

B.E.2nd Semester Examination, May -2007

Paper : PHY-102-E

Time Allowed : 3 hrs.

Maximum Marks : 100

Note:- Attempt **five** questions in all, selecting atleast **two** questions from each part.

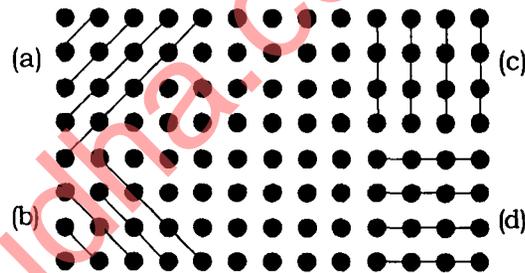
PART-A

1. (a) Define lattice planes. How would you represent them by Miller indices? Also state the method of calculating the density of lattice points in a lattice plane.

Ans. Lattice plane : A crystal lattice may be considered as an aggregate of a set of parallel equidistant planes passing through the lattice points. These equidistant planes are known as lattice planes.

For a particular lattice these sets of planes and the density of lattice points vary accordingly. These planes are specified as follows :

(i) Let us consider that the intercepts by the given lattice plane on the three crystal axis X, Y and Z are in the ratio $pa : qb : rc$ where a, b and c may be either small integers or simple fractions.



(ii) Take the reciprocal of p, q, r, i.e., $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}$.

(iii) Finally determine the smallest possible integers h, k, l such that $h : k : l = \frac{1}{p} : \frac{1}{q} : \frac{1}{r}$.

The numbers h, k, l are called the **Miller indices** of a given set of planes and the plane is specified as (hkl).

Density of lattice points in a lattice plane :

Let us consider N successive parallel lattice planes having spacing d and area of cross-section A. Then the volume of the part of lattice under consideration = N.A.d.

If V is the volume of each unit cell, then the number of unit cells, in this part of lattice = N.A.d/V. Further, if n is the number of lattice points per unit cell, then the total number

of lattice points in this part of lattice = $\frac{N.A.d}{V}n$. If ρ is the number of lattice points per unit area i.e. the density of lattices points in these planes, then the total number of lattice points in the same part of lattice = $A\rho N$.

$$\therefore A\rho N = \frac{N.A.d}{V}n$$

$$\text{or } \rho = \frac{nd}{V} \quad \dots(1)$$

In the case of tetragonal orthorhombic and cubic lattices the volume of each unit cell, $V = abc$.

$$\therefore \rho = \frac{nd}{abc} \quad \dots(2)$$

Also for primitive lattice in each of these system, there is one lattice point, per unit cell, i.e., $n = 1$. Therefore in such case

$$\rho = \frac{d}{abc}$$

Q.1. (b) A crystal is mounted on an X-ray spectrometer the glancing angles in incidence for three reflections are $5^{\circ}58'$, $12^{\circ}01'$ and $18^{\circ}12'$. Show that these are successive orders of reflections from the same crystal plane. Also find the spacing (Given λ of X-rays used in 0.586 \AA). **10**

Solution. Successive orders of reflections for three glancing angles are

$$\sin q_1 = \frac{\lambda}{2d} \text{ first maximum}$$

$$\sin q_2 = \frac{\lambda}{d} \text{ second maximum}$$

$$\sin q_3 = \frac{3\lambda}{2d} \text{ third maximum}$$

here $q_1 = 5^{\circ} - 58'$, $q_2 = 12^{\circ} - 01'$, $q_3 = 18^{\circ} - 12'$ and $\lambda = 0.586 \text{ \AA}$, $d = ?$

from first maximum
$$d = \frac{\lambda}{2\sin q_2} = \frac{0.586}{2\sin 5-58} = \frac{0.586}{2 \times 0.104} = 2.817 \text{ \AA}$$

from second maximum
$$d = \frac{\lambda}{2\sin q_2} = \frac{0.586}{0.204} = 2.817 \text{ \AA}$$

from third maximum
$$d = \frac{3\lambda}{2\sin q_3} = \frac{3 \times 0.586}{2 \times 0.312} = 2.817 \text{ \AA}$$

As the value of d , obtained from 1st, 2nd and 3rd maxima is the same hence the given glancing angles corresponds to successive order of reflections.

2. (a) Discuss the various kind of defects in crystals and find out the concentration of Schottky defects in a crystal. **10**

Ans. Defects in crystals :

A perfect crystal is the periodic arrangement of atoms in the lattice arrays in all directions. "Any deviation in the crystal from the perfect atomic periodicity is called lattice defect. Thus the lattice defect is a state in which the lattice arrays of atoms are not continuous through the crystal.

The defects can change the properties of the solids, for example

(a) The defects may scatter conduction electrons in a metal, increasing its electrical resistance by several percent in many pure metals and much more in alloys.

(b) Some defects decrease the strength of the crystal.

(c) Properties like crystal growth, magnetic hysteresis, dielectric strength are sensitive to the state of perfection of the crystal.

1. The point defects in solids can be divided into three classes :

- (i) Lattice - site defects , e.g. vacancies and interstitials
- (ii) Compositional defects i.e. substitutional impurity and interstitial impurity.
- (iii) Electronic defects.

2. Line defects are classified as edge dislocation and screw dislocation.

3. Surface defects are the defects on external surface due to discontinuity at the surface and external surface e.g. grain boundaries.

4. Volume defects, such as crack, may arise when there is only small electrostatic dissimilarity.

Concentration of Schottky defects in the crystals :

Please answer 1(b), Paper May 2008

2. (b) Discuss the short comings of the classical physics and derive the Plank's radiation Law. 10

Ans. Shortcomings of the classical physics :

- (i) It does not hold in the region of atomic demensions i.e., it can not explain the non relativistic motion of atoms electrons, protons etc.
- (ii) It couldnot explain the stability of atoms.
- (iii) It could not explain observed spectrum of black body radiations.
- (iv) It could not explain the observed variation of specific heat of metals and gases.
- (v) It could not explain the origin of discrete spectra of atoms since according to classical mechanics the energy changes are always continuous.

