"Combinatorics":-

1. Permutation & combination

2. Summation → Binomial summations → $\sum_{1}^{n}$
   → Generating functions → $\sum_{0}^{\infty}$

3. Solution of Reccurence Relation

:- keywords to identify :-

Combination	Permutation
Select	Arrange
Collection	ordered
unordered	
Set	Sequence
Committee	numbers
Interview selection	words
Set, subset	
Team selection	

	No Repeation	unlimited Repeation	Limited Repeation
Permutation	^nP_r	n^r	$\frac{n!}{n_1!, n_2! \dots n_r!}$
Combination	^nC_r	$^{n-1+r}C_r$	Generating functh (Procedure)

*Note :- By default take the objects as Distinct.

10 coins :- 10 different coins (by default) unless not given specifically.

7. Distribution Problem or Dividing Problem < OP (ordered Partition), UP (unordered partition)

8. Inclusion - Exclusion Principle :-

1) $n(A \cup B)$
2) $n(A \oplus B)$
3) $n(A-B)$
4) $n(B-A)$
5) $n(A^c \cap B^c)$
6) $n(A^c \cup B^c)$

9. Pigeon - Hole Principle :- (P-H-P)

Keywords :- "atleast" & "same"

Permutation :-

$$^nP_r = {}^nC_r * r!$$

$$= \frac{n!}{(n-r)!\,r!} * r!$$

$$\boxed{^nP_r = \frac{n!}{(n-r)!}}$$

eg :- :— 10 People 3 ~~chairs~~ chairs

$$10C_3 * 3! = \underline{10P_3} = \frac{10!}{7!}$$

(or)

:— 3 People 10 ~~chairs~~ chairs

$$\underline{10P_3}$$

Que: How many 5 letters passwords can make with distinct letters. using A-Z letters.

A - Z = 26

26 × 25 × 24 × 23 × 22 → $^{26}P_5$

26 ways, 25 ways

$$^{26}P_5$$

If

A				B
1	24	× 23	× 22	1

1) Start with A

& Not end with B

use end with B

$$n(A-B) = n(A) - (n(A \cap B))$$

$25 \times 24 \times 23 \times 22$ −

A				
1	25	24	23	22

2) Start with A or end with B

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$25 \times 24 \times 23 \times 22 + 25 \times 24 \times 23 \times 22 - 24 \times 23 \times 22$

A				B
1	24	23	22	1

3) Not starting with A

$$n(A^c) = \underbrace{n(U)}_{\text{universal set}} - n(A)$$

$$= {}^{26}P_5 - {}^{25}P_4$$

:- unconditional permutation:-

1) 5 letter password from A - Z
(Repeatation allow)

26	26	26	26	26

$\rightarrow 26^5$

eg :- outcome of tosses :-

1) 10 coin tossed

1 coin - 2 Possibility

2	2	2	-	-	

$\Rightarrow 2^{10}$

2) 10 dice tossed

only even

3

1, 2, 3, 4, 5, 6

last dice

3	6	6	6		1

$3 \times 6^8 \times 1 = 3 \times 6^8$

:- If identical (Toss or dice) are tossed then use n^r $^{n-1+r}C_r$

:- Limited Repeatation Permutation :-

MISSISSIPPI

M - 1
I - 4
S - 4
P - 2
11

$$\frac{n!}{n_1!\, n_2! \cdots n_r!}$$

$$\Rightarrow \frac{11}{4!\,4!\,2!}$$

:- Jumbles of Mississipi Not start with 's'

| S | |

$\underbrace{\qquad\qquad}_{10}$

M - 1
I - 4
S - 3
P - 2
10

$$\frac{10}{4!\,3!\,2!}$$

$$\Rightarrow \frac{11}{4!\,4!\,2!} - \frac{10}{4!\,3!\,2!}$$ Not starts with 'S'

