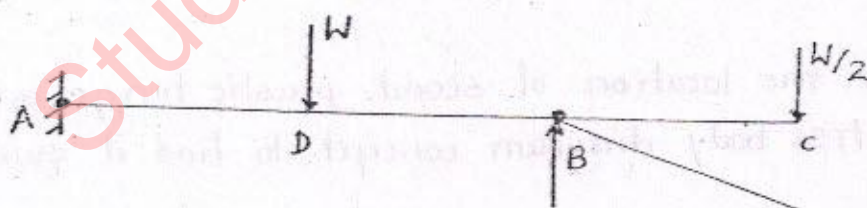
(-ve because force and displacement are in opposite direction)

$$W = W_u = \frac{12 M_p}{l}$$

So correct failure mechanism is 1st Mechanism and true collapse load is $\frac{4 M_p}{l}$.

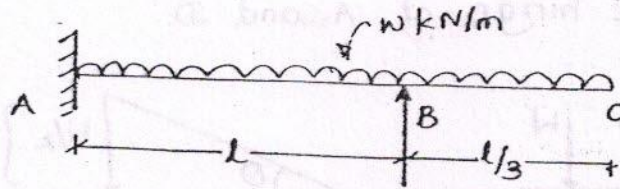
5th October
Saturday.

Note:

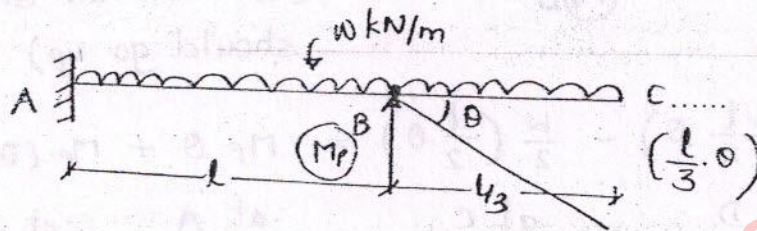


- (i) If hinges are developed at A and B, M_p will not do any work at A. Therefore plastic hinge at A will be useless. (AB should remain straight)
- (ii) Whenever there is overhang, we must check the probability of partial collapse.

Q Find Collapse load W_u :



possibility I: plastic hinge at B along C (partial collapse)



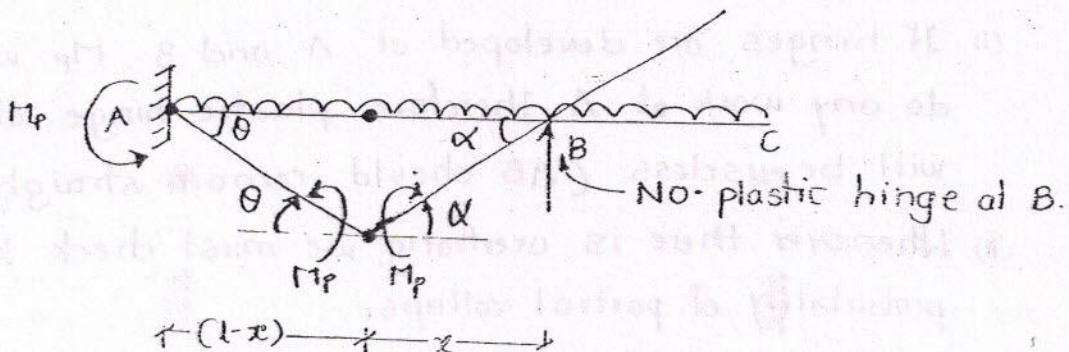
$$w \left(\frac{1}{2} \times \frac{l}{3} \times \frac{l}{3} \cdot \theta \right) = M_p \cdot \theta$$

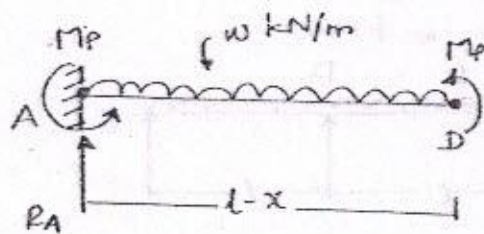
$$w = w_u = \frac{18 \cdot M_p}{l^2} \text{ kN/m} \quad \text{--- (i)}$$

possibility II plastic hinge at A and somewhere between A and B.

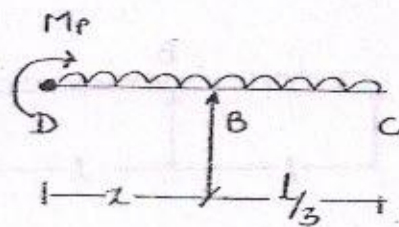
Note:

- (i) Second plastic hinge is not developed at distance of $0.414 l$ from propped end B, because of the overhang BC.
- (ii) Since the location of second plastic hinge is not known, use free body diagram concept to find it quickly.





F.B.D. of AD



F.B.D. of DC

No reaction at D, because B.M. is max. at D and at the location of max. B.M. shear force is zero. (changes its sign)

from F.B.D. of AD.

$$\sum M_{@A} = 0$$

$$-M_p - M_p + w \cdot (l-x) \cdot \frac{(l-x)}{2} = 0$$

$$M_p = \frac{w(l-x)^2}{2}$$

→ +ve ← -ve
—— (i)

from F.B.D of DBC.

$$\sum M_{@B} = 0$$

$$+M_p - w \cdot x \cdot \frac{x}{2} + w \cdot \frac{l}{3} \cdot \frac{l}{6} = 0$$

$$M_p = \frac{wx^2}{2} - \frac{wl^2}{18} \quad \text{—— (ii)}$$

from — (i) & — (ii)

$$\frac{w(l-x)^2}{2} = \frac{wx^2}{2} - \frac{wl^2}{18}$$

$$l^2 + x^2 - 2lx = x^2 - \frac{l^2}{9}$$

$$x^2 + 2lx - \frac{4l^2}{18} - l^2 = 0$$

$$x = 0.49l$$

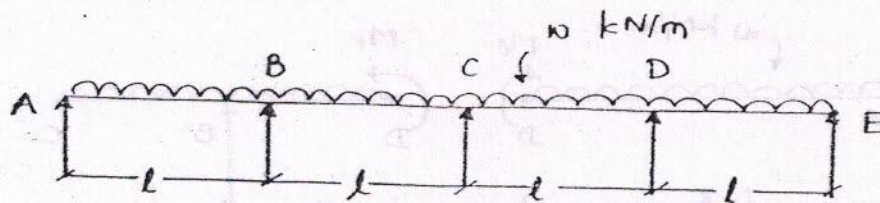
from proped end.

from — (i)

$$M_p = \frac{w(l - 0.49l)^2}{2}$$

True collapse is, $w = w_u = \frac{15.42 M_p}{l^2}$

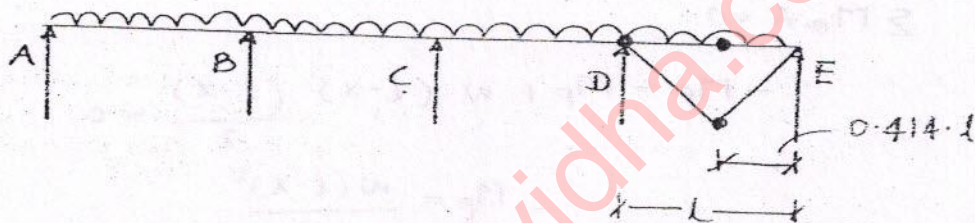
Q. Find the collapse load :



Note:

For continuous beams subjected to downward loading, failure mechanism of individual span will decide the true collapse.....($n = D_s + 1$ need not be followed)

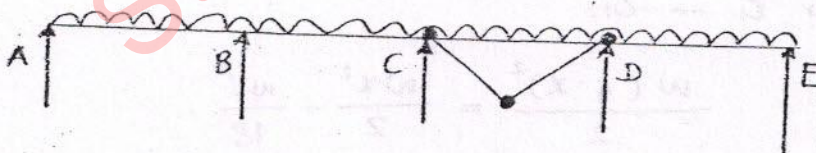
Possibility I: Mechanism in span DE



The failure mechanism is similar to propped cantilever subjected to UDL.

$$w_u = \frac{11.65 M_p}{l^2}$$

Possibility II: Mechanism in span CD.



