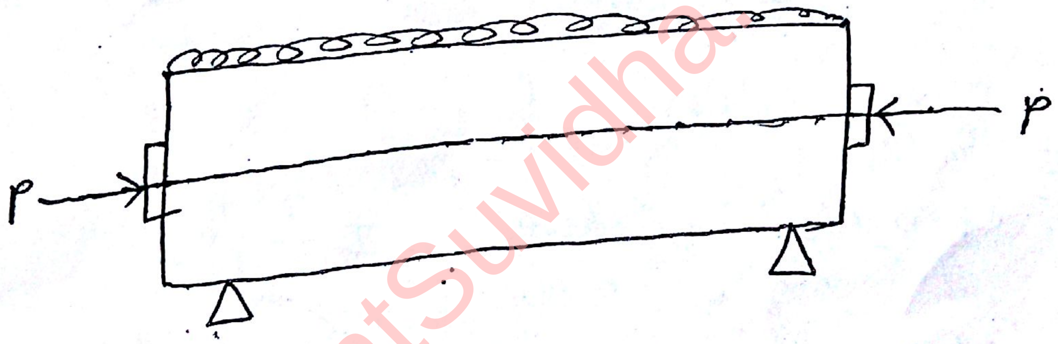
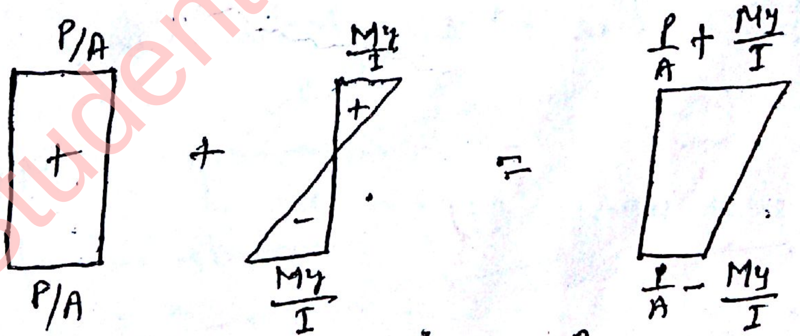
Basic Concept :

(i) stress concept
(ii) strength concept
(iii) load balancing concept.

(1) Stress Concept :-

Case (1) Beam provided with a concentric tendon





Compression
Tension

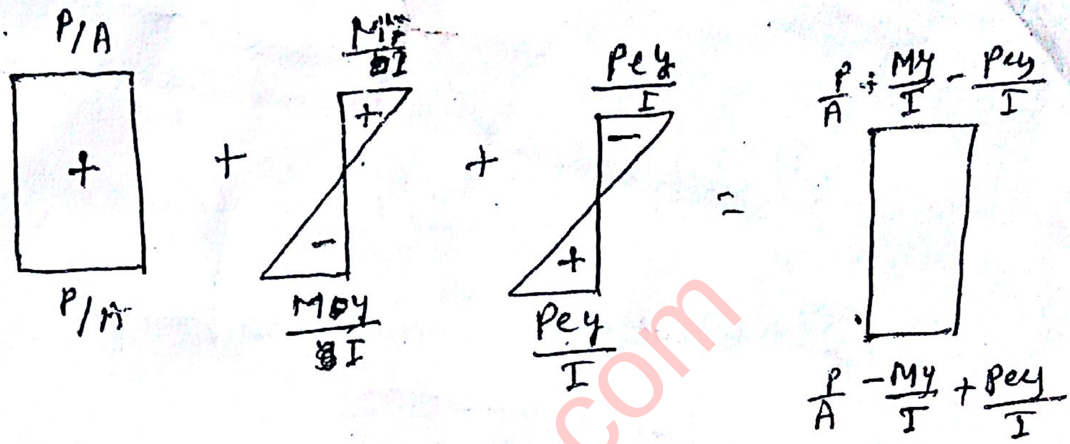
∴ Direct compressive stress $= \frac{P}{A}$

