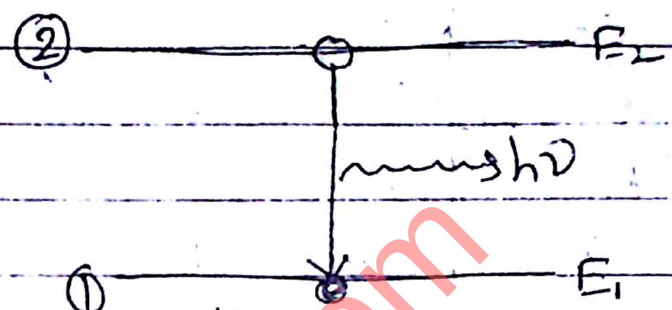
Spontaneous Emission:-



When energy given to the atom its electron are jump from energy level ① to the ②. The electron in the energy level E_2 will remain only 10^{-8} sec. After that they jump back to the ground energy level E_1 by releasing energy in the form of Photon. This process are called spontaneous emission.

Thus the probability of electron going from state ② to ① depends only characteristic of state ② and ① but does not depend upon the energy density.

Thus the probability of electron going from state ② to ① be P_{21} that is given by

$$P_{21} = A_{21} \quad \text{—— ②}$$

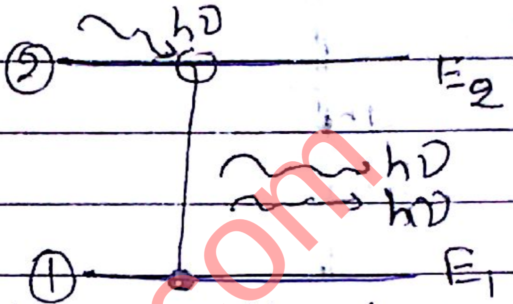
Where A_{21} is proportionality constant that depends

Upon the characteristic of state 2 and 1.

Stimulated Emission:-

There are some materials in which life time of upper state is longer. These materials are called active materials and these state are called meta stable state.

Then in such if the atom will still in the higher energy level. Then in such a case a photon of frequency ($\nu = \frac{E_2 - E_1}{h}$) is incident on the atom.



Then due to attraction force electron in the higher level loose the energy and jump back to the ground energy level by emitting an additional photon. Thus there are two photon of the same frequency are present in such a case. This type of emission are called stimulated emission.

Thus in such a case probability of electron going from state 2 to 1 depend upon the energy density of the incident photon. That is

$$P''_{21} \propto u(\nu) \quad \text{(3)}$$

or $P''_{21} = B_{21}\, u(\nu)$ Where B_{21} are called proportionality constant that depend upon the properties of state 1 and 2.

Thus total probability of electron going from state 2 to 1 is denoted by P_{21} that is given by $P_{21} = P'_{21} + P''_{21}$ — (4).

Relation between spontaneous and stimulated emission:-

Let us consider an assembly of atoms in the thermal equilibrium at temperature T with radiation of frequency ν and energy density $u(\nu)$

Let N_1 and N_2 be the number of atom in energy levels E_1 and E_2. The probability that number of atoms in state ①, absorb a photon and going to state ②

Thus :-

$$P_{12} = \frac{\text{number of atom going from state ① to ②}}{\text{Total number of atom in state ①}}$$

$\Rightarrow$ number of atom going from state ① to ② = $P_{12} \times$ Total number of atom in state ①

= No. of atom going from state ① and ② = $N_1 \times P_{12}$.

putting value of P_{12} from equation ①

that is

No. of atom going from state ① to ② = $N_1 \times B_{12} u(\nu)$

eq. ①

The probability that the number of atom in state ② to ① is given by:-

$$P_{21} = \frac{\text{number of atom going from state ② to ①}}{\text{Total number of atom in state ②}}$$

Thus. No. of atom going from state ② to ① = $P_{21} \times$ Total no. of atom in state ②

No. of atom going from state ② to ① = $B_{21} \times N_2$.

Putting value of $P_{21} = P_{21}' + P_{21}''$
$= [A_{21} + B_{21} u(\nu)]$

Thus No of atom going from state ② to ① = $[A_{21} + B_{21} u(\nu)] N_2$ — ②

In thermal equilibrium emission and absorption must be equal. Thus.

~~No~~ No. of atom going from state ① to ② =
= No of atom going from state ② to ①

$N_1 B_{12} u(\nu) = N_2 [A_{21} + B_{21} u(\nu)]$

or

$N_1 B_{12} u(\nu) = N_2 A_{21} + N_2 B_{21} u(\nu)$

or $u(\nu) [N_1 B_{12} - N_2 B_{21}] = N_2 A_{21}$

or $$\boxed{u(\nu) = \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}}}$$

or $$u(\nu) = \frac{N_2 A_{21}}{N_2 \left[\frac{N_1}{N_2} B_{12} - B_{21} \right]}$$

or $$\frac{A_{21}}{B_{21} \left[\frac{N_1}{N_2} \frac{B_{12}}{B_{21}} - 1 \right]} = u(\nu) \quad \text{— ③}$$

According to maxwell Boltzman law: Consider assembly of atoms at an absolute temperature T in which the atom's are in different energy states. Let N_0 be the number of atoms per unit volume in ground state.

then the number of atoms per unit volume in excited state of energy E is given by.
If N_1 and N_2 be the number of atom per unit volume in the state of energies E_1 and E_2 are given as.

$$N_1 = N_0 e^{-E_1/KT} \quad - (4)$$

or $$N_2 = N_0 e^{-E_2/KT} \quad - (5)$$

By dividing eq. (4) by (5) we get.

$$\frac{N_1}{N_2} = \frac{N_0 e^{-E_1/KT}}{N_0 e^{-E_2/KT}}$$

$$\frac{N_1}{N_2} = e^{-(E_1-E_2)/KT} = e^{(E_2-E_1)/KT} \quad — (6)$$

Putting value of $E_2 - E_1 = h\nu$ in eq. (6) then we get.

$$\frac{N_1}{N_2} = e^{h\nu/KT}$$

Putting value of N_1/N_2 in eq (3) we get.

$$u(\nu) = \frac{A_{21}}{B_{21}\left[\frac{B_{12}}{B_{21}} e^{h\nu/KT} - 1\right]} \quad - (7)$$

According to Planks Hypothesis

$$u(\nu) = \frac{8\pi h\nu^3}{c^3} \frac{1}{(e^{h\nu/KT}-1)} \quad - (8)$$

Comparing eq. (7) and (8) we get.

$$\boxed{\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}} \text{ and } \boxed{\frac{B_{12}}{B_{21}} = 1}$$

Thus $\frac{A_{21}}{B_{21}} \propto \nu^3$.

This is the formula for ~~ratio~~ ratio between the spontaneous emission and induced emission coefficients. The ratio is proportional to ν^3. This shows that the probability of spontaneous emission increases with the energy difference between two states.