Collapse mechanism is similar to fixed beam subjected to UDL

$$w_u = \frac{16 M_p}{l^2}$$

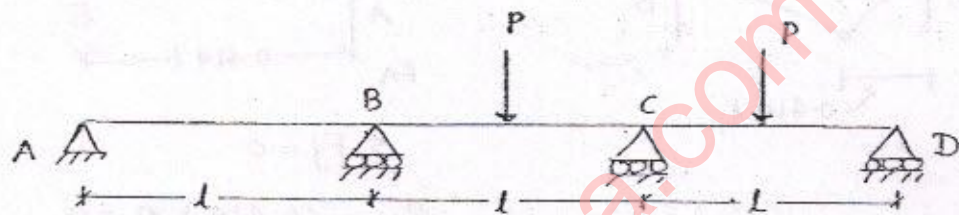
So true collapse load is,

$$w_u = \frac{11.65 M_p}{l^2}$$

Note:

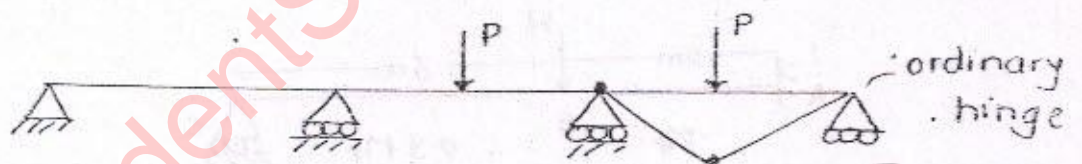
At the intermediate supports of a continuous beam negative moments are developed due to continuity of the beam over those supports. So plastic hinges can be developed at those supports. At the end supports, the plastic hinges will not develop because there is no continuity of beam over those ends. supports. (A & E)

Q. Find collapse load



ABCD is single piece of beam.

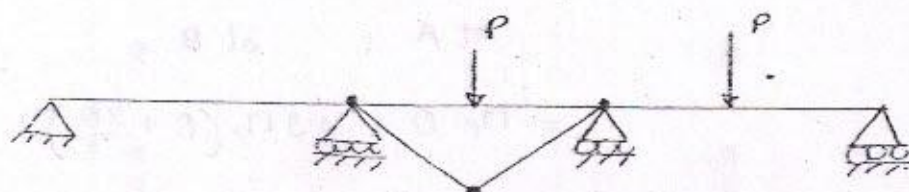
Possibility I: Mechanism in CD



Mechanism is similar to propped cantilever subjected to central point load.

$$P_u = \frac{6 M_p}{L}$$

Possibility II: Mechanism in BC

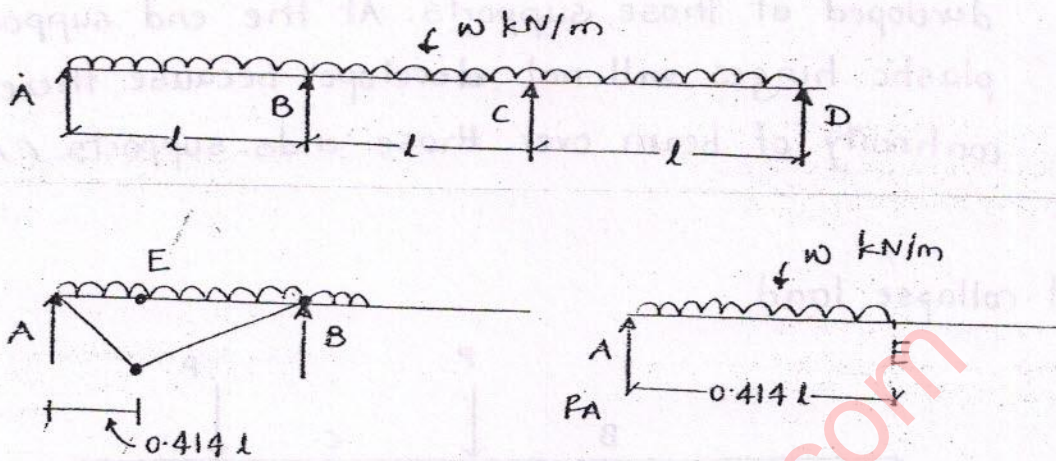


Mechanism is similar to fixed beam subjected to central point load.

$$P = P_u = \frac{8 M_p}{L}$$

So, true collapse load is. $P_u = \frac{6M_p}{L}$

Q. A continuous beam is subjected to collapse load system as shown in fig. reaction at A is —



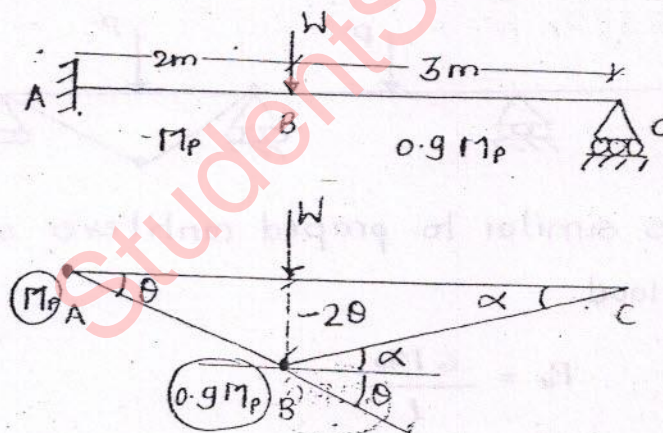
$$\sum F_y = 0$$

$$P_A - (0.414) \cdot w = 0$$

↑ +ve ↓ -ve

$$P_A = 0.414w$$

Q. The correct virtual work equation for collapse load for the beam shown in fig. is —



$$\alpha = \frac{2\theta}{3}$$

$$W \times 2\theta = M_p \cdot \theta + 0.9M_p (\theta + \alpha)$$

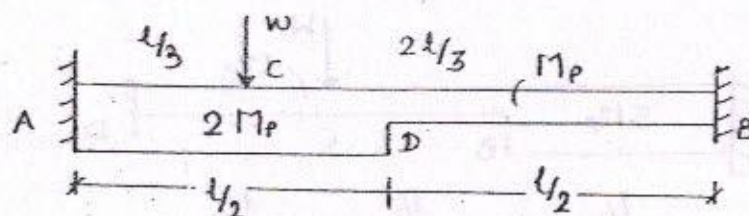
at A at B

$$= M_p \cdot \theta + 0.9M_p \left(\theta + \frac{2\theta}{3} \right)$$

$$W \cdot 2\theta = M_p \cdot \theta + 0.9M_p \left(\theta + \frac{2\theta}{3} \right)$$

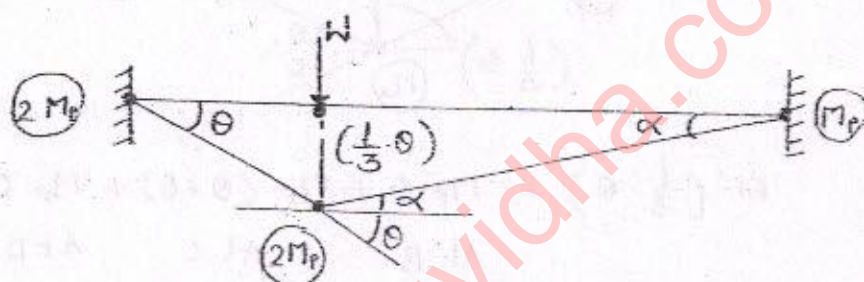
is the virtual work equation.

Q. Find collapse load W_u :



First two plastic hinges are developed at A and B and third plastic hinge can develop at point load or at point where W changes i.e. two possibilities of plastic hinge development are at C and D.

