

Roll No.

Total No. of Questions : 09]

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B.Tech. (Sem. - 4th)
MATHEMATICS - III
SUBJECT CODE : CS - 204
Paper ID : [A0495]

[Note : Please fill subject code and paper ID on OMR]

Time : 03 Hours

Maximum Marks : 60

Instruction to Candidates:

- 1) Section - A is **Compulsory**.
- 2) Attempt any **Four** questions from Section - B.
- 3) Attempt any **Two** questions from Section - C.

Section - A

Q1) (10 x 2 = 20)

- a) Define the Rolle's Theorem and Mean Value Theorem.
- b) Find the poles from $\frac{e^z}{1+z^2}$.
- c) How Taylor series is different from Laurent series?
- d) Using Picard's method to find first approximation of $\frac{dy}{dx} = x + y^2, y(0) = 1$.
- e) Explain Crank-Nicholson difference scheme for PDEs.
- f) Find the inverse Laplace transform of $\frac{1}{(s-2)^2 + 3^2}$.
- g) Define Translational transformation with example.
- h) Name few methods used to solve boundary value problems.
- i) Change $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dydx$, into polar co-ordinates.
- j) Show that the function $f(x) = \sin \frac{1}{x}$ is continuous in $\left(0, \frac{2}{\pi}\right)$.

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Section - B

(4 x 5 = 20)

Q2) Solve the equation $\frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ by the method of separation of variables.

Q3) Use residue calculus to evaluate the integral $\int_0^{2\pi} \frac{d\theta}{5 - 4 \sin \theta}$.

Q4) Prove that the function $\sin z$ is analytic and find its derivative.

Q5) Evaluate $\iint_R e^{2x+3y} dx dy$ over the triangle bounded by $x=0$, $y=0$ and $x+y=1$.

Q6) Employ Taylor's method to obtain approximate of y at $x=0.02$ for differential equation $\frac{dy}{dx} = 2y + 3e^x$, $y(0) = 0$.

Section - C

(2 x 10 = 20)

Q7) (a) Expand $f(z) = \frac{1}{z(z^2 - 3z + 2)}$ in a Laurent's series valid for the regions $|z| > 2$.

(b) Give an example of bounded function which is not Riemann integrable on $[0, 1]$.

Q8) (a) Using Runge Kutta method of order 4, find $y(0.1)$ and $y(0.2)$ given that $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$. (Take $h = 0.1$).

(b) If $A \frac{\partial^2 u}{\partial x^2} + B \frac{\partial^2 u}{\partial x \partial y} + C \frac{\partial^2 u}{\partial y^2} + D \frac{\partial u}{\partial x} + E \frac{\partial u}{\partial y} = 0$, then what are the conditions for this equation to be elliptic, parabolic and hyperbolic.

Q9) (a) Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions $u(0, y) = u(1, y) = u(x, 0) = 0$ and $u(x, a) = \sin n \pi x$.

(b) Find all singularities of the function $f(z) = \frac{z-i}{(z^2+1)(z-3)}$.