Derivation of Plank's Radiation Law :

Plank found an empirical formula to explain the experimentally observed distribution of energy in the spectrum of a black-body. The formula may be deduced using following assumptions which may be called as Plank's hypothesis :

1. A black-body radiation chamber is filled up not only with radiation, but also with simple harmonic oscillators (or resonators) of molecular dimensions (called as Plank's oscillators or resonators) which can vibrate with all possible frequencies. The vibrations of the resonator entails one degree of freedom only.

2. The oscillators or resonators can not radiate or absorb energy continuously; but an oscillator of frequency ν can only radiate or absorb energy in units or quanta of magnitude $h\nu$, where h is a universal constant, called Plank's constant, its value being equal to $6.62 \cdot 10^{-34}$ joule-sec.

In other words, this states that the exchanges of energy between radiation and matter can not take place continuously, but ar elimited to discrete set of values $0, h\nu, 2h\nu, 3h\nu, \dots, \nu h\nu$, i.e., multiples of some small unit, called the quantum.

In order to derive Plank's radiations law, we shall first derive the number of resonators per unit volume lying in the frequency range ν and $\nu + d\nu$ and the average energy of Plank's resonator. The number of modes of vibration per unit volume with frequency range ν and

$$\nu + d\nu = \frac{4\pi\nu^2 d\nu}{c^3}$$

Now the black body radiation travel with velocity of light c and are transverse in character unlike sound waves in the string which are longitudinal.

As there are two possible polarisation states for each transverse wave, the number of modes of vibration or transverse waves is double as for longitudinal wave. Therefore, for black body radiations or electromagnetic waves, the number of modes of vibrations per unit volume within frequency range ν and $\nu + d\nu$.

$$= \frac{2 \times 4\pi\nu^2 d\nu}{c^3} = \frac{8\pi\nu^2 d\nu}{c^3} \quad \dots(1)$$

Average energy of Plank's Oscillator : If N is the total number of Plank's resonators and E their total energy, then average energy per Plank oscillator is given by

$$\bar{\epsilon} = \frac{E}{N} \quad \dots(2)$$

According to Maxwell's law of molecular motion if ϵ is is a certain amount of energy, the probabilities that a system will have energies $0, \epsilon, 2\epsilon, \dots, r\epsilon, \dots$ are the ratio $1 : 1 : e^{-\epsilon/KT} : e^{-2\epsilon/KT} : \dots$ et.

In N_0 is the number of resonators having energy zero, then the number of resonators N_1 having energy ε will be $N_0 e^{-\varepsilon/kT}$, the number of resonators N_2 having energy 2ε will be $N_0 e^{-2\varepsilon/kT}$ and in general, the number of resonator N_r having energy $r\varepsilon$ will be $N_0 e^{-r\varepsilon/kT}$ and so on

$$\begin{aligned} N &= N_0 + N_1 + N_2 + \dots + N_r + \dots \\ &= N_0 + N_0 e^{-\varepsilon/kT} + N_0 e^{-2\varepsilon/kT} + \dots + N_0 e^{-r\varepsilon/kT} + \dots \\ &= N_0 [1 + e^{-\varepsilon/kT} + e^{-2\varepsilon/kT} + \dots + e^{-r\varepsilon/kT} + \dots] \end{aligned}$$

Putting

$$e^{-\varepsilon/kT} = y \quad \dots(3)$$

$$N = N_0 [1 + y + y^2 + \dots + y^r + \dots] \quad \dots(4)$$

or

$$N = \frac{N_0}{1 - y}$$

The total energy of Plank's resonators will be

$$\begin{aligned} E &= 0 \times N_0 + \varepsilon \times N_1 + 2\varepsilon \times N_2 + \dots + r\varepsilon \times N_r + \dots \\ &= 0 + \varepsilon N_0 e^{-\varepsilon/kT} + 2\varepsilon N_0 e^{-2\varepsilon/kT} + \dots + r\varepsilon N_0 e^{-r\varepsilon/kT} + \dots \\ &= N_0 \varepsilon [e^{-\varepsilon/kT} + 2e^{-2\varepsilon/kT} + \dots + r e^{-r\varepsilon/kT} + \dots] \\ &= N_0 \varepsilon [y + 2y^2 + \dots + r y^r + \dots] \end{aligned}$$

Let

$$S = y + 2y^2 + \dots + r y^r + \dots \quad \text{[using equation(3)]}$$

\therefore

$$S y = y^2 + 2y^3 + \dots + (r-1)y^r$$

Subtracting

$$S - S y = y + y^2 + \dots + y^r + \dots$$

$$S(1 - y) = \frac{y}{1 - y} \Rightarrow S = \frac{y}{(1 - y)^2}$$

\therefore

$$E = N_0 \varepsilon S = N_0 \varepsilon \cdot \frac{y}{(1 - y)^2} \quad \dots(5)$$

Therefore the average energy of a resonator will be

$$\bar{\varepsilon} = \frac{E}{N} = \frac{N_0 \varepsilon \cdot \frac{y}{(1 - y)^2}}{\frac{N_0}{1 - y}} = \frac{\varepsilon y}{1 - y} = \frac{\varepsilon e^{-\varepsilon/kT}}{1 - e^{-\varepsilon/kT}}$$

or

$$\bar{\varepsilon} = \frac{\varepsilon}{e^{\varepsilon/kT} - 1} \quad \dots(6)$$

According to Plank's hypothesis of quantum theory $\varepsilon = h\nu$; therefore the average energy of Plank's oscillator is given by

$$\bar{\varepsilon} = \frac{h\nu}{e^{h\nu/kT} - 1} \quad \dots(7)$$

Now the energy density belonging to the range $d\nu$ can be obtained by multiplying the average energy of a Plank's oscillator by the number of oscillators per unit volume, in the frequency range ν and $\nu + d\nu$, i.e.

$$E_\nu d\nu = \left(\frac{8\pi\nu^2}{c^3} d\nu \right) \left(\frac{h\nu}{e^{h\nu/kT} - 1} \right)$$

or

$$E_\nu d\nu = \frac{8\pi h\nu^3}{c^3} \cdot \frac{1}{e^{h\nu/kT} - 1} \quad (8)$$

where $e_\nu d\nu$ is energy density (i.e., total energy per unit volume) belonging to the range $d\nu$.