Possibility I: plastic hinges at A, B and C



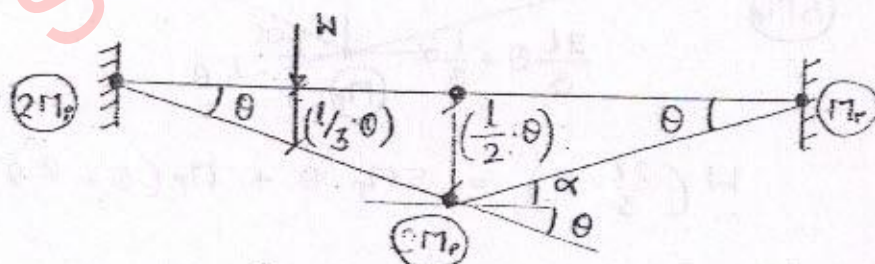
$$\therefore \frac{1}{3} \theta = \frac{2l}{3} \alpha$$

$$\alpha = \frac{\theta}{2}$$

$$W \left(\frac{1}{3} \theta \right) = 2M_p \cdot \theta + 2M_p (\theta + \alpha) + M_p \cdot \alpha$$

$$W_u = \frac{16.5 M_p}{l}$$

Possibility II: plastic hinges at A, B and D



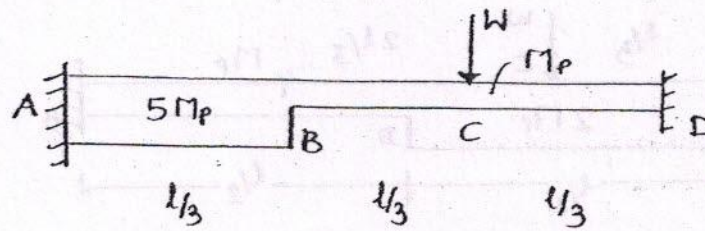
$$W \cdot \left(\frac{1}{2} \theta \right) = 2M_p \cdot \theta + M_p (\theta + \theta) + M_p \cdot \theta$$

$$W = W_u = \frac{15 M_p}{l}$$

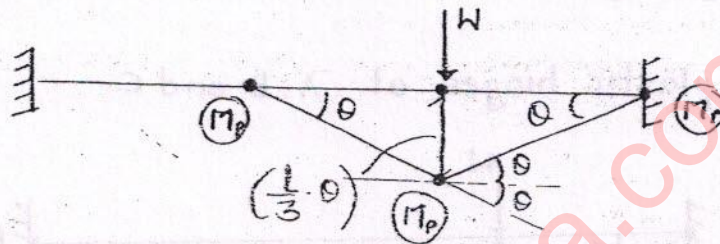
$$\theta = \alpha$$

So, true collapse load is $\frac{15 M_p}{l}$

Q. Find collapse load W_u .



Since the plastic moment capacity of member AB is $5M_p$, it behaves like a rigid member, so, three plastic hinges are developed at B, C, and D. (since at B section is weak)

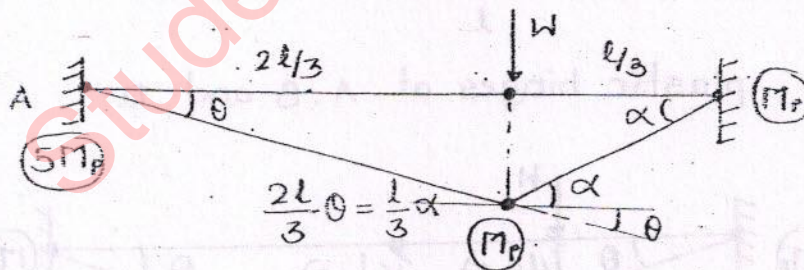


$$W \cdot \left(\frac{l}{3} \cdot \theta\right) = M_p \cdot \theta + M_p \cdot (\theta + \theta) + M_p \cdot \theta.$$

At B At C At D

$$W = W_u = \frac{12 M_p}{l}$$

Possibility II : Hinges at A, C and D.

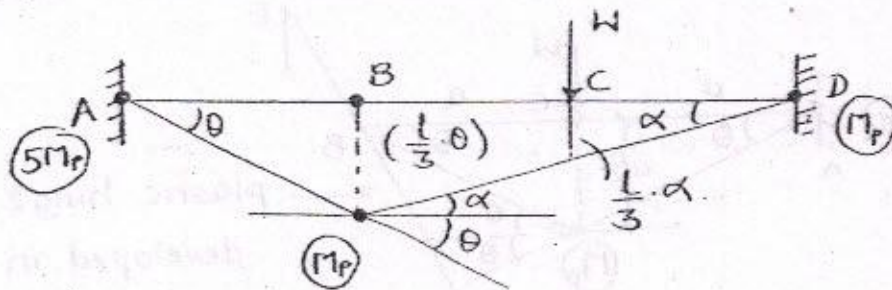


$$W \left(\frac{2l}{3} \cdot \theta\right) = 5M_p \cdot \theta + M_p (\theta + 2\theta) + M_p \cdot 2\theta$$

$$\therefore \alpha = 2\theta$$

$$W = W_u = \frac{15 M_p}{l}$$

Possibility III : plastic hinges at A, B and D



$$\frac{l}{3} \theta = \frac{2l}{3} \alpha \quad \therefore \alpha = \frac{\theta}{2}$$

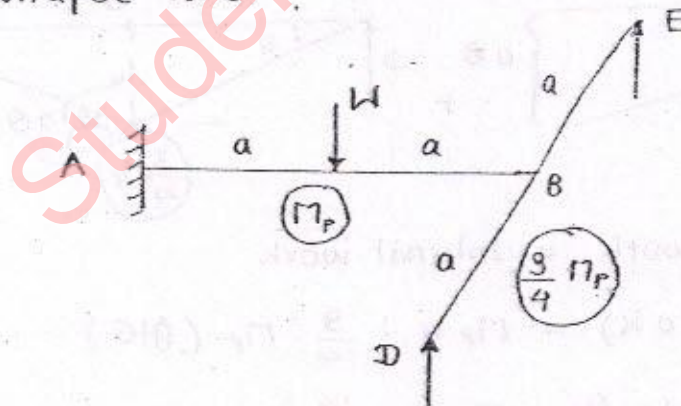
$$W \left(\frac{l}{3} \cdot \alpha \right) = 5M_p \cdot \theta + M_p (\theta + \alpha) + M_p \alpha$$

$$W \cdot \frac{l \cdot \theta}{6} = 5M_p \cdot \theta + M_p \cdot \frac{3\theta}{2} + \frac{M_p \cdot \theta}{2}$$

$$W = W_u = \frac{42 M_p}{l}$$

Thus, true collapse load is $W_u = \frac{12 M_p}{l}$

Q. A load W is supported by cantilever resting on simply supported beam as shown in fig. If $(M_p)_{SSB} = \frac{3}{4} (M_p)_{cantilever}$ find collapse load.

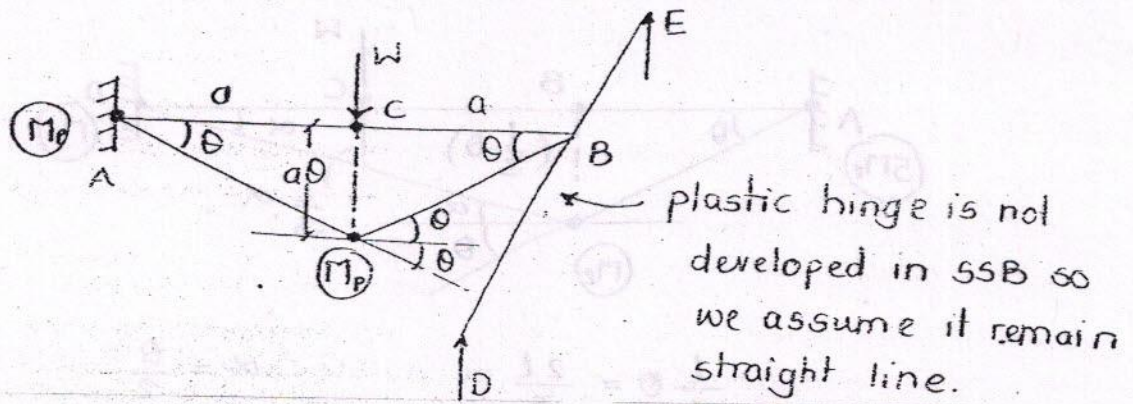


(looking upside down, cantilever forms, propped cantilever due to the reaction at B from SSB)

Note:

In the cantilever beam, at the free end B, plastic hinge will not form because BM is zero at the free end in cantilever beam. But in simply supported beam plastic hinge can form at B because BM is max in simply supported beam at centre.

