★ Q.1 $\sum_{r=0}^{\infty} \left(4 \cdot 3^r + (-1)^r \, {}^{(5+r)}C_r \right) x^r$

$4 \cdot 3^r \quad (-1)^r \, {}^{6-1+r}C_r$

$\Rightarrow \left\{ \frac{4}{1-3x} + \frac{1}{(1+x)^6} \right\}$

becoz of $(-1)^r$

$\Rightarrow A(x) = \frac{x^3}{(1-x)^8} \Rightarrow \frac{1}{(1-x)^8} \times x^3$

$\Rightarrow {}^{8-1+r}C_r$

$a_0 \quad a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5$

$0, 0, 0, {}^7C_0, {}^8C_1, {}^9C_2 \text{ --- ---}$

$a_3 - a_0 \Rightarrow {}^7C_0 - 0 = 1$

Sequence	Numeric func^n	Generating func^n
★ $\Rightarrow$	$\frac{1}{r!}$	e^x
	r^2	$\Rightarrow \frac{x(1+x)}{(1-x)^3}$

$e^{-8x} \Rightarrow \cancel{a_r} \; a_r = \frac{1}{r!} \Rightarrow (-8)^r \Rightarrow \frac{(-1)^r 8^r}{r!}$

$\star$ $\sum_{r=0}^{\infty} \left(\frac{(-1)\,3^r}{r!} \right) x^r$

$= \underline{e^{-3x}}$

eg e^{-8x}

$\Rightarrow$

$\therefore \quad a_r = \frac{(-8)^r}{r!} = \frac{(-1)^r\, 8^r}{r!}$

$\underline{\underline{a_{12}}} = \frac{(-1)^{12}\, 8^{12}}{12!} = \frac{2^{36}}{12!}$

$\therefore \quad \sum_{r=0}^{\infty} {}^{n-1+r}C_r\, x^r = \frac{1}{(1-x)^n}$

$\Downarrow$

$\sum_{r=0}^{\infty} {}^{1-1+r}C_r\, x^r = \frac{1}{(1-x)^1}$

$\sum_{r=0}^{\infty} {}^{2-1+r}C_r\, x^r$

$\Downarrow$

$\sum_{r=0}^{\infty} {}^{(r+1)}C_r\, x^r = \frac{1}{(1-x)^2}$

$\Downarrow$

$\therefore$ or $\boxed{(r+1) = \frac{1}{(1-x)^2}}$

$\sum_{r=0}^{\infty} {}^{3-1+r}C_r\, x^r = \frac{1}{(1-x)^3}$

$\sum_{r=0}^{\infty} {}^{r+2}C_r\, x^2 \rightarrow$

$$r+2C_2 \Rightarrow \frac{1}{(1-x)^3}$$

$$\frac{(r+2)(r+1)}{2} \Rightarrow \frac{1}{(1-x)^3}$$

$$r^2+3r+2 \Rightarrow \frac{2}{(1-x)^3}$$

$$r^2+3r+2-(3r+2) \Rightarrow \frac{2}{(1-x)^3} - \left(\frac{3x}{(1-x)^2}+\frac{2}{1-x}\right)$$

$$\Rightarrow \frac{2}{(1-x)^3} - \frac{3x}{(1-x)^2} - \frac{2}{(1-x)}$$

$$\frac{2-3x(1-x)-2(1-x)^2}{(1-x)^3} \Rightarrow \frac{x+x^2}{(1-x)^3}$$

$$= \frac{x(1+x)}{(1-x)^3}$$

Que:- $\sum_{i=0}^{\infty} (i+1) z^i = ?$

a) $\frac{z}{(1-z)^2}$ b) $\frac{1}{1+z}$ c) $\frac{1}{(1-z)^2}$

$\sum_{i=0}^{\infty} i z^i + \sum_{i=0}^{\infty} z^i$ Put $i=r$ $z=x$

$$\sum_{i=0}^{\infty} (r+1) x^r = \frac{1}{(1-x)^2}$$

upper constraint Problem :-

eg :- $x_1 + x_2 + x_3 = 12$

$1 \le x_1 \le 6$, $1 \le x_2 \le 6$ $1 \le x_3 \le 6$

$$A(x) = (x^1 + x^2 + x^3 + \ldots + x^6) + (x^1 + x^2 + x^3 + \ldots + x^6) + (x^1 + x^2 + x^3 \ldots + x^6)$$