:- # Jumbles If

| S | | P |

M - 1
I - 4
S - 4
P - 1
10

M = 1
I - 4
S - 3
P - 1
9

$$n(A \cup B) = \frac{10}{4!\,3!\,2} + \frac{10}{4!\,4!} - n(A \cap B)$$

$$\frac{9}{4!\,3!}$$

Note:- Answersheet → 5 Possibilities

Answer key → 4 Possibilities

12 books on shelf

3E 4P 5M

12! becoz identical word not used

$$\frac{12!}{3!\,4!\,5!}$$

of 3 identical E

4 identical P

5 identical M

3 Red 4 green 6 Purple flags:

$$\frac{13!}{3!\,4!\,6!}$$

type of mississippi Problems

also with Persons

5- Pair identical twins

3 Pair id triplate

A - 2

B - 2

C - 2

D - 2

E - 2

~~AA~~ FFF - 3

GGG - 3

III - 3

$$\frac{19}{(2!)^5(3!)^3}$$

:- Permutation constraints

1) Start with, End with

2) not start with $n(A^c) = n(U) - n(A)$

3) Together

Together Contraints :-

word :- "EQUATION"

Vowels	Consonent
E U A I O	Q, T, N
V	C
↓	↓
5	3

:- 1st take all vowels together :

as V(5) C

1 3 ⇒ 4!

4! × 5! → for inside vowel bundle

word :- "MISSISSIPPI"

all :- SSSS coming together

SSSS → 1

M - 1, I - 4, P - 2 } = 8 ⇒ $\frac{8!}{4!\,2!}$

$$\frac{8!}{4!\,2!\,1!} * (1) \rightarrow \text{all 'S' are same}$$

so no inside bundle Permutation

:- In MISSISSIPPI all similar letter together

SSSS - 1
IIII - 1
PP - 1
M - 1

4

$$\Rightarrow 4! \times (1) \times (1) \times (1)$$

'S' same
I same
P same

:- In EQUATION all 'V' and 'C' together

V	C
1	1
5	3
1	1
1	1

$(1+1) = 2$

$$2! \times 5! \times 3!$$

vowels Consonant

4.) all vowels not together = at least 2 seperate

:- EQUATION = 8 letters

V	C
5 (1)	3

$1 + 3 = 4!$

= 7 (all 'V' together)

= $4! \times 5!$

universal

$= (8!) - 4! \times 5!$

eg :- all vowels together and in the starting EQUATION

$\underbrace{\boxed{E \mid Q \mid U \mid A \mid I \mid O}}\ \underbrace{\boxed{Q \mid T \mid N}}$

$5! \times 3!$

5.) No two objects of the certain type together :-

:- No two consonants are together

'EQUATION'

:→ 1st arrange the vowel & put 'C' in gap

5!

O E O I O O O U O A O

6 places → (3 Consonents)

$5! \times {}^6P_3$

:- No two vowel together

_ Q _ T _ N _ only 4 Places so

No two vowels together not possible

so '0' ways.

1— 5 B, 6 Girl's

No two boys together

Method
1) Arrange girls
2) Put boys in gap

$6! \times {}^{7}P_{5}$ → 7 gaps

1— No two girls together

Arrang boys first

$5! * {}^{6}P_{6}$ → 6 gaps

eg:

a a a b b b b c c c c c

No two b's together

Put bbbb

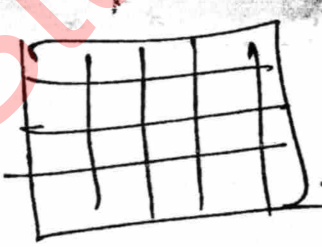
Put aaa ccccc first

gaps

$\frac{8!}{3!\,5!} * {}^{9}C_{4}$

becoz choices are same

No arrangement



→ 64 cells

Put {1, 2, - - 15}

${}^{64}P_{15}$

Put {1, 1, 1 - . . } 10's ⇒ ${}^{64}C_{15}$

⑧ ~~No~~ two Particular object ✓ together never

5 B, 4 G

~~No~~ 2 B together always

$(b_1 b_2)$ (1+1) = 1 , 3 = 4 B

4 B 4 G

$(9! - 8! \times 2!)$

Universal ... 2 boys always together

⑥ "Alternate" :-

n Boys, n girls

Let

3B 3G

B_1G_1 B_2G_2 B_3G_3
G_1B_1 G_2B_2 G_3B_3

2 ways

$= 3! \times 3! \times 2$

Boys are different ... G are diff.