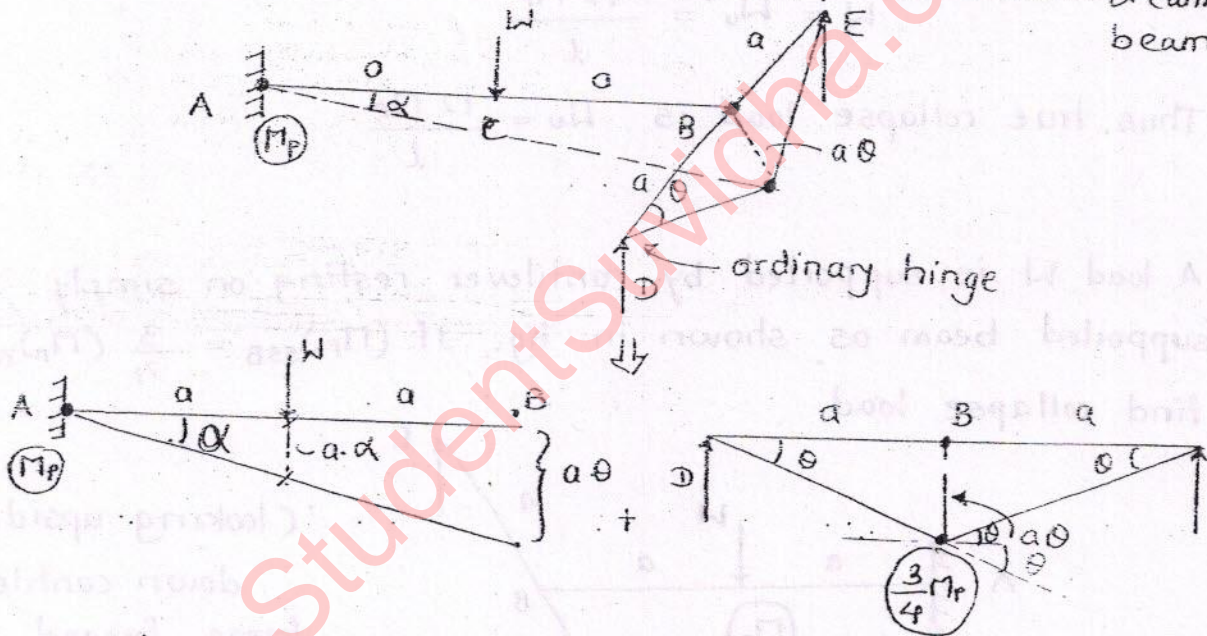
Possibility I: Mechanism in cantilever beam:



$$W \cdot (a\theta) = M_p \cdot \theta + M_p \cdot (\theta + \theta)$$

$$W_u = \frac{3M_p}{a}$$

Possibility II: Mechanism in simply supported beam: & cantilever beam.



External work = Internal work

$$W \cdot (a\alpha) = M_p \cdot \alpha + \frac{3}{4} \cdot M_p \cdot (\theta + \theta)$$

$$\therefore 2a \cdot \alpha = a\theta$$

$$\alpha = \frac{\theta}{2}$$

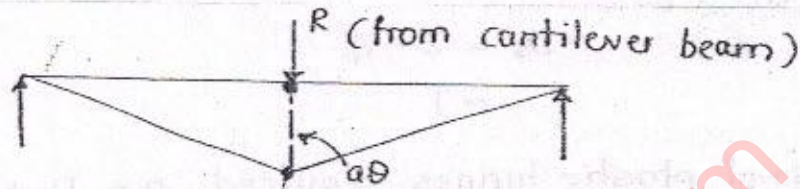
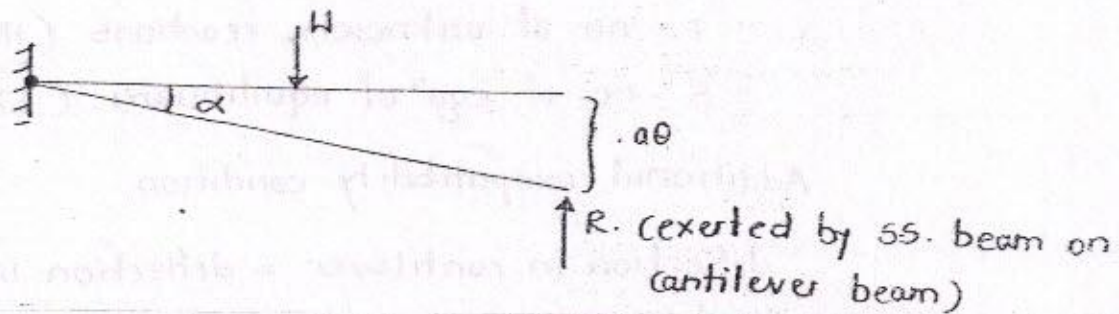
$$W \cdot a \cdot \frac{\theta}{2} = M_p \cdot \frac{\theta}{2} + \frac{3}{4} \times 2 M_p \theta$$

$$W = W_u = \frac{4M_p}{a}$$

$$\text{So true collapse load} = \frac{3M_p}{a}$$

Note:

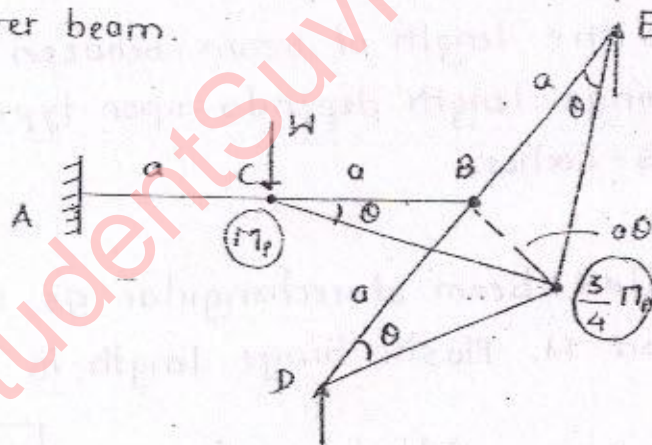
(i) Work done by internal reaction forces is always zero.



Total work done by reaction = $-[R \times (a \cdot \theta)] + [R \times (a \cdot \theta)]$
 $= 0$

on cantilever on simply support

(ii) If plastic hinge is developed under point load in the cantilever beam.



External work = Internal work

$$H \times 0 = M_p \cdot \theta + \frac{3}{4} M_p \cdot (\theta + \theta)$$

$$0 \cdot M_p = 0$$

$\therefore M_p$ cannot be zero $\therefore \theta = 0$

It means, it is an impossible mechanism.

So, if external loads, don't do any work, mechanism will not happen.

(iii) Degree of static indeterminacy, (D_s)

$$D_s = r - S$$

r - no. of unknown reactions (H_A, R_A, M_A, R_D, R_E)

S - no. of eq^s of equilibrium ($\sum X=0, \sum Y=0, \sum M=0$)

Additional compatibility condition.

deflection in cantilever = deflection in SS beam.

$$\therefore D_s = 5 - 4 = 1$$

$$\therefore \text{No. of plastic hinges required, } n = D_s + 1 = 1 + 1 = 2$$

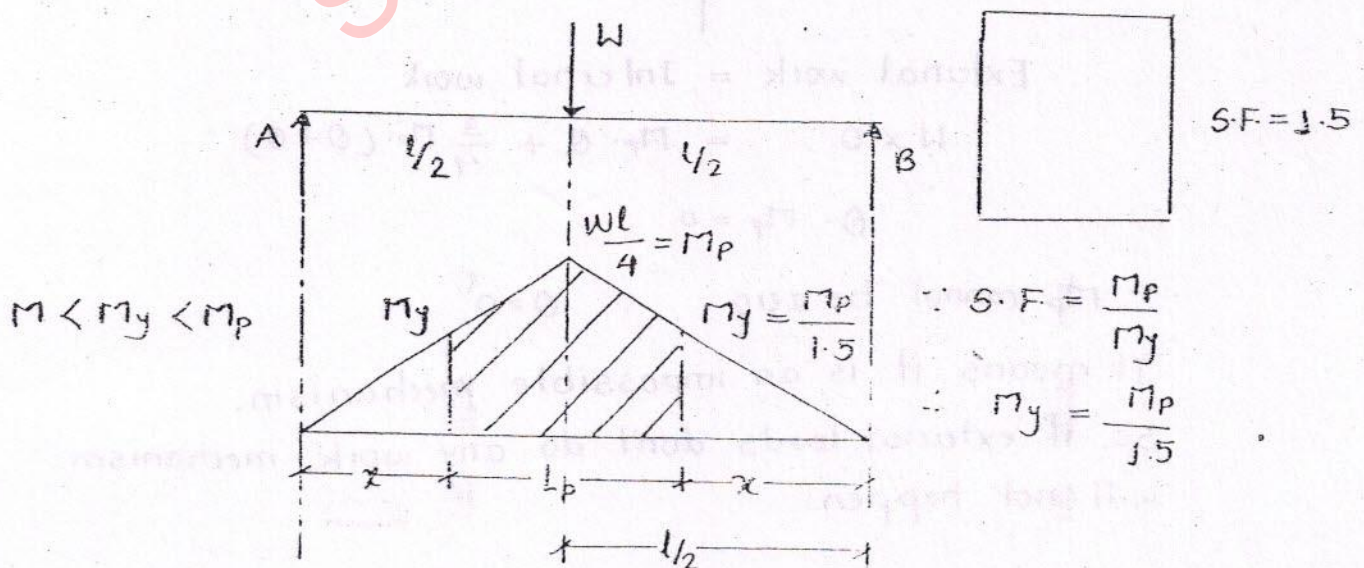
Compatibility means relationship between deformations.