Bending stress $= \pm \frac{My}{I}$

Final stress at top fibres $= \frac{P}{A} + \frac{My}{I}$

Final stress at bottom fibres $= \frac{P}{A} - \frac{My}{I}$

Case II :- Beam with eccentric tendon :-



$$\text{Direct stress} = \frac{P}{A}$$

$$\text{stress due to bending} = \pm \frac{My}{I}$$

$$\text{Stress due to eccentricity} = \mp \frac{Pey}{I}$$

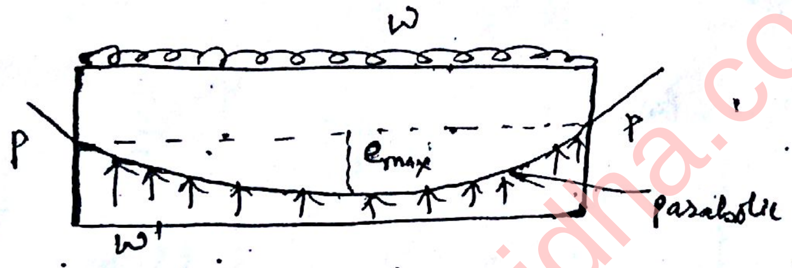
$$\text{final stress} = \frac{P}{A} \pm \frac{My}{I} \mp \frac{Pey}{I}$$

$$\text{Final stress at top fibre} = \frac{P}{A} + \frac{My}{I} - \frac{Pey}{I}$$

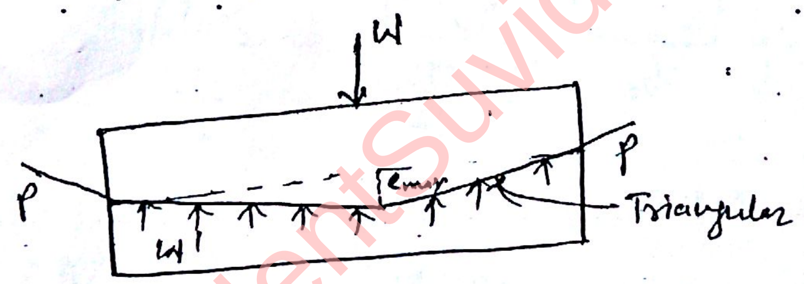
$$\text{Final stress at bottom fibre} = \frac{P}{A} - \frac{My}{I} + \frac{Pey}{I}$$

Load Balancing Concept :-

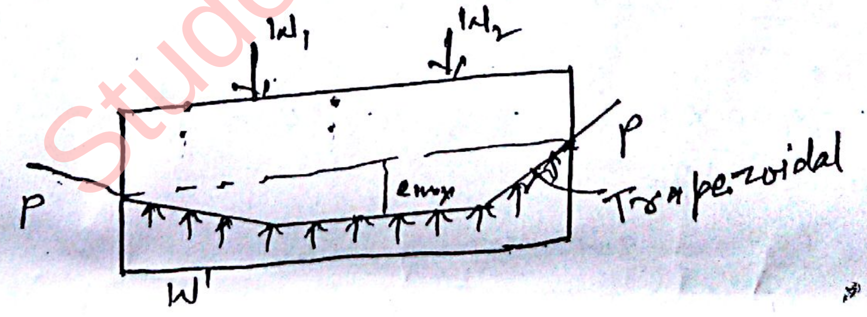
In this concept, prestressing is treated primarily as a process of balancing loads on the members. The tendons are placed so that eccentricity of prestr force varies in the same fashion as moments from the applied loads. The bending stress would be zeros, if this could be exactly achived. The section would then be subjected to axial stress P/A.