Eqn. (8) is called Planck's radiation law in terms of frequency.

The energy density $E_\lambda d\lambda$ belonging to range $d\lambda$ can be obtained by using the relation

$$\nu = \frac{c}{\lambda} \text{ and hence } |d\nu| = \left| -\frac{c}{\lambda^2} d\lambda \right|, \text{ we get}$$

$$\begin{aligned} E_\lambda d\lambda &= \frac{8\pi h}{c^3} \left(\frac{c^3}{\lambda^3} \right) \cdot \frac{1}{e^{hc/\lambda kT} - 1} \cdot \left| -\frac{c}{\lambda^2} d\lambda \right| \\ &= \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{hc/\lambda kT} - 1} \quad \dots(9) \end{aligned}$$

This is Planck's radiation law in terms of wavelength.

3. (a) Derive time dependent Schrodinger equation and discuss the concept of stationary states wave packets and the significance of wave function.

Ans. Time dependent Schrödinger equation : Time-dependent Schrödinger equation may be obtained by eliminating E from time independent Schrodinger equation i.e.,

$$\nabla^2 \psi + \frac{2m}{\hbar^2} (E - V) \psi = 0 \quad \dots(1)$$

Differentiating equation $\psi(r, t) = \psi_0(r) e^{-i\omega t}$ with respect to t , we get

$$\begin{aligned} \frac{\partial \psi}{\partial t} &= -i\omega \psi_0(\mathbf{r}) e^{-i\omega t} \\ &= -i(2\pi\nu) \psi_0(\mathbf{r}) e^{-i\omega t} && \text{(since } \omega = 2\pi\nu) \\ &= -2\pi i\nu \omega(\mathbf{r}) && \text{[Using } \psi(r, t) = \psi_0(r) = e^{-i\omega t}] \\ &= -\frac{2\pi i E}{\hbar} \psi && \text{(since } E = h\nu, \text{ i.e., } \nu = \frac{E}{h}) \\ &= -\frac{iE}{\hbar} \psi \frac{i}{i} && \left[\text{Using } h = \frac{\hbar}{2\pi} \right] \end{aligned}$$

which gives $E\psi = i\hbar \frac{\partial \psi}{\partial t} \quad \dots(2)$

Substituting value of $E\psi$ from above equation in

$$\nabla^2 \psi + \frac{2m}{\hbar^2} (E - V) \psi = 0$$

we get $\nabla^2 \psi + \frac{2m}{\hbar^2} \left[i\hbar \frac{\partial \psi}{\partial t} - V\psi \right] = 0$

or
$$\nabla^2 \psi = -\frac{2m}{\hbar^2} \left[i\hbar \frac{\partial \psi}{\partial t} - V\psi \right]$$

i.e.
$$-\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi = i\hbar \frac{\partial \psi}{\partial t} \quad \dots(3)$$

This equation may be written as

$$\left(-\frac{\hbar^2}{2m} \nabla^2 + V \right) \psi = i\hbar \frac{\partial \psi}{\partial t} \quad \dots(4)$$

This equation contains the time and hence is called **time dependent Schrödinger Equation**.

Significance of wave function :

The wave function (r, t) which is a solution to the Schrödinger equation, is assumed to provide information about the behaviour of the particle.

The fundamental postulate was formulated by Max Born, which states that the quantity $|\psi|^2$ may be interpreted as probability density of the particle associated with de-Broglie wave described by the wave function ψ . The wave function is a complex quantity and expressed as

$$\psi(r, t) = \psi(x, y, z, t) = a + ib$$

The complex conjugate of ψ is ψ^* , which is given as

$$\psi^*(x, y, z, t) = a - ib$$

and

$$\psi \psi^* = (a + ib)(a - ib) = a^2 + b^2$$

This may be denoted by P i.e. $P = |\psi(x, y, z, t)|^2 = a^2 + b^2$

The product of ψ and ψ^* is real and positive if $\psi \neq 0$. The positive square root is denoted by $|\psi(x, y, z, t)|$ and is called modulus of ψ . The quantity $|\psi(x, y, z, t)|^2$ is called 'probability density'

The probability of finding the particle in a volume element $d\tau = dx dy dz$ at the position (x, y, z) is

$$P d\tau = \psi(x, y, z, t) \psi^*(x, y, z, t) d\tau = |\psi(x, y, z, t)|^2 d\tau$$

For motion of particle in one dimension

$$P dx = |\psi(x, t)|^2 dx$$

is the probability that the particle will be found over a small distance dx at position x , at time τ and $|\psi(x, t)|^2$ is called probability per unit distance i.e. position probability density.

Since
$$\psi(x, t) = \psi(x) e^{\frac{iEt}{\hbar}} = \psi(x) e^{-i\omega t}$$

Then
$$\psi(x, t) = \psi^*(x) e^{i\omega t}$$

$$P dx = \psi(x, t) \psi^*(x, t) dx = |\psi(x, t)|^2 dx$$

or

$$P dx = \psi(x) \psi^*(x) = |\psi(x)|^2 dx$$

Similarly for three dimensional wave function $\psi(x, y, z, t)$

$$P d\tau = |\psi(x, y, z)|^2 d\tau$$

Thus for wave function satisfying the time dependent Schrodinger equation, the probability is always independent of time. "A state with well defined energy having wave function of

the form given by eqn. $\psi(x, t) = \psi(x) \exp\left(-\frac{1}{\hbar} Et\right)$, for which probability density is dependent of time is called stationary state."