Plastic hinge length (L_p)

It is the length of beam between M_y and M_p .

The plastic hinge length depends upon type of loading & shape of cross-section.

Q. A simply supported beam of rectangular cs is subjected to central point load W . Plastic hinge length is —



from similar triangles,

$$\frac{x}{M_y} = \frac{l/2}{M_p}$$

$$\frac{x}{M_{p/1.5}} = \frac{l/2}{M_p}$$

$$x = \frac{l}{3}$$

$$L_p = l - 2x$$

$$= l - 2\left(\frac{l}{3}\right) = 0.333 l$$

$$L_p = \frac{l}{3}$$

Q. If shape of c/s in above problem is J-section, plastic hinge length is -

$$M_y = \frac{M_p}{S.F.}$$

$$= \frac{M_p}{1.12}$$

$$\therefore (S.F.)_{J\text{-section}} = 1.12$$

$$\frac{x}{(M_p/1.12)} = \frac{l/2}{M_p}$$

$$x = \frac{l}{2.24}$$

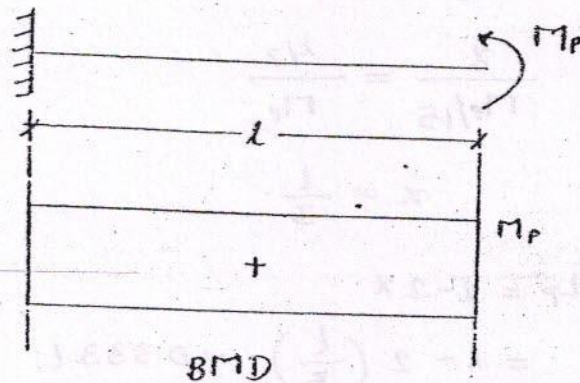
$$L_p = l - 2x$$

$$= l - \left(\frac{l}{2.24}\right) \times 2 = 0.107 l$$

$$L_p = \frac{l}{9}$$

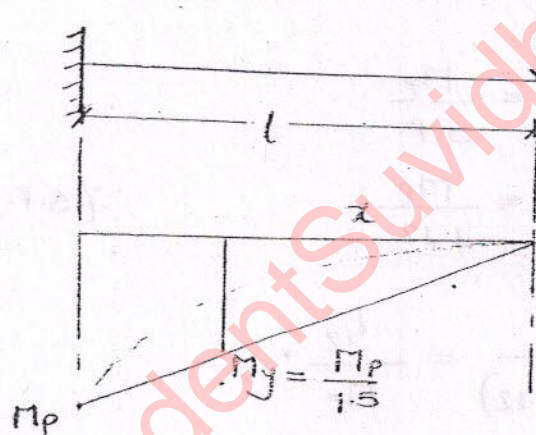
\therefore If shape factor is less, plastic hinge length is also less.

Q. A cantilever is subjected to a moment M_p at the free end as shown in fig. plastic hinge length is —



Plastic hinge length, $L_p = l$.

Q. Find plastic hinge length:



$$\square \quad 6 \cdot F = 1.5$$

$$\frac{x}{M_y} = \frac{l}{M_p}$$

$$x = \frac{l}{M_p} \times \frac{M_p}{1.5}$$

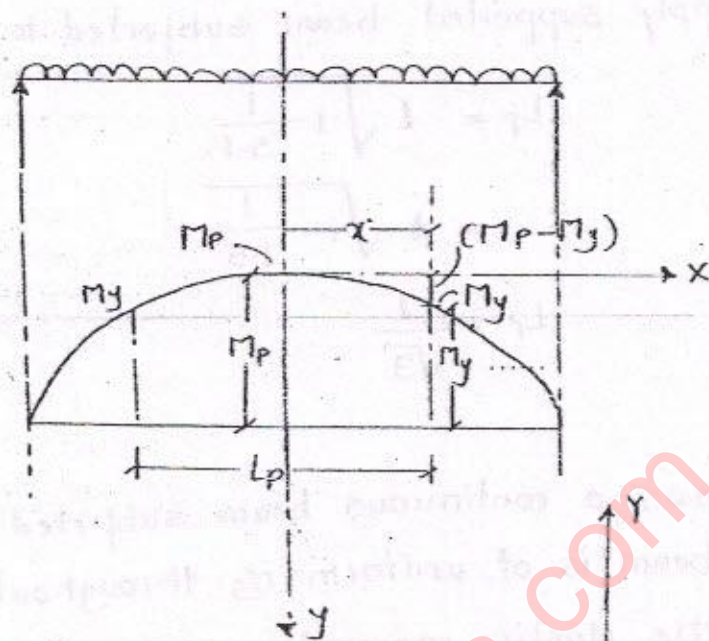
$$x = \frac{2l}{3}$$

$$L_p = l - x$$

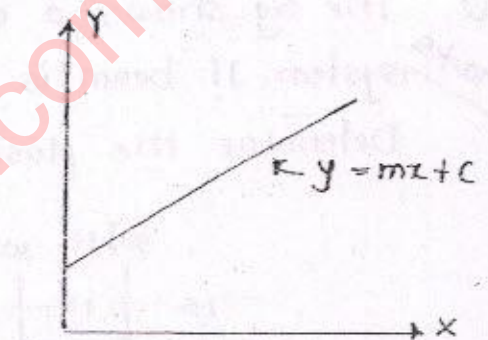
$$= l - \frac{2l}{3}$$

$$= \frac{l}{3}$$

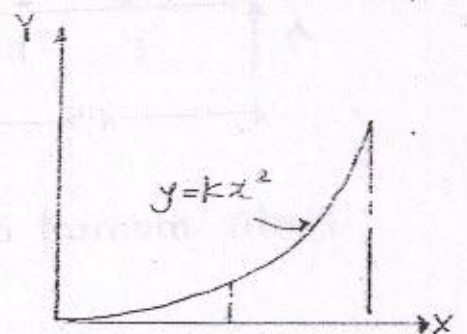
Q. A simply supported beam is subjected to UDL. Plastic hinge length is



$y = mx + c$ - eqn of straight line



$y = kx^2$ - eqn of parabola



$$\frac{y}{x^2} = k - \text{constant}$$

from similar parabola,

$$\frac{M_p - M_y}{x^2} = \frac{M_p}{\left(\frac{l}{2}\right)^2}$$

$$x^2 = \left(\frac{l}{2}\right)^2 \cdot \frac{M_p - M_y}{M_p}$$

$$\therefore S.F. = \frac{M_p}{M_y}$$

$$x = \frac{l}{2} \sqrt{1 - \frac{1}{S.F.}}$$

$$\therefore \text{Plastic hinge length } (L_p) = 2x = l \sqrt{1 - \frac{1}{S.F.}}$$

Q. A simply supported beam of rectangular c/s is subjected to UDL throughout its length. Length of plastic hinge is —

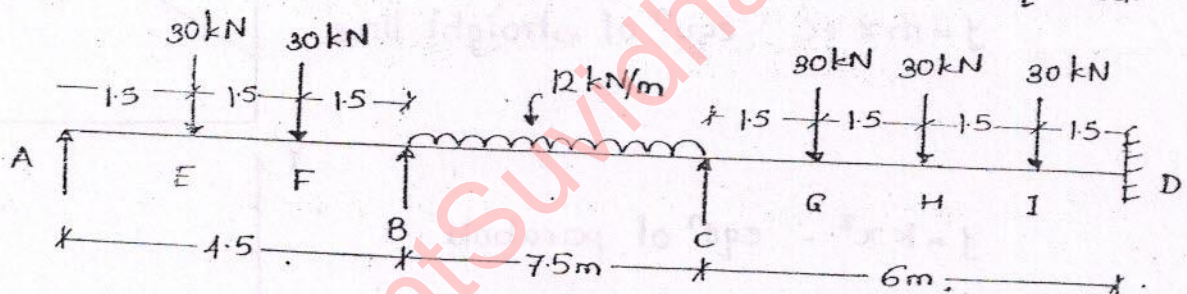
For simply supported beam subjected to U.D.L.

$$L_p = l \sqrt{1 - \frac{1}{S.F.}}$$

$$= l \sqrt{1 - \frac{1}{1.5}}$$

$$L_p = \frac{l}{\sqrt{3}}$$

20 Marks Q. The fig. shows a continuous beam subjected to collapse load system. If beam is of uniform c/s throughout its length. Determine the plastic moment capacity of beam required.