$$\frac{w'l^2}{8} = P \cdot e_{max}$$



$$\frac{W'l}{4} = P \cdot e_{max}$$

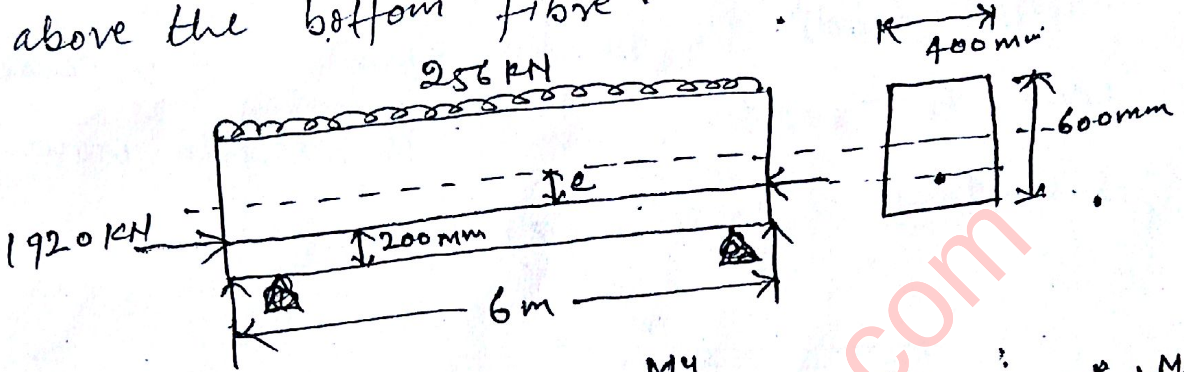


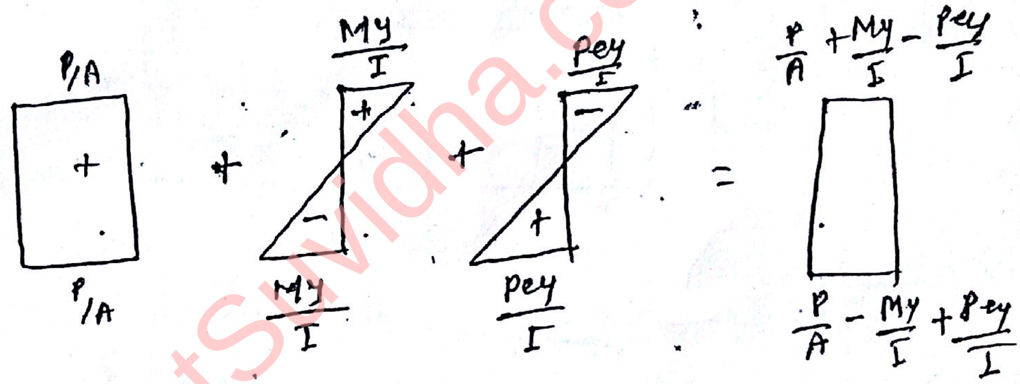
→ if $W' (= w'l)$ is equal to W, transverse load on the beam is balanced and the beam is subjected to only the axial force T which produce uniform stress in concrete $= T/A$

→ If $W' (= w'l) > W$

Resulting stress $= \frac{T}{A} \pm \frac{My}{}$

Prob:- A simply supported prestressed concrete beam of rectangular cross-section 400 mm × 600 mm is loaded with total uniformly distributed load of 256 KN over a span of 6m. Sketch the distribution of stresses at mid span and end sections if the prestressing force is 1920 KN and the tendon is located at 200 mm above the bottom fibre.





$P = 1920\ KN$

$e = \frac{600}{2} - 200 = 100\ mm$

$A = 400 \times 600 = 240000$

$I = \frac{400 \times 600^3}{12} = 72 \times 10^8\ mm^4$

$y = \frac{600}{2} = 300\ mm$

$M = \frac{wl^2}{8} = \frac{Wl}{8} = \frac{256 \times 6^2}{8} = 192\ KN\text{-}m = 192 \times 10^6\ N\text{-}mm$

Resultant stress at top fibre $= \frac{P}{A} + \frac{My}{I} - \frac{Pey}{I}$

$= \frac{1920 \times 1000}{240000} + \frac{192 \times 10^6 \times 300}{72 \times 10^8} - \frac{1920 \times 1000 \times 100 \times 300}{72 \times 10^8}$

$= 8 + 8 - 8$

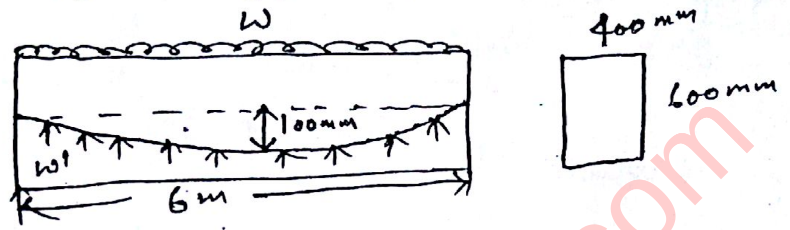
Resultant stress at bottom fibre

$= P/A - \frac{My}{I} + \frac{Pey}{I}$

$= 8 - 8 + 8$

$= 8\ N/mm^2$

Prob:- A simply supported prestressed concrete beam of rectangular cross-section 400 mm × 600 mm is prestressed by a parabolic tendon with a prestressing force of 1920 KN. The tendon has a sag of 100 mm at mid span. Find the extreme fibre stresses by load balancing concept, if it is subjected to (a) total udl of 256 KN (b) total udl of 360 KN.



$e_{max} = 100$ mm

$A = 400 \times 600 = 240000$ mm^2

$I = \frac{400 \times 600^3}{12} = 72 \times 10^8$ mm^4

$P = 1920$ KN $= 1920 \times 1000$ N.