3. (b) Calculate the energy difference between the ground state and first excited state for an electron in one dimensional box of length 10^{-10}m , assuming $V = 0$ for $0 \leq x \leq a$ and $V = \infty$ for $0 > x > a$ 10

Solution : Under the specified conditions energy for the n th level is given as :

$$E_n = \frac{\hbar^2}{2m} \left(\frac{n\pi}{a} \right)^2$$

for $n = 1$
$$E_1 = \frac{\hbar^2}{2m} \left(\frac{\pi}{a} \right)^2$$

$n = 2$
$$E_2 = \frac{\hbar^2}{m} \left(\frac{\pi}{a} \right)^2$$

Difference between first excited state and ground state is

$$E_2 - E_1 = \frac{3\hbar^2}{m} \left(\frac{\pi}{a} \right)^2 = \frac{3\hbar^2}{8ma^2}$$

Here

$$a = 10^{-10}\text{m}, m = 9.1 \times 10^{-31} \text{ kg}, \hbar = 6.626 \times 10^{-34} \text{ J-s}$$

$$\therefore E_2 - E_1 = \frac{3 \times (6.626 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times 10^{-20}} = 1.809 \times 10^{17} \text{ J. Ans.}$$

4. Considering the free electron in a metal to form an electron gas obeying Fermi-Dirac statistics obtain Richardson equation for thermionic emission of electrons. 20

Ans. Thermionic Emission :

Please see answer 4 (ii), Paper May 2008

Richardson's Equation :

Please see answer 4 (iii), Paper December 2007

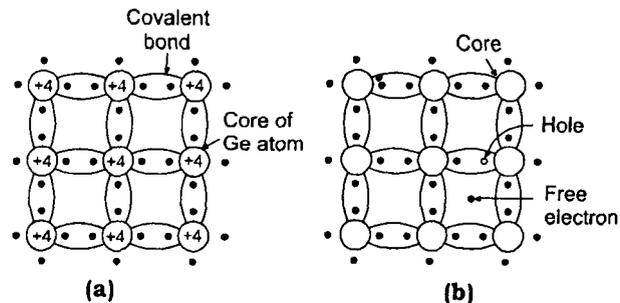
PART - B

5. What do you mean by intrinsic semiconductor? Obtain an expression for the intrinsic carrier concentration in an intrinsic semiconductor under what conditions will Fermi level be in the middle of the forbidden gap. 20

Ans. Intrinsic semiconductor :

Pure Germanium or silicon are known as intrinsic semiconductors. A pure Germanium atom has 32 electrons out of which 28 are tightly bound to the nucleus and the remaining four revolve in the outermost level, these are called as valence electrons. Nucleus with 28 tightly bound electrons forms the positive core of the atom.

When two atoms of Germanium are brought close to each other the positive core of one atom interacts with one of the valence electrons of other atom. Each core will attract the electron of the other atom and the two electrons will be shared by two atoms. Two such electrons form an electron pair fig. (a). Equilibrium state



will reach when the attractive force is balanced by the repulsive force between two positive cores and a covalent bond is thus formed. These covalent bonds are the binding force between the adjacent atoms of the crystal and two electrons contributed by the two atoms are

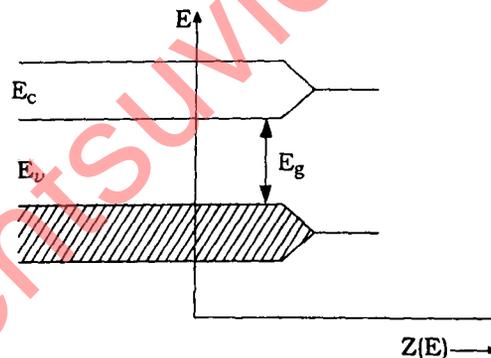
still not available for conduction. It means electron pair is bond to the cores of the atom. For their disruption, considerable amount of energy is required. Because of those covalent bonds crystal of pure Germanium at OK is a non-conductor of electricity as there are no free carriers of charge.

At ordinary temperature, because of thermal excitation, crystal lattice is in continuous random agitation. As a result, an individual electron of a covalent bond acquires sufficient energy even at room temperature to break the bond and becomes free. These escaped electrons which are neither repelled nor attracted by the core of the atom move freely about the crystal in random way.

The empty space left behind by the escaped electron from covalent bond is called a hole Fig. (b). The tendency of second electron in covalent bond is to form again an electron pair for which it steals one electron from an adjacent pair due to favourable situation arising out of thermal agitation. The previous hole is now shifted to another place from which electron is stolen. Thus when a hole is created it moves about in the crystal in a random way in the same manner as the free electrons. Thus in a semiconductor for every covalent bond that is broken, there are two carriers of electricity available i.e., the electron and the hole.

Carrier Concentration and Fermi Level of Intrinsic Semiconductor :

It is assumed that the width of allowed energy band is comparable to forbidden gap. The electrons in the conduction band may have energy lying between E_c and ∞ while the electrons in valence band have energy lying from $-\infty$ to E_v , E_g is the width of the forbidden gap, as shown in the fig.



(a) Density of electrons in conduction band :

The density of electrons in the conduction band, i.e., total number of electrons per unit volume is given by

$$n_c = \int_{E_p}^{\text{top}} D(E)F(E)DE \quad \dots(1)$$

where $D(E)$ is the energy density of states. $D(E) DE$ gives the total number of available states in the range E and $E + dE$; $F(E)$, is the Fermi function giving the probability of occupation. The Fermi function $F(E)$ decreases as we move up in the conduction band because $F(E) \rightarrow 0$ for $E \gg E_g$, hence we can assume the upper limit $E = \infty$ for the condition band. To evaluate eqn. (1) it is sufficient to know $D(E)$ near the bottom of conduction band and integrating from $E = E_c$ to $E = \infty$.