$n! \times n! \times 2!$

$= 2(n!)^2$

$n_b, n+1\ G$

3B, 4G

$G_1B_1G_2B_2G_3B_3G_4$

$3! \times 4!$

$n! * (n+1)!$

eg:- How many Binary string n 0's, n's with no two 1's & no two 0's together

1's and 0's are same

so

$2 \times {}^nC_n \times {}^nC_n$

2 ways

eg:- How many strings with atmost 3 1's

${}^8C_0 + {}^8C_1 + {}^8C_2 + {}^8C_3$

atleast 3 1's

${}^8C_3 + {}^8C_4 + {}^8C_5 + \text{------} + {}^8C_8$

$2^8 - {}^8C_0 + {}^8C_1 + {}^8C_2$

Note

$\sum {}^nC_r = 2^n$

Circular permutation :-

n objects in circle :-

so they can be rotate in n ways

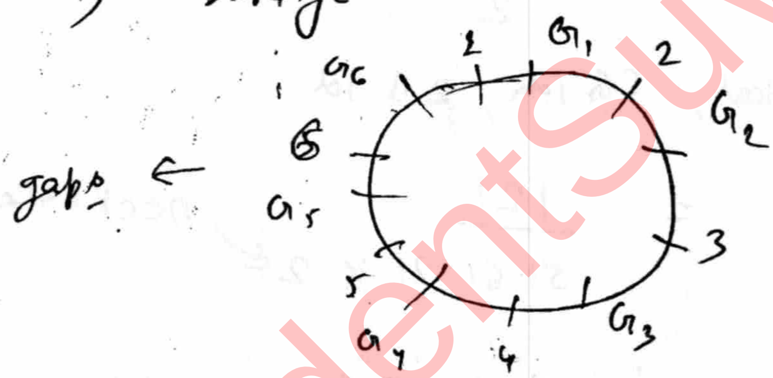
so :- Permutation $= \frac{n!}{n} = (n-1)!$

Que ? ways to arrange 5B, 6G around circular table

$= 10!$

Q ? way to " so No two Boys together

1) Arrange Girls 1st.



$5! \times {}^6P_5$

Que ? way so two girls G_1, G_2 always together

1) Put them together

so $(1+1) \overset{+4}{\Rightarrow}$ $\underline{5G + 5B}$

universal

$10! - \frac{9!}{(n-1)!} \times \frac{2!}{G_2G_1 \text{ or } G_1G_2}$

Necklace Problems :-

$$\frac{(n-1)!}{2} \Rightarrow$$ divided by two because Necklace can be ~~seen~~ seen on both side.

So clock-wise & Anti clock wise become same.

$\therefore$ so divide by 2

Que:- ? necklaces possible with 5R, 6G, 2B beans

$$\underbrace{5R,\ 6G,\ 2B}_{13} \Rightarrow \frac{12!}{2}$$

but 5R identical, 6G ide, 2B ide

$$= \frac{12!}{5!\,6!\,2! \times 2}$$ ← necklace

1— 1st Arrange them in straight line

so $\frac{13!}{5!\,6!\,2!}$ but connect the end

so $\frac{12!}{5!\,6!\,2! \times 2}$

^nC_r (Combination) :-

Combination		
Non rept	Unlimited repetition	Limited repeat
^nC_r	$^{n-1+r}C_r$	Generating funct

1) ^nC_r

Committee making / Team making / interview selection :-

★ $\boxed{^nC_r = {^nC_{n-r}}}$

$\boxed{^{n-1}C_{n-k} = {^{n-1}C_{k-1}}}$

eg: $\boxed{^{10}C_3 = {^{10}C_7}}$

10 People 3 select ↑ ↑ 10 People 7 reject

Note :- some Combination also become Permutation.

select 3 People in 10 → $^{10}C_3$

select 3 People for P, VP, Minist post ⇒ $^{10}P_3$

P	VP	M

← Arranging (Permutation)

:- Arrange in ascending order

Que:.

- A
  - 20