$$= (x^1 + x^2 + x^3 + \ldots x^6)^3$$

~~we know~~

*Note :-

$$\boxed{\begin{array}{l} 1 + x + x^2 + \ldots = \dfrac{1}{1-x} = (1-x)^{-1} \\[2ex] 1 + x + x^2 + \ldots + x^n = \dfrac{(1 - x^{n+1})}{1-x} \end{array}}$$

$$A(x) = x^3 (1 + x + x^2 + x^3 \ldots + x^5)^3$$

$$= x^3 \left[\frac{(1 - x^6)}{1 - x} \right]^3$$

$$= \frac{x^3 (1 - x^6)^3}{(1-x)^3} \Rightarrow x^3 (1-x^6)^3 * \frac{1}{(1-x)^3}$$

$$\Rightarrow x^3 (1-x^6)^3 \sum_{r=0}^{\infty} {}^{3-1+r}C_r x^r$$

$$\Rightarrow x^3 (1-x^6)^3 \sum_{r=0}^{\infty} {}^{r+2}C_r x^r$$

$$\Rightarrow \underline{x^3}(1-x^6)^3 \sum_{r=0}^{\infty} {}^{r+2}C_2 x^r$$

we want 12th term so after Removing x^3 we need 9th term

$$\Rightarrow \left(1-\cancel{x^{18}}-3x^6+\cancel{3x^{12}}\right) \underline{\underline{\sum_{r=0}^{\infty} {}^{r+2}C_2 x^r}}$$

$$\Rightarrow (1-3x^6) - \cancel{2\times {}^{2}C_2}$$

Q:-

5R	6B	12G

? ways can you select 12 chocolates If you must take ≥ 2 Red & $\geq$ green chocolates ?

$$x_1+x_2+x_3=12$$

$$\underline{2 \leq x_1 \leq 5} \quad \underline{0 \leq x_2 \leq 6} \quad 1 \leq x_3$$

$$(x^2+x^3+x^4+x^5) + (1+x+x^2 \ldots x^6) + (1+x+x^2 \ldots)$$

$$\Rightarrow x^2(1+x+x^2+x^3) + (1+x+x^2 \ldots x^6) + \text{''}$$

$$\left(\frac{1-x^4}{1-x}\right) * \frac{1-x^7}{1-x} * \frac{1}{1-x}$$

$$= (1-x^4)(1-x^7) \sum_{r=0}^{\infty} {}^{r+2}C_2 x^r$$

$\leq x^9$

$$= (1 - x^7 - x^4) \quad \sum {}^{r+2}C_2 x^r \qquad r=9$$

$$=$$

x^9

$r=9$

$r=2$

$r=5$

$$= {}^{11}C_2 - {}^4C_2 - {}^7C_2$$

$$55 - 6 - 21$$

Que:- ? Distribute $3n$ identical balls into 2 boxes so that each contain $\leq 2n$ balls

$$x_1 + x_2 = 3n$$

$$0 \leq x_1 \leq 2n$$

$$0 \leq x_2 \leq 2n$$

$$(1 + x + x^2 - - + x^{2n})^2$$

$$= \left[\frac{1 - x^{2n+1}}{1-x}\right]^2$$

$$x^{3n} \longrightarrow = (1 - x^{2n+1})^2 \sum_{r=0}^{\infty} {}^{2-1+r}C_r x^r$$