$$\therefore n_c = \int_{E_c}^{\infty} D(E)F(E)DE \quad \dots(2)$$

The energy density of states at the bottom of conduction band of the semiconductor is

$$D(E) = \frac{4\pi}{h^3} (2m_e^*)^{3/2} (E - E_c)^{1/2} \quad \dots(3)$$

where m_e^* is the effective mass of the electron in conduction band. Now

$$F(E) = \frac{1}{\exp\{(E - E_F)/kT\} + 1} \quad \dots(4)$$

Putting (6.39) and (6.40) in (6.38), the density of electrons in conduction band is

$$n_c = \frac{4\pi}{h^2} (2m_e^*)^{3/2} \int_{E_c}^{\infty} \frac{(E - E_c)^{1/2} dE}{\exp\left\{\frac{(E - E_F)}{kT}\right\}} \quad \dots(5)$$

If $E - E_c \gg kT$, the unit term in the denominator is negligible and (5) can be written as

$$\begin{aligned} n_c &= \frac{4\pi}{h^2} (2m_e^*)^{3/2} \int_{E_c}^{\infty} \frac{(E - E_c)^{1/2} dE}{\exp\left\{\frac{(E - E_F)}{kT}\right\}} \\ &= \frac{4\pi}{h^2} (2m_e^*)^{3/2} \int_{E_c}^{\infty} (E - E_c)^{1/2} \exp\left\{-\frac{(E_F - E)}{kT}\right\} dE \end{aligned}$$

or

$$\begin{aligned} n_c &= \frac{4\pi}{h^2} (2m_e^*)^{3/2} \int_{E_c}^{\infty} (E - E_c)^{1/2} \exp\left\{-\frac{(E_F - E)}{kT}\right\} \\ &\quad \exp\left\{-\frac{(E_c - E)}{kT}\right\} dE \quad \dots(6) \end{aligned}$$

Putting

$$\frac{E - E_c}{kT} = x$$

or

$$E - E_c = m kT$$

or

$$dE = kT dx$$

and

$$(E - E_c)^{1/2} = x^{1/2} (kT)^{1/2}$$

Putting these values in eqn. (4.42), we have

$$n_c = \frac{4\pi}{h^2} (2m_e^*)^{3/2} \int_{E_c}^{\infty} x^{1/2} (kT)^{1/2} e^{-x} kT \exp\left\{-\frac{(E_F - E_c)}{kT} = x\right\} dx$$

or

$$n_c = \frac{4\pi}{h^2} (2m_e^*)^{3/2} (kT)^{1/2} \exp\left\{-\frac{(E_F - E_c)}{kT}\right\} \int_0^{\infty} x^{1/2} e^{-x} dx$$

But

$$\int_0^{\infty} x^{1/2} e^{-x} dx = \left(\frac{\pi}{4}\right)^{1/2}$$

∴

$$n_c = \frac{4\pi}{h^2} (2m_e^*)^{3/2} (kT)^{1/2} \exp\left\{-\frac{(E_F - E_c)}{kT}\right\} \left(\frac{\pi}{4}\right)^{1/2}$$

or

$$n_c = 2 \left(\frac{2\pi m_e^* kT}{h^2} \right) \exp \left\{ \left(\frac{E_F - E_c}{kT} \right) \right\} \quad \dots(7)$$

(b) Density of holes in valence band : To calculate the density of holes n_h , in the valence band, we shall use $[1-F(E)]$ instead of $F(E)$ as the probability for a state of energy E to be unoccupied [$F(E)$ is the probability for a state of energy E to be occupied]. Thus density of holes in the valence band can be put as

$$n_h = \int_{\text{bottom}}^{E_v} D(E)[1 - F(E)]dE \quad \dots(8)$$

where the lower limit is from $-\infty$, to be taken for convenience and it will include certainly all the holes in valence band Now

$$1 - F(E) = 1 - \frac{1}{\left\{ \frac{(E - E_F)}{kT} \right\} + 1} = \frac{\exp \left\{ \left(\frac{E - E_F}{kT} \right) \right\}}{\exp \left\{ \left(\frac{E - E_F}{kT} \right) \right\} + 1}$$

Since $E < E_F$ being in valence band, so the exponential term is less than unity and can be neglected in the denominator, thus

$$1 - F(E) = \exp \left(\frac{E - E_F}{kT} \right) \quad \dots(9)$$

Thus in the valence band, $[1-F(E)]$ decrease exponentially *i.e.* as we move down below the top of valence band, the probability of finding holes decreases exponentially as the holes reside near the top of the valence band. The value of $D(E)$ near the top of the valence band can be written as

$$D(E)dE = \frac{4\pi}{h^3} (2m_h^*)^{3/2} (E_v - E)^{1/2} dE \quad \dots(10)$$

Where m_h is the effective mass of holes near the top of valence band. Putting (9) and (6.46) in (6.44) and integrating between $-\infty$ and E_v we get

$$n_h = \frac{4\pi}{h^3} (2m_h^*)^{3/2} \int_{-\infty}^{E_v} (E_v - E)^{1/2} \exp \left(\frac{E - E_F}{kT} \right) dE$$

evaluating intergral, we get

or

$$n_h = 2 \frac{(2\pi m_h^* kT)^{3/2}}{h^2} \exp \left(\frac{E - E_F}{kT} \right) \quad \dots(11)$$

(c) **Fermi Level**

In an intrinsic semiconductor, we know $n_c = n_h$

$$\therefore 2 \left(\frac{2\pi m_e^* kT}{h^2} \right)^{3/2} \exp \left(\frac{E_F - E_c}{kT} \right) = 2 \left(\frac{2\pi m_h^* kT}{h^2} \right)^{3/2} \exp \left(\frac{E_v - E_F}{kT} \right)$$

or

$$m_e^{3/2} \exp \left(\frac{E_F - E_c}{kT} \right) = m_h^{3/2} \exp \left(\frac{E_v - E_F}{kT} \right)$$

or
$$\exp\left(\frac{E_F - E_c - E_v}{kT}\right) = \left(\frac{m_h^*}{m_e^*}\right)^{3/2}$$

Taking log
$$\frac{2E_F - E_c - E_v}{kT} = \frac{3}{2} \log\left(\frac{m_h^*}{m_e^*}\right)$$

or
$$E_F \frac{E_c - E_v}{2} + \frac{3}{4} kT \log\left(\frac{m_h^*}{m_e^*}\right) \dots(12)$$

when $m_h^* = m_e^*$ then $\log\left(\frac{m_h^*}{m_e^*}\right) = \log\left(\frac{m_h^*}{m_e^*}\right)$

Thus
$$E_F = \frac{E_c + E_v}{2} \dots(13)$$

i.e. Fermi level lies exactly half way between top of valence band and conduction band. But for finite temperature $m_h^* > m_e^*$ and the Fermi level is slightly raised with increase of T. But for all practical purpose the Fermi level in intrinsic semiconductors can be assumed to be constant over a wide range of temperatures.

