Dispersion:- The dispersion is defined as the spreading of light pluse as it travels from one end of the fibre to the other end of the fibre.

In case of a digital communication system, the information to be transmitted through fibre is first coded in the form of pluses. After this these pluses are transmitted through the fibre. Finally these pluses are recived at the reciver end and decoded. The light pluses are entering at different angle so that they take different time to reach at the output end. the pluses are broden at the output end. This brodening are called dispersion. One of the most important dispersion is due to the material of the core of the fibre optics.

The refractive index of material depends on wavelength. Therefore velocity of light in material medium ($v = c/n$) is different for different wavelength, that is various wavelength in a light pluse travel with different velocity in optical fibre. This leads to material dispersion.

In order to calculate material dispersion let us consider a plane electromagnetic wave of spectral width $\Delta\lambda_0$ is travelling through the fibre. Thus the wave may be expressed as.

$$\phi = E_0 e^{i(kx - \omega t)}$$

Where k is propagation constant whose value is $k = \frac{2\pi}{\lambda}$ where n is refractive index of material. ω is angular frequency

$$\omega = 2\pi\nu = \frac{2\pi v}{\lambda} \quad \left[\nu = \frac{v}{\lambda}\right]$$

$k = \frac{2\pi}{\lambda}$, $\omega = 2\pi\nu = 2\pi\frac{v}{\lambda}$

$\Rightarrow \omega/v = \frac{2\pi}{\lambda} = k$

$$\Rightarrow \frac{\omega}{v} = \frac{2\pi}{\lambda} = k$$

Thus $k = \frac{\omega}{v}$ or $k = \frac{\omega}{c/n} = \frac{n\omega}{c}$ — (1).

The group velocity is given by:- $V_g = \frac{d\omega}{dk} \rightarrow$ (2)

Diff. eq. (1) with respect to ω we get

$$\frac{dk}{d\omega} = \frac{n}{c} + \frac{\omega}{c}\frac{dn}{d\omega} = \frac{1}{c}\left[n + \omega\frac{dn}{d\omega}\right] \quad — (3)$$

Putting value of eq. (3) in (2) we get.

$$V_g = \frac{d\omega}{dk} = \frac{c}{n + \omega\frac{dn}{d\omega}} \quad — (4)$$

we know that $\omega = 2\pi\nu_0 = \frac{2\pi c}{\lambda_0}$ — (5)

$$\lambda_0 = \frac{2\pi c}{\omega} \quad — (6)$$

Now value of $\frac{dn}{d\omega}$ can be written as.

$$\frac{dn}{d\omega} = \frac{dn}{d\lambda_0} \cdot \frac{d\lambda_0}{d\omega} \quad [\text{multiply and divide by } d\lambda_0]$$

$$\frac{dn}{d\omega} = \frac{dn}{d\lambda_0}\left[-\frac{2\pi c}{\omega^2}\right] = \frac{dn}{d\lambda_0}\left[-\frac{2\pi c}{\frac{(2\pi c)^2}{\lambda_0^2}}\right] = \frac{dn}{d\lambda_0}\left[-\frac{\lambda_0^2}{2\pi c}\right]$$

$$\frac{dn}{d\omega} = \frac{-\lambda_0^2}{2\pi c} \cdot \frac{dn}{d\lambda_0}. \quad — (7)$$

Thus the group velocity

$$V_g = \frac{c}{n + \omega\frac{dn}{d\omega}}$$ Putting value of $\frac{dn}{d\omega}$ in this eq.

$$V_g = \left(\frac{c}{n - \frac{\omega\lambda_0^2}{2\pi c}\frac{dn}{d\lambda_0}}\right) — (8)$$

Time taken by the light Pulse to travel through a distance L is given by.

$$\tau = \frac{L}{V_g} = \frac{L}{c}\left[n - \frac{\lambda_0^2\omega}{2\pi c}\frac{dn}{d\lambda_0}\right].$$

Now putting value of $\frac{\omega}{2\pi c} = \frac{1}{\lambda_0}$ from eq. (5) we get.

$$\tau = \frac{L}{c}\left[n - \frac{\lambda_0^2}{\lambda_0}\frac{dn}{d\lambda_0}\right] = \frac{L}{c}\left[n - \lambda_0\frac{dn}{d\lambda_0}\right] — (9)$$

This equation shows that different wave length components take different time to travel distance L of the fibre. Therefore the pluse dispersion is given by:-

$$\frac{\Delta\tau}{\Delta\lambda_0} = \frac{d\tau}{d\lambda_0} \quad \text{or}$$

$$\Delta\tau = \frac{d\tau}{d\lambda_0}\Delta\lambda_0. \quad — (10)$$

Now find out value of $\frac{d\tau}{d\lambda_0}$ from eq. (9). By Diff.

$$\frac{d\tau}{d\lambda_0} = \frac{L}{c}\left[\frac{dn}{d\lambda_0} - \left[\frac{dn}{d\lambda_0} + \lambda_0\frac{d^2n}{d\lambda_0^2}\right]\right]$$

$$= \frac{L}{c}\left[\cancel{\frac{dn}{d\lambda_0}} - \cancel{\frac{dn}{d\lambda_0}} - \lambda_0\frac{d^2n}{d\lambda_0^2}\right] = -\frac{L}{c}\lambda_0\frac{d^2n}{d\lambda_0^2}.$$

∴ Putting value of $\frac{d\tau}{d\lambda_0}$ in eq. (10) we get.

$$\boxed{\Delta\tau = -\frac{L}{c}\lambda_0 \frac{d^2 n}{d\lambda_0^2}\Delta\lambda_0}$$

This is the expression for material dispersion.

Attenuation:- The signal attenuation with in optical fibre is expressed in the logarithmic unit of decibel as in metallic conductors. If P_i is input Power and P_o is output power then number of decibels is given by:-

$$\text{Number of decibels (dB)} = 10 \log_{10} \frac{P_i}{P_o} \quad -(1)$$

The logarithmic unit has the advantage that the operation of multiplication and division are converted into addition and subtraction while power and roots are converted into multiplication and division.

from eq. (1) $\frac{P_i}{P_o} = 10^{+dB/10}$ or $P_o = P_i\, 10^{-dB/10}$.

The output power get attenuated.

In optical fibre communication the attenuation is expressed in decibels per unit length that is denoted by α_{dB}. is signal attenuation per unit length in decibles. and L is the length of optical fibre then

$$\alpha_{dB} = \frac{dB}{L} = \frac{1}{L}\left[10 \log_{10} \frac{P_i}{P_o}\right]$$

$$\boxed{\alpha_{dB} = \frac{1}{L}\left[10 \log_{10} \frac{P_i}{P_o}\right]}$$