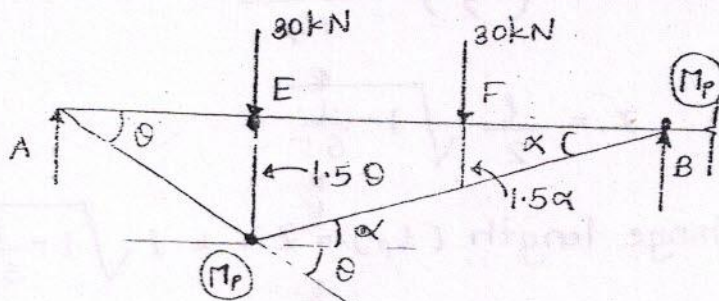
Plastic moment capacity for given c/s = M_p (constant)

$$M_p = P_y \cdot Z_p$$

$$= P_y \times \frac{A}{2} (\bar{y}_1 + \bar{y}_2)$$

(i) Mechanism in AB

Possibility I: plastic hinges at E and B



$$1.5 \theta = 3 \alpha$$

$$\alpha = \frac{\theta}{2}$$

$$\begin{array}{cccc}
 30(1.5\theta) & + & 30(1.5\alpha) & = & M_p(\theta + \alpha) & + & M_p(\alpha) \\
 \text{At E} & & \text{at F} & & \text{at E} & & \text{at B}
 \end{array}$$

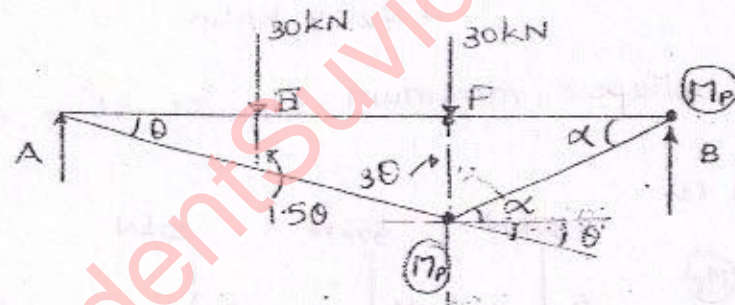
$$45\theta + 45 \frac{\theta}{2} = M_p \left(\frac{3}{2}\theta \right) + \frac{\theta}{2} \cdot M_p$$

$$M_p = \frac{2}{2} \times 45 \times \frac{1}{2}$$

$$= 33.75 \text{ kNm}$$

It means that to prevent this failure mechanism, the size of the beam should be such that, it should have a minimum plastic moment capacity of 33.75 kNm (size of beam should be more than this)

possibility II: plastic hinges at F and B.



$$3\theta = 1.5\alpha$$

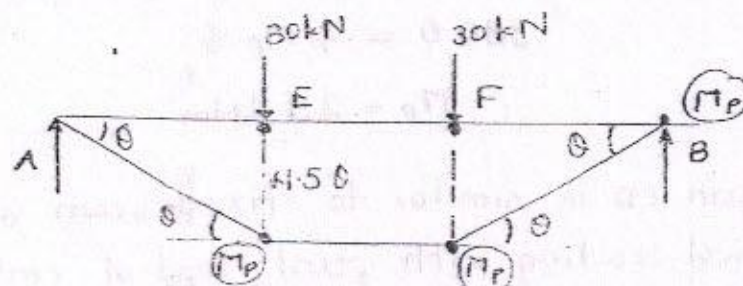
$$\alpha = 2\theta$$

$$(30 \times 1.5\theta) + (30 \times 3\theta) = M_p(\theta + \alpha) + M_p \cdot \alpha$$

$$135 \cdot \theta = M_p \left(\frac{3}{2}\theta \right) + 2M_p \cdot \theta \quad \therefore \alpha = 2\theta$$

$$M_p = 27 \text{ kNm}$$

possibility III: plastic hinges at E, F and B.

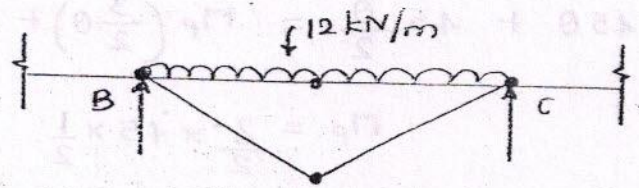


$$(30 \times 1.5\theta) + (30 \times 1.5\theta) = M_p \cdot \theta + M_p \cdot \theta + M_p \cdot \theta$$

$$90 \cdot \theta = 3 M_p \cdot \theta$$

To prevent collapse of beam AB the size of beam must be such that it should have minimum M_p of 33.75 kNm

(ii) Mechanism in BC:



Failure mechanism is similar to fixed beam subjected to UDL.

$$W_u = \frac{16 M_p}{l^2}$$

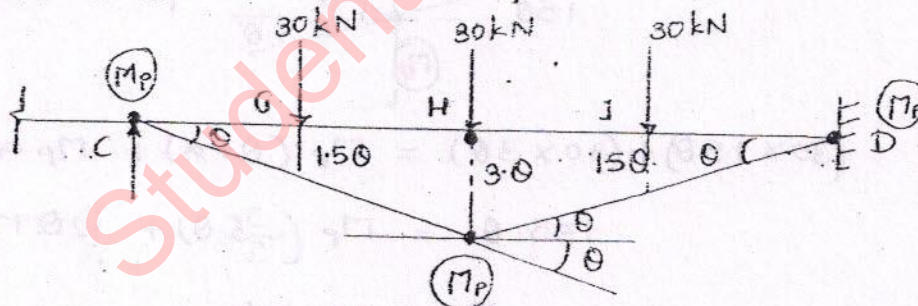
$$M_p = \frac{W_u \cdot l^2}{16}$$

$$= \frac{12 \times 7.5^2}{16}$$

$$= 42.19 \text{ kNm}$$

To prevent collapse, minimum M_p for BC = 42.19 kNm

(iii) Mechanism in CD



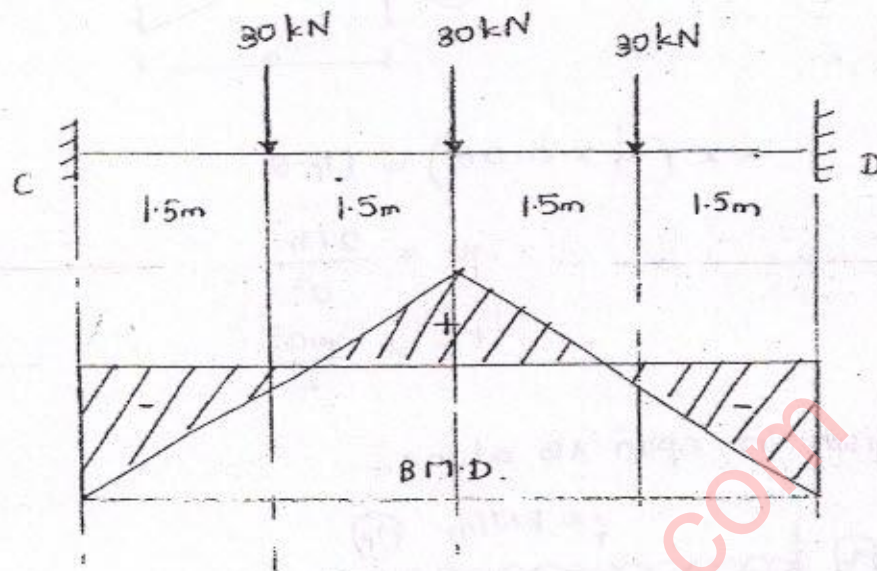
$$30(1.5\theta) + 30(3\theta) + 30(1.5\theta) = M_p \cdot \theta + M_p(\theta + \theta) + M_p \cdot \theta$$