The knowledge of E_F allows to calculate the number of carrier electrons and holes in terms of band gap E_g , where $E_g = E_c - E_v$. Thus eqns. (6.47) after simplification become.

$$n_c = 2 \left(\frac{2\pi kT}{h^2}\right)^{3/2} (m_h^* m_e^*) \exp\left(-\frac{E_g}{2kT}\right) \Rightarrow \exp\left(-\frac{E_g}{2kT}\right) \dots(14)$$

$$n_h = 2 \left(\frac{2\pi kT}{h^2}\right)^{3/2} (m_h^* m_e^*) \exp\left(-\frac{E_g}{2kT}\right) \Rightarrow \exp\left(-\frac{E_g}{2kT}\right) \dots(15)$$

eqns. (6.50) and (6.51) show that

$$n_c = n_h = n_i = 2 \left(\frac{2\pi kT}{h^2}\right)^{3/2} (m_h^* m_e^*) \exp\left(-\frac{E_g}{2kT}\right)$$

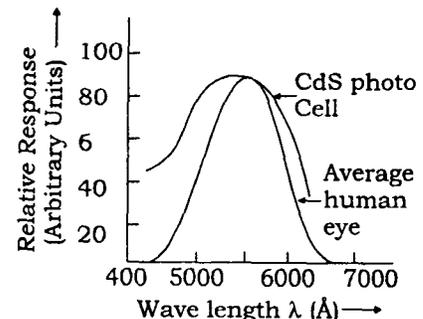
where n_i is the number of electrons or holes per unit volume in the material.

6. (a) Discuss the photo conductive effect and spectral response. Explain the salient features of photodiode and its characteristics.

Ans. Spectral Response :

Figure shows the spectral response of CdS cell. The response is sensitive to visible light. It is maximum over the visible region and tapers off towards the ultraviolet and infrared. It closely matches the response of human eyes. Since the photo current increase linearly with the intensity of illumination and the spectral response of CdS cell is similar to that of the human eye, a photoconductive cell may be used for the following purposes :

1. To measure the intensity of illumination i.e., to work as light meters.
2. To work as ON-OFF switch.
3. In street lighting control.

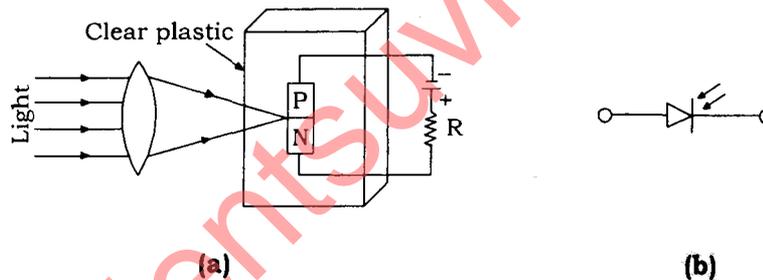


4. In camera exposure settings.
5. In counting applications.
6. For relay control.
7. In aircraft and missile tracking system.
8. In burglar alarm
9. As voltage regular.

Photo Diode : A photo diode is essentially a reverse biased p.N. junction diode which is desgined to respond to photon absorption.

A reverse biased *P-N* junction diode has a reverse saturation current which is mainly due to the flow of the minority carriers. If light is allowed to fall on such a reverse biased *p-N* junction diode, additional electron-hole pairs are generated in both *P* and *N* regions. It produces a very large change in minority carrier concentration and hence increases the reverse current through the diode. It is found that the current through the diode varies almost linearly with the light flux. hence light can be detected using a reverse biased *P-N* junction diode known as photo diode.

Construction : A photo diode consists of a *P-N* junction embedded in a clear plastic capsule as whown in fig. Radiation is allowed to fall upon one surface across the junction. Sometimes, a lens is placed on the junction side of the photo diode to focus the incident light on the surface for maximum activity. all the sides of the plastic capsule, except the illuminated one, are either painted black or enclosed in ametallic case. The semiconductor photo diode is extremely small and has dimensions of the order of a few mm fig shows the schematic symbol for a photo-diode. The inward arrows represent the incoming light.



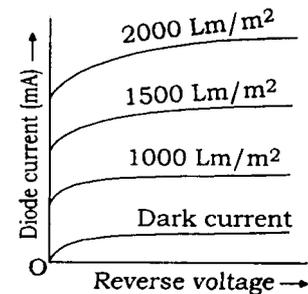
Characteristics of photo diode :

Characteristics of photo diode is given by

$$I = I_s + I_0(1 - e^{Ve/\eta kT})$$

where η is equal to 1 for Ge and 2 for Si.

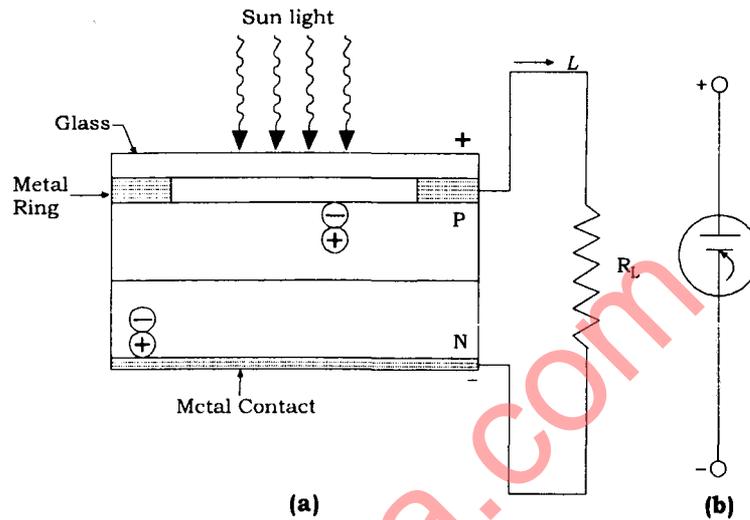
Fig. represents the volt-ampere characteristics curve for three different values of illumination for Ge photo diode. Only the curve representing the dark current pass through the origin. It may be seen that the current increases with increase in the level of illumination for given reverse voltage. Thus, the reverse current can be changed by changing the level of illumination. Thus reverse resistance of the diode can be changed likewise.