Date
28/08/2017

~~2n+1+r=~~

$2n+1+r = 3n$

$r = n-1$

$\Rightarrow (3n+1) - 2n$

$= n+1$

Sol^n of Recurrence Relation :-

eg :- $a_n = 4a_{n-1} + 5a_{n-2} + 6$

$\Downarrow$

$$\boxed{a_n = f(n)}$$

$a_1 = 2,\ a_2 = 3$

$a_{31} = f(n)$

:- Characteristic method :-

- Recurrence Relation
  - Determinant order R.R
    - Linear
      - LRCC
        - LRCC H (rHS = 0)
        - LRCC IH (R.H.S ≠ 0)
      - LRVC
    - Non-Linear
  - Indeterminant order R.R

Finding order :-

$a_n = 5a_{n/2} + 6$

$a_n = 4a_{n-1} + 5a_{n-2} + 6$

Highest − lowest

$n - (n-2) = n - n + 2 = 2$ nd order

$a_n = 5a_{n-1} + w$

$n - n + 1 = 1^{st}$ order

$a_n = 5a_{n/d} + 6$

order $= n - n/d \Rightarrow$ can't determine as it also dep^n of ~~w~~ d

If the Power of "a" is other than '1', then R.R is Non linear.

$$a_n^1 = 4a_{n-1}^1 + 5a_{n-2}^1 + 6 \Rightarrow \text{Linear}$$

$$a_n = 4\sqrt{a_{n-1}} \Rightarrow \text{Non-Linear}$$

$$= 4a_{n-1}^2$$

LRCC :-

$$a_n = 4a_{n-1} + 5a_{n-2} + 6$$

$$\Downarrow$$

$$\Rightarrow a_n - 4a_{n-1} - 5a_{n-2} = 6 \Leftarrow \text{LRCC}$$

a terms to left side

LRVC :-

$$a_n = n a_{n-1}$$

this is not constant

LRCCH :-

R.H.S = 0

$$a_n = 5a_{n-1} + 6a_{n-2}$$

$$a_n - 5a_{n-1} + 6a_{n-2} = 0$$

LRCC - IH :- R.H.S $\neq 0$

$$a_n = 4a_{n-1} + 5a_{n-2} + 6$$

$$a_n - 4a_{n-1} - 5a_{n-2} = 6$$

$\Rightarrow$ Questions are only on LRCC-H & LRCC-IH

Conversions :-

Non-linear to linear :-

$$a_n^2 = a_{n-1}^2 + 5 \leftarrow \text{Non-Linear}$$

$$b_n = a_n^2$$

so $b_n = b_{n-1} + 5$ linear

~~Indet~~ Inderminant to determinant :-

$$a_{2^k} = b_n$$

$$b_n = 5 \cdot b_{n-1} + 6$$

$$a_n = 5a_{n/2} + 6$$

$$n = 2^k$$

$$a_{2^k} = 5a_{2^{k-1}} + 6$$

" LRCCH " :-

(Homogeneous)

$$a_n = 5a_{n-1} - 6a_{n-2} \qquad a_1 = 2,\ a_2 = 5$$

characteristics Rules method :

Step 1st :- all a terms to left side

$$\boxed{a_n - 5a_{n-1} + 6a_{n-2}}$$

⇒ Every term, Replace by Power of t

$$t^2 - 5t + 6 = 0$$

$$t = 2, 3$$ ⇐ as it was 2nd order Polynomial so 2 roots

$$\boxed{a_n = c_1 2^n + c_2 \cdot 3^n}$$

If initial condition not given this is ans.