$$90\theta = 4 M_p \cdot \theta$$

$$M_p = 22.5 \text{ kNm}$$

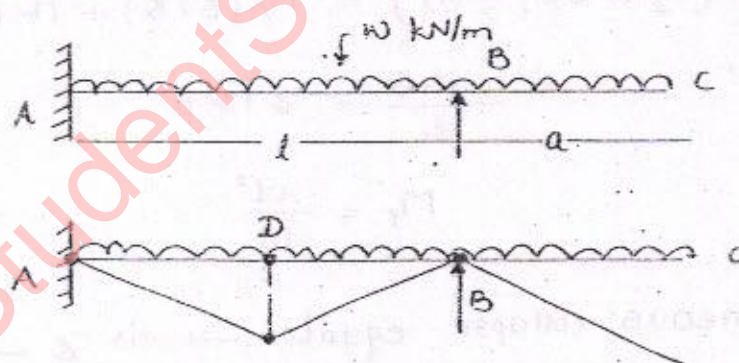
Since span CD is similar to fixed beam subjected to symmetrical loading with point load at centre, max. +ve B.M. will occur at H only. so a third plastic hinge will develop only at centre.

Since the beam is of uniform section throughout the length of beam ABCD, minimum M_p required for beam is 45 kNm



Monday
7th October 2015

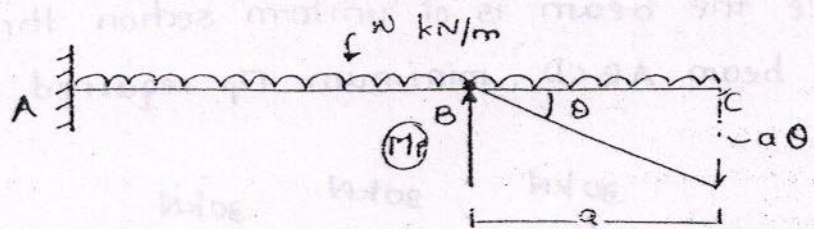
Q. A propped cantilever is loaded as shown in fig. What is the value of overhang for simultaneous collapse of spans AB and BC



For the simultaneous collapse of spans AB and BC, plastic hinges must develop at A and B. A third hinge will develop at the centre of span AB. (Because after the two hinges are formed at A and B, the beam behaves like a simply supported beam. Since the loading is symmetric on the span, third hinge must develop at the centre only.)

For a propped cantilever the no. of plastic hinges required at collapse is "two," but for simultaneous collapse of overhang and AB span we require three plastic hinges.

(i) Mechanism in BC alone:

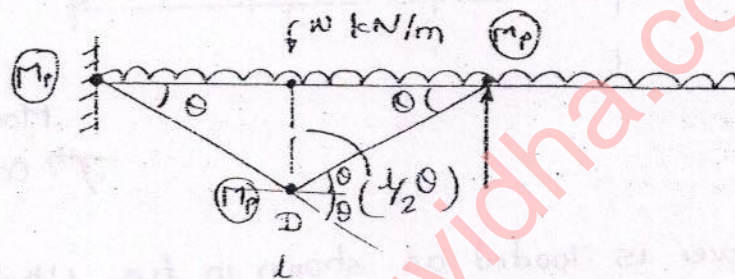


$$w \times \left(\frac{1}{2} \times a \cdot a\theta \right) = M_r \cdot \theta$$

$$w = \frac{2M_r}{a^2}$$

$$M_r = \frac{wa^2}{2} \quad \text{--- (i)}$$

(ii) Mechanism in span AB alone:



$$w \left(\frac{1}{2} \times l \times \left(\frac{1}{2} l\theta \right) \right) = M_r (\theta + \theta) + M_r \cdot \theta + M_r \cdot \theta$$

$$\frac{wl^2 \theta}{4} = 4 M_r \cdot \theta$$

$$M_r = \frac{wl^2}{16} \quad \text{--- (ii)}$$

For simultaneous collapse, equate --- (i) & --- (ii)

$$\frac{wa^2}{2} = \frac{wl^2}{16}$$

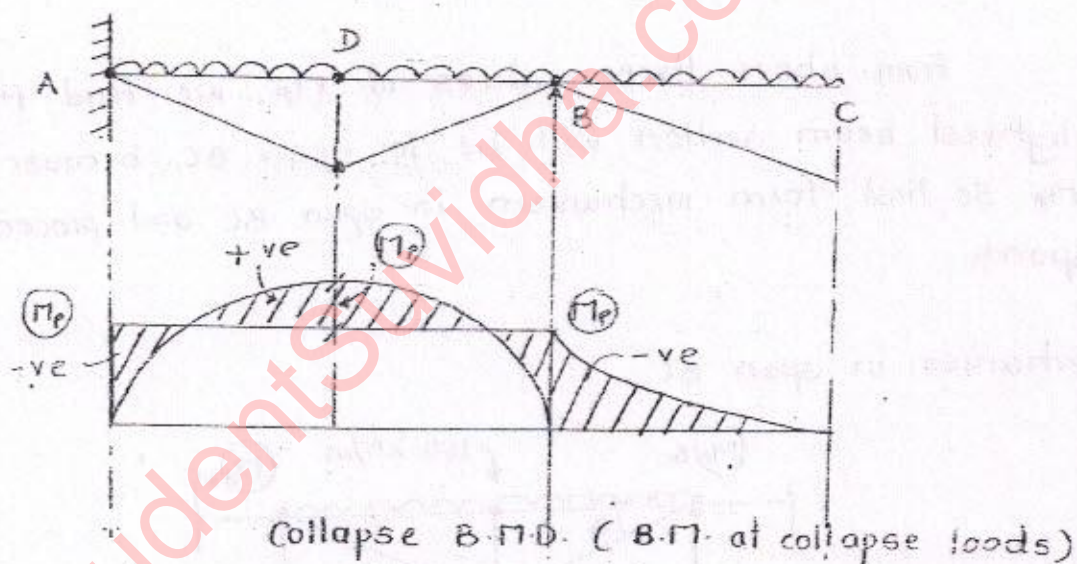
$$a^2 = \frac{l^2}{8}$$

$$a = \frac{l}{2\sqrt{2}} = 0.35 l$$

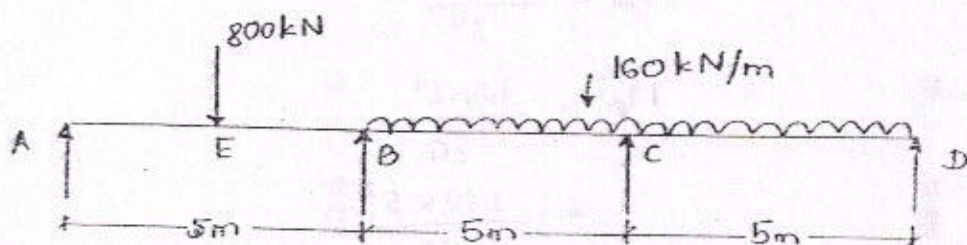
So, for simultaneous collapse of spans AB and BC, the value of overhang 'a' is 0.35l

Note:

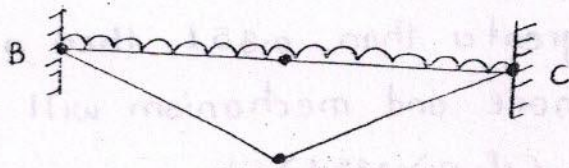
- (i) If overhang 'a' is greater than $0.35L$, then negative B.M. at B will be more and mechanism will occur in the span BC alone. (if $a > 0.35L$)
- (ii) If overhang 'a' is less than $0.35L$, mechanism will happen in the span AB alone as in the previous problem. (AB collapses first) (if $a < 0.35L$)
- (iii) If overhang 'a' is equal to $0.35L$, spans AB and BC will fail simultaneously. (if $a = 0.35L$)



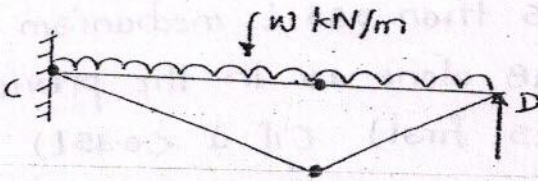
Q A continuous beam shown in fig is subjected to collapse load system. If each span is having uniform c/s, and the beam should collapse simultaneously find m_p required for each span.