6. (b) What are solar cells? Describe in detail their construction, working Characteristics and their applications.

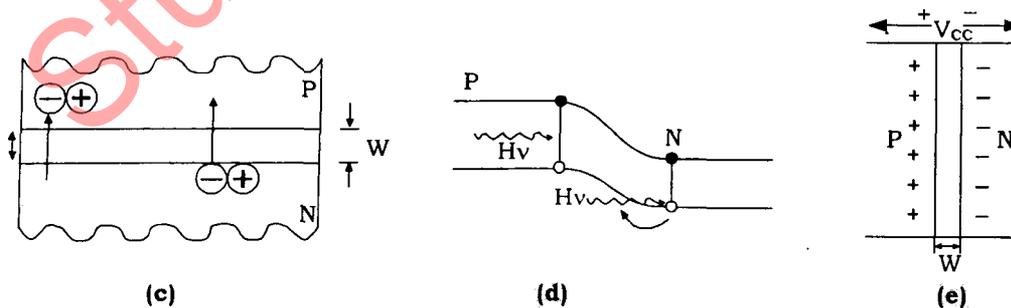
Ans. A solar or solar battery is basically a *P-N* junction diode which cconverts solar energy into electrical energy. It is also called a solar energy converter and is simplify a photo diode operated a zero bias voltage.

Constructin : A solar cell consist of a P-N junction diode generally made of Ge or Si. It may also be constructed with many other semiconducting materials like gallium arsenide (GaAs), indium arsenide (InAs) and cadmium arsenide (CdAs). The P-N diodes so formed is packed in a can with glass window on top so that light may fall upon P and N type materials. The thickness of the P-region is kept very small so that electrons generated in this region can diffuse to the junction before recombination takes place. Thickness of N-region is also kept small to allow holes generated near the surface to diffuse to the junction before they recombine. A heavy doping of P and N region is recommended to obtain a large photo voltage. A nickel plate ring is provided around the P-layer which acts a the positive output terminal. A metal contact at the bottom services as the negative output terminal. The schematic symbol of a solar cell is shown in fig. (b). The inward arrow indicates the incoming light.



Working : The working of solar cell may be understood with refrence to figures (a) to (e) and When light is allowed to fall on P-N junction diode, photons collide with valence electrons and impart them sufficient energy enabling them to leave their parent atoms. Thus electron hole pairs are generated in both the P and N sides of the junction as showing in fig. (a). These electrons and holes reach the depletion region W by diffusion fig. (c) and then separated by the strong barrier field existing there. However the minority carriers, electrons in the P-side, slide down the barrier potential to reach the N-side and the holes in the N-side move to P-side, as shown in fig. (d). There flow constitutes the minority current which is directly proportional to the illumination and also depends on the surface area bring exposed to light.

The accumulation of electrons and holes on the two sides of the junction fig. (e) gives rise to an open circuit voltage V_{oc} , which is a function of illumination

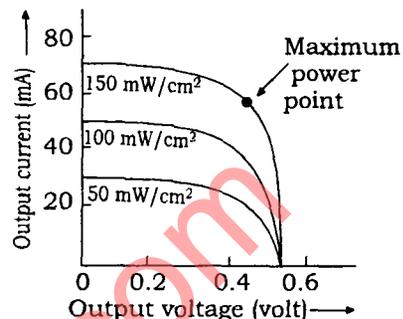


The open-circuit voltage produced for a silicon solar cell is typically 0.6 volt and the short circuit current is about 40 mA/cm² in bright noon day sunlight Power conversion efficiency of about 15% is obtained with a thin N-diffused layer into a P-wafer. Many such

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cells are interconnected to provide large quantities of electrical power the greater is the number of solar cells in a panel, the greater will be power output. solar panels providing 5 watt at 12 volt have been built to operate 24 hours and recharging of the panel is done during day light.

Characteristics : Voltage versus current characteristics of a solar cell, corresponding to different levels of illumination are shown in fig. It may be seen that 100 mW/cm² illumination, the open circuit voltage is about 0.57 volt while the short circuit current is 50 mA. Maximum power output is obtained when the cell is operated at the knee of the curve.



Uses : Solar cells are used in Satellites and space vehicles to supply power to electronic and other equipments or to charge storage batteries. They are receiving attention even for terrestrial electric power storage batteries. They are receiving attention even for terrestrial electric power generation. For this, it is planned to orbit big panels of solar cells outside the earth atmosphere for converting solar energy into electrical energy.

7. (a) Give an account of Langevin's theory of paramagnetism of a gas and point out its limitations. 15

Ans. Langevin's theory of paramagnetism of a gas :

Please see answer 6 (a), Paper December 2008.

Limitations of Langevin's theory :

(i) It could not explain the complicated dependence of susceptibility upon temperature exhibited by several paramagnetics such as highly compressed and cooled gases, very concentrated solution of salts, solid salts and crystals. Such substances obey the modified law

$$\chi = \frac{\lambda}{T - \theta}; \theta \text{ being a constant.}$$

(ii) It could not explain the intimate relation between para and ferromagnetism.

Langevin's theory applied strictly to gases where molecules are sufficiently far apart so that mutual interactions are negligible.