Now, $\frac{w' l^2}{8} = P . e_{max}$

$\Rightarrow w' = \frac{P . e_{max} \times 8}{l^2} = \frac{1920 \times 1000 \times 100 \times 8}{6^2} = 42.67 \times 10^3$ N/m

Also, $w = \frac{256}{6} = 42.67$ KN/m $= 42.67 \times 10^3$ N/m

$\therefore w = w'$

Hence external load is balanced so beam is subjecte to only direct compressive load

$\therefore$ Resultant stress at extreeme fibres $= \frac{P}{A} = \frac{1920 \times}{24 \times 10}$
$= 8$ N/$_{\text{r}}$

Case II :- $w = \frac{360 \times 1000}{6} = 60 \times 10^3$ N/m

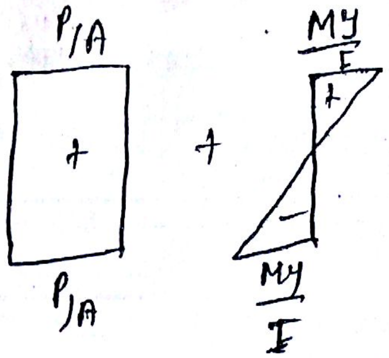
$\because w > w'$ bending stress is developed on the be

Resultant downward force, $w_r = 60 \times 10^3 - 42.67 \times 10^3 = 17.33 \times 1$

$M = \frac{w_p l^2}{8}$

$= \frac{17.33 \times 10^3 \times 6^2}{8}$

$= 78 \times 10^6$ N-mm



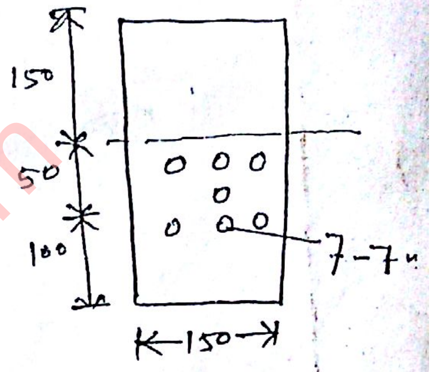
Bending stress $= \pm \frac{My}{I} = \pm \frac{78 \times 10^6 \times 300}{72 \times 10^8} = \pm 3.25$ N/mm^2

Direct stress $= \frac{P}{A} = 8$ N/mm^2

$\therefore$ Resultant stress at top fibre $= 8 + 3.25 = 11.25$ N/mm^2

Resultant stress at bottom fibre $= 8 - 3.25 = 4.75$ N/mm^2

Prob:- A concrete beam 150 mm x 300 mm is pre-tensio by 7 wires of 7 mm diameter at an initial stress of 100 with their centroid located at an eccentricity of 50 mm as shown in figure. Find loss of prestress due to elastic shortening of concrete, creep and shrinka of concrete if there is a relaxation of 4% of st stress. Use M40 concrete and creep coefficient = 1.6.



$A = 150 \times 300 = 45000\ mm^2$

$I = \dfrac{150 \times 300^3}{12} = 3.375 \times 10^8\ mm^4$

$E_s = 200\ KN/mm^2$

$E_c = 5700\sqrt{f_{ck}}\ N/mm^2$

$= 5700\sqrt{40} = 36050\ N/mm^2$

Prestressing force, $P = 1000 \times \cancel{450000}\ 7 \times \frac{\pi}{4} \times 7^2$

$= 270000\ N$

modular Ratio, $m = \dfrac{E_s}{E_c} = \dfrac{200 \times 1000}{36050} = 5.55$

: Stress in concrete at the level of steel :

$$\sigma_c = \frac{P}{A} + \frac{Pey}{I}$$

$$= \frac{270000}{450000} + \frac{270000 \times 50 \times 50}{3.375 \times 10^8}$$

$$= 8\ N/mm^2$$

i) Loss of prestress due to elastic shortening of concrete

$= m\sigma_c$

$= 5.55 \times 8$

$= 56.2\ N/mm^2$

% loss $= \frac{56.2}{1000} \times 100$

$= 5.62\%$

ii) Loss of prestress due to creep of concrete

σ_c' ~~= stress in concrete at the level of steel~~

$= m \theta \sigma_c$

$= 5.56 \times 1.6 \times 8$

$= 71.168\ N/mm^2$

$\%\ Loss = \frac{71.168}{1000} \times 100 = 7.116\ \%$

iii) Loss due to shrinkage of concrete

$= \varepsilon_{sh} \cdot E_s$

$= 3 \times 10^{-4} \times 200 \times 1000$

$= 60\ N/mm^2$

$\%\ loss = \frac{60}{1000} \times 100 = 6\ \%$

iv) Loss due to relaxation of steel

$= 4\ \%$ of total stress

$= \frac{4}{100} \times 1000$

$= 40\ N/mm^2$

Total loss $= 56.2 + 71.168 + 60 + 40$

$= 227.368\ N/mm^2$

$\%\ loss = \frac{227.368 \times 100}{1000}$

$= 22.73\ \%$