but $a_1 = 2$, $a_2 = 5$ given

$$a_1 = 2c_1 + 3c_2 = 2 \quad \text{--- (1)}$$

$$a_2 = 4c_1 + 9c_2 = 5 \quad \text{--- (2)}$$

$$3c_2 = 1$$

$$c_2 = 1/3 \qquad c_1 = 1/2$$

$$a_n = 1/2 \, 2^n + 1/3 \, 3^n$$

$$\boxed{a_n = 2^{n-1} + 3^{n-1}}$$

Qu ⇒ $a_n = 5a_{n-1}$, $a_1 = 3$

$$a_n - 5a_{n-1} = 0$$

$$t - 5 = 0$$

$$t = 5$$

$$a_n = c5^n \Rightarrow 3/5 \; 5^n = 3 \cdot 5^{n-1}$$

$$a_1 = 5c = 3 \Rightarrow c = 3/5 \Rightarrow a_n = 3 \cdot 5^{n-1}$$

Note :- If roots are same

$t = 2, 2$

then

$a_n = c_1 2^n + c_2 2^n$ ✗

$$\boxed{a_n = c_1 \cdot 2^n + c_2 \cdot n 2^n}$$ ✓

or If $t = 3, 3, 3$

$$\boxed{a_n = c_1 \cdot 3^n + c_2 \cdot n \cdot 3^n + c_3 n^2 3^n}$$ ✓

If

$(t-2)(t-3)^2 = 0 \quad t = 2, 3, 3$

$$\boxed{a_n = c_1 2^n + c_2 3^n + c_3 n 3^n}$$

LRCC - IH :-

$a_n = a_{n-1} + n^2 + n \quad , \quad a_1 = 1$

for 2nd degree or more, characteristic method will be lengthy so use:

Substitution method :-

$\sum n = \frac{n(n+1)}{2}$

$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$

$\sum n^3 = \left(\frac{n(n+1)}{2}\right)^2$

$a_2 = 1 + 2^2 + 2$

$a_2 = 7$

$a_3 = a_2 + 3^2 + 3$

$= 7 + 9 + 3$

$a_3 = 19$

LRCC-IH :-
- Que:

$$a_n = 5a_{n-1} - 6a_{n-2} + 5$$

$$\boxed{a_n - 5a_{n-1} + 6a_{n-2} = 5}$$

1st degree polynomial

given $a_1 = 2,\ a_2 = 8$

$$\boxed{a_n = a_n^h + a_n^p}$$

Homogeneous

Particular

Homogeneous :-

$$a_n - 5a_{n-1} + 6a_{n-2} = 0$$

Put t

$$t^2 - 5t + 6 = 0$$

$$t = 2, 3$$

$$\boxed{a_n^h = c_1 2^n + c_2 3^n}$$

can't put here, get a_n^p

Particular solution:-

$$a_n - 5a_{n-1} + 6a_{n-2} = 5$$

$$a_n^p = d$$

take a_n^p as a_n

$$a_n = d$$

Put

$$d - 5d + 6d = 5$$

$$2d = 5$$

$$d = 5/2$$

$$\boxed{a_n^p = 5/2}$$

R.H.S	a_n^p
c_0	d_0
$c_0 + c_1 n$	$d_0 + d_1 n$
$c_0 + c_1 n + c_2 n^2$	$d_0 + d_1 n + d_2 n^2$
but c_0 & d_0, c_1 & d_1, d_2 & c_2 need not to be equal	
$c \cdot a^n$	$d \cdot a^n$
$(c_0 + c_1 n)\, a^n$	$(d_0 + d_1 n)\, a^n$

Put :-

$$a_n = a^h_n + a^p_n$$

$$\boxed{a_n = c_1 2^n + c_2 3^n + 5/2}$$

given $a_1 = 2,\ a_2 = 5$

$$a_1 = 2c_1 + 3c_2 + 5/2 = 5$$

$$a_2 = 4c_1 +$$

Let :-

$$a^p_n \Rightarrow a_n - 5a_{n-1} + 6a_{n-2} = 3n+2$$

$$a^p_n = d_0 + d_1 n$$

Note:- If one of the Root is '1' then Put $a^p_n = dn$ or dn^2. By checking that a^p_n not collide with a^h_n