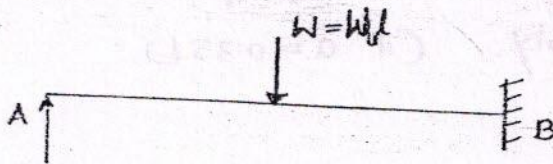
Each span has uniform section means the different spans can have different uniform cross sections. We have to find the size of each span so that all the spans fail simultaneously.



$$W_u = \frac{16 M_p}{l^2} \quad \therefore M_p = \frac{W_u \cdot l^2}{16}$$



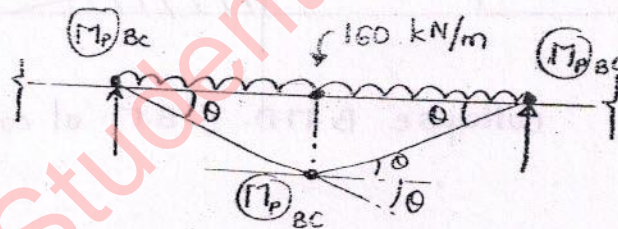
$$W_u = \frac{11.56 M_p}{l^2} \quad M_p = \frac{W_u \cdot l^2}{11.56}$$



$$W_u = \frac{6 M_p}{l} \quad M_p = \frac{W_u \cdot l}{6} = \frac{W_u \cdot l^2}{6}$$

From above three values of M_p , we find that the lightest beam section will be for span BC because of least M_p . So first form mechanism in span BC and proceed to other spans.

c) Mechanism in span BC:



Failure mechanism is similar to fixed beam subjected to UDL.

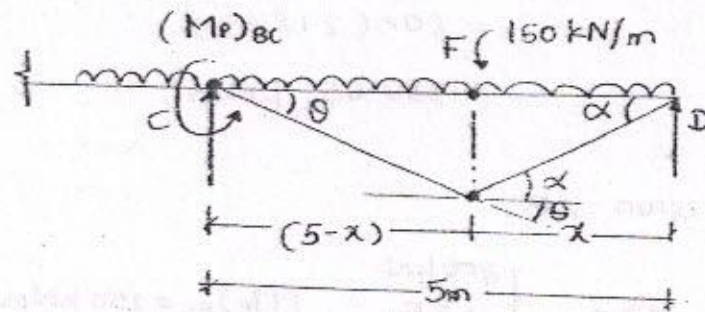
$$W_u = \frac{16 M_p}{l^2}$$

$$M_p = \frac{W_u \cdot l^2}{16}$$

$$= \frac{160 \times 5^2}{16}$$

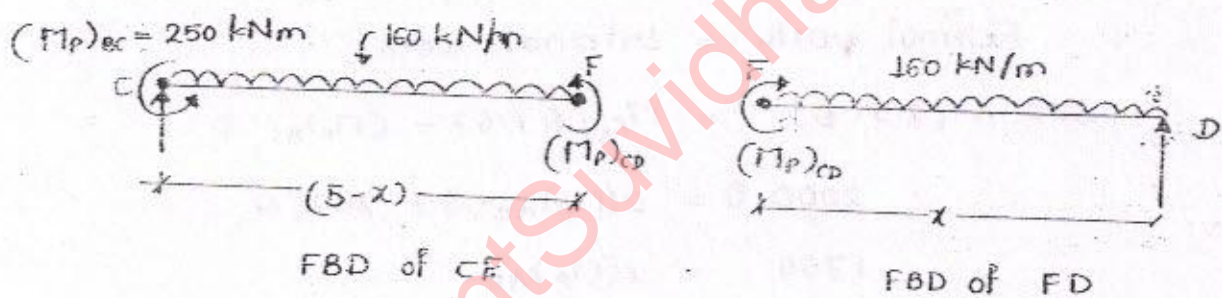
$$(M_p)_{BC} = 250 \text{ kNm}$$

Mechanism in span CD



The failure mechanism is similar to propped cantilever subjected to UDL, but second plastic hinge is not developed at a distance of $0.414 L$ from propped end, because M_p at C is fixed as 250 kNm.

Since the location of second plastic hinge is not known use free body diagram concept to find it quickly.



From FBD of CE

$$\sum M_a C = 0$$

$$-250 - (M_p)_{CD} + (160 \times (5-x) \cdot \frac{(5-x)}{2}) = 0$$

↑ +ve ↺

$$(M_p)_{CD} = 80(5-x)^2 - 250$$

— (i)

From FBD of FD

$$\sum M_D F = 0$$

$$+ (M_p)_{CD} - (160 \cdot x \cdot \frac{x}{2}) = 0$$

$$(M_p)_{CD} = 80x^2$$

— (ii)

from — (i)

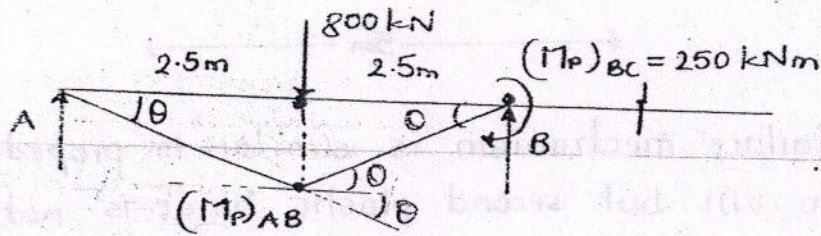
$$80x^2 = 80(25 + x^2 - 50x) - 250$$

$$0 = 2000 - 400x - 250$$

$$x = 2.1875 \text{ m from propped end.}$$

$$\begin{aligned}
 (M_p)_{CD} &= 80 \lambda^2 \\
 &= 80 \times (2.1875)^2 \\
 &= 382.63 \text{ kNm}
 \end{aligned}$$

(ii) Mechanism in span AB.



The failure mechanism is similar to propped cantilever subjected to central point load but $W_u \neq \frac{6M_p}{l}$ because M_p at B is fixed as 250 kNm

∴ External work = Internal work.

$$800 \times (2.5 \cdot \theta) = (M_p)_{AB}(\theta + \theta) + (M_p)_{BC} \cdot \theta$$

$$2000 \cdot \theta = 2(M_p)_{AB} \cdot \theta + 250 \cdot \theta$$

$$1750 = 2(M_p)_{AB}$$

$$(M_p)_{AB} = 875 \text{ kNm}$$

Conclusion :

So we provide sizes of beams such that,

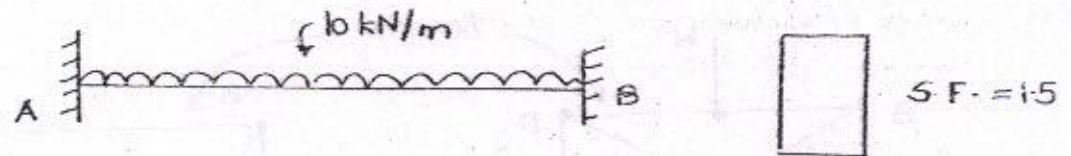
$$(M_p)_{AB} = 875 \text{ kNm}$$

$$(M_p)_{BC} = 250 \text{ kNm}$$

$$(M_p)_{CD} = 382.63 \text{ kNm}$$

then, all spans fail simultaneously.

- Q. A steel beam of rectangular section is clamped at both ends. Deformation is just observed when UDL on beam is 10 kN/m . At the instant of collapse, the load on the beam will be _



M_y (yield moment) happened at $w = 10 \text{ kN/m}$.

Plastic hinge will form at A and B at plastic moment M_p

$$M_p = S.F. \times M_y \\ = 1.5 \times M_y$$

$$w = \frac{12 M_p}{l^2}$$

At $w = 15 \text{ kN/m}$, two hinges will be formed at A & B. At collapse, third hinge will be formed at centre.

\therefore At $w_u = \frac{16 M_p}{l^2}$, third hinge is formed.

For $\frac{12 M_p}{l^2} \rightarrow 15 \text{ kN/m}$

For $\frac{16 M_p}{l^2} \rightarrow ?$

$$w_u = \frac{\frac{16 M_p}{l^2} \times 15}{\frac{12 M_p}{l^2}} \\ = 20 \text{ kN/m}$$

At $w_u = 20 \text{ kN/m}$ collapse mechanism will be formed.