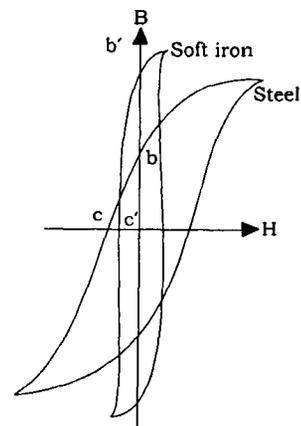
7. (b) How would you use hysteresis curve to select material for the construction of permanent magnets?

Ans. Use of hysteresis curve to select material for the construction of permanent magnets : The materials used for permanent magnet must have the following characteristics :

(i) high retentivity so that the magnet may cause strong magnetic field (even in the presence of mechanical disturbance)

(ii) high coercivity so that the magnetisation is not wiped out by strong external fields, mechanical ill-treatment and temperature changes. The loss due to hysteresis is immaterial because the magnet in this case is never put to cyclic changes.

According to these considerations steel is better for permanent magnets than soft iron. The fact that the retentivity of iron is a little greater than steel is completely outweighed by its much smaller coercivity which makes it very easy to demagnetise. The materials used for permanent magnets are cobalt-steel, tungsten-steel, chromium steel and a number of other alloys



Recently various alloys such as Alnico (Ni 18%, Al 10%, Co 12%, Cu 6%, Fe 54%) and vicalloy (composed of iron, cobalt, vanadium) etc. have been developed which are most suitable for permanent magnets.

Alnico has a coercivity of 500 oersteds = 0.05 Tesla and retains 70% of its magnetism at even a high temperature of 600°C and more than 80% in alternating fields which will be sufficient to demagnetise cobalt-steel.

Q. 8. Write short notes on the following :

- (a) Quarks and gluons (b) Group velocity and phase velocity
 (c) Meissner effect (d) London Equation

Ans. (a) Quarks and gluons :

Please see answer 4 (a), Paper December 2008.

Ans. (b) Group velocity and phase velocity :

Please see answer a (a), Paper December 2007.

Ans. (c) Meissner effect :

Meissner Effect : Meissner and Ochsenfeld (1933) showed that, if a long superconductor is cooled in a longitudinal magnetic field down to the transition temperature, then at the transition the lines of induction B are pushed out fig. (a). This phenomenon is called Meissner Effect. On the other hand if the material is cooled first below transition temperature and then placed in a magnetic field, a flux will not penetrate the material fig. (b).

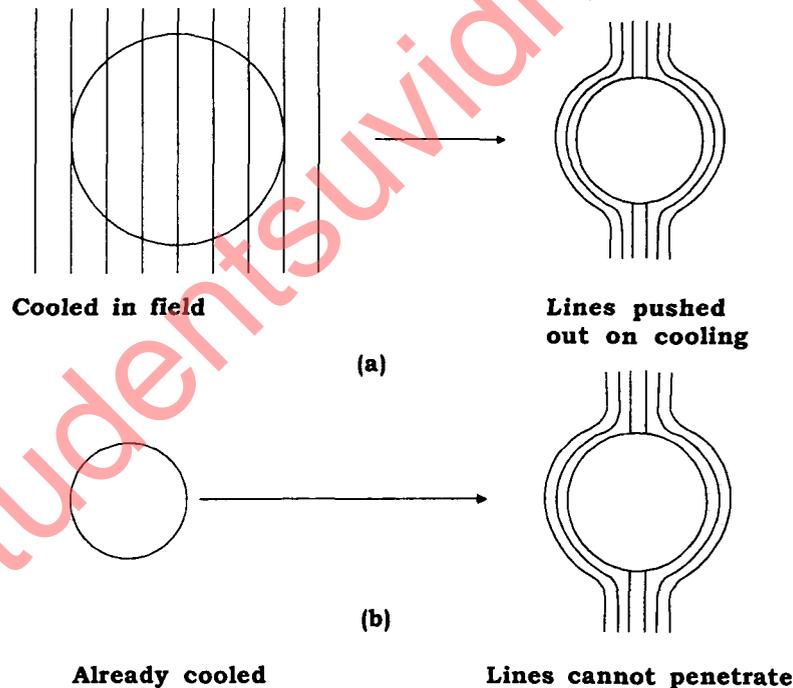


Fig. Meissner effect, a sphere (a) cooled in a constant applied magnetic field and then passes below T_c , (b) initially cooled below T_c and then placed in constant magnetic field.

The Meissner effect is a reversible process. The Meissner Effect shows that in an external applied field, a bulk superconductor behaves as if inside the specimen $B = 0$. Thus from eqn.

$$B = H + 4\pi M$$

we get

$$H = -H\pi M$$

So that susceptibility in CGS

$$\chi = \frac{M}{H} = -\frac{1}{4\pi} \quad \dots(1)$$

In MKS system

$$\chi = \frac{M}{H} = -1 \quad \dots(2)$$

a negative susceptibility *i.e.* a superconductor exhibits perfect diamagnetism.

This important result cannot be derived merely from the characterization of a superconductor as a medium of zero resistivity. From Ohm's law, $\mathbf{E} = \rho\mathbf{J}$, if the resistivity ρ goes to zero while \mathbf{J} is held finite then \mathbf{E} must be zero.

From Maxwell's equation

$$\frac{d\mathbf{B}}{dt} = -\nabla \times \mathbf{E}, \text{ we get}$$

$$\frac{d\mathbf{B}}{dt} = 0 \quad (\text{for } \mathbf{E} = 0)$$

or

$$\mathbf{B} = \text{constant}$$

which predicts that the flux through the metal cannot change on cooling through the transition. The Meissner Effect contradicts this result and suggests that perfect diamagnetism is an essential property of superconducting state. Thus the conditions defining superconducting state are

$$\mathbf{E} = 0 \text{ (absence of resistivity)}$$

$$\mathbf{B} = 0 \text{ (Meissner Effect)}$$

i.e., perfect diamagnetism and zero resistivity are two independent properties of superconducting state.

Ans. (d) London Equation :

Please see answer 7 (a), Paper December 2008.