GRAPH THEORY AND COMBINATORICS

Paper–CSE–322

Time allowed : 3 hours] *[Maximum marks : 100*

Note : *Attempt **five** questions in all, selecting at least **one** question from each unit. All questions carry equal marks.*

Unit–I

1. (a) What is a Graph ? Write and explain any two applications of graph. 6

(b) Show that the maximum number of edges in a simple graph with n vertices is n (n – 1)/2. 7

(c) Define :

(i) Isomorphism (ii) Subgraph (iii) Walk (iv) Path (v) Circuit. 7

2. (a) Prove that a given connected graph is an Euler graph if and only if all vertices of G are of even degree. 10

(b) Prove that in a complete graph with n vertices there are (n – 1)/2 edge-disjoint Hamiltonian circuits, if n is an odd number >= 3. 10

Unit–II

3. (a) Prove that a graph of n vertices is a complete graph if and only if its chromatic polynomial is

$$P_n(\lambda) = \lambda(\lambda - 1)(\lambda - 2) \ldots (\lambda - n + 1).$$ 8

(b) Prove that vertices of every planar graph can be properly colored with five colors. 12

4. (a) Prove that complete graph of five vertices is nonplanar. 10

(b) Prove that a connected planar graph with n vertices and e edges has e – n + 2 regions. 10

Unit–III

5. (a) Write down the Dijkstra's algorithm for finding the shortest path in a graph and explain with suitable example. 10

(b) Write and explain the Prim's algorithm for minimal spanning tree. 10

6. (a) What is planarity testing ? Write an algorithm for planarity testing. 10

(b) Compare the performance of various graph-theoretic algorithms. 10

Unit–IV

7. (a) Predict formula for following recurrence relation by substitution method.

$$t_n = 1, n = 2 \text{ and } tn = 2t_{n-1} + 1, n > 2$$ 10

(b) State and prove inclusion and exclusion principles. 10

8. Explain following in detail :

(i) Hadamard matrices

(ii) Error correcting codes. 20