$$(d_0 + d_1 n) - 5[d_0 + d_1(n-1)] + 6[d_0 + d_1(n-2)] = 3n+2$$

$$d_0 - 5d_0 + 5d_1 + 6d_0 - 12d_1 = 2$$

coefficients of n $\quad d_1 - 5d_1 + 6d_1 = 3$

$$2d_0 - 7d_1 = 2 \qquad d_1 = 3/2$$

$$d_1 \qquad d_0 = 5/6$$

eg $\quad a_n^p = d.5^n$

$$a_n - 5a_{n-1} + 6a_{n-2} = 1.5^n$$

$$t^2 - 5t + 6 = 0$$

$$t = 2, 3$$

$$a_n^h = c_1 2^n + c_2 3^n$$

$$a_n^p = d.5^n$$

$$d.5^n - 5d\,5^{n-1} + 6d\,5^{n-2} = 5^n$$

divide by 5^{n-2} (lowest Power)

$$25d - 25d + 6d = 25$$

$$d = 25/6$$

$$a_n^p = \frac{25}{6}5^n$$

$$a_n = c_1 2^n + c_2 3^n + \frac{25}{6}5^n$$

$$a_n = c_1 2^n + c_2 3^n + \frac{5^{n+2}}{6}$$

eg :- $n a_n = 5n a_{n-1} - 5a_{n-1} + 6$ $a_1 = 2$

LRVC

$$n a_n = 5(n-1)\, a_{n-1} + 6$$

$$n a_n - 5(n-1) a_{n-1} = 6$$

Convert to LRCC

Put $b_n = n a_n$

$$b_n - 5 b_{n-1} = 6$$

Put t $b^{(h)}$ $t - 5 = 0$

$$t = 5$$

$$b_n^{(h)} = c \cdot 5^n$$

$b'_n = d = -3/2$ ← $d - 5d = 6$

$d = -\frac{6}{4} = -3/2$

$$b_n = c5^n - 3/2$$

$$\boxed{a_n = \frac{c5^n - 3/2}{n}}$$

=

$$a_n = \frac{\frac{7}{10} \cdot 5^n - 3/2}{n} = \frac{\frac{7}{2} 5^{n-1} - 3/2}{n}$$

$$\boxed{a_n = \frac{7 \cdot 5^{n-1} - 3}{2n}}$$

$$a_1 = \frac{5c - 3/2}{1} = 2$$

$$5c = 2 + 3/2 = 7/2$$

$$c = 7/10$$

eg:- $a_n^2 = 8 a_{n-1}^2 + 6$

$$a_n = 8 (a_{n-1})^{1/2}$$

take log If power not matching

$$\log_2 a_n = 3 + 1/2 \log a_{n-1}$$

Put $b_n = \log_2 a_n$ ~~$3 + 1/2\, b_{n-1}$~~

$$b_n - 1/2\, b_{n-1} = 3$$

:- Indeterminant order :-

$$a_n = 5a_{n/2} + 6$$

$\boxed{n = 2^K}$

$$a_{2^K} = 5a_{2^{K-1}} + 6$$

let

$$b_K = a_{2^K}$$

$$\boxed{b_K = 5b_{K-1} + 6}$$ → become IRCC-IH

put t

$$b_K - 5b_{K-1} \cancel{+6} = 6$$

$$t - 5 = 0$$

$$t = 5$$

$$b_K^h = C \cdot 5^n$$

$$b_K^p = d$$

$\cancel{d = 5}d$

$$d = -3/2$$

$$b_K = c5^n - 3/2$$

$$a_n = c5^K - 3/2$$

$$a_n = c \cdot 5^{\log_2 n} - 3/2$$

$$a_n = c\,5^{\log_2 n} - 3/2$$

$$a_n = c \cdot n^{\log_2 5} - 3/2 \Rightarrow \underline{O(\log_2 5)}$$

given $a_1 = 2$ $\Rightarrow$ $a_1 = c - 3/2 = 2$

$$c = 2 + 3/2 = 7/2$$

$$\boxed{a_n = 7/2\, n^{\log_2 5} - 3/2}$$

If.

$$a_n = 5\,a_{n/2} + n^2 \qquad a_1 = 2$$

$$a_{2^k} = 5\,a_{2^{k-1}} + n^{2k}$$

If $a_n = 5\,a_{n/2} + \cancel{3}n + n^2$

then take

$$\boxed{a_n = a^h_n + a^{p_1}_n + a^{p_2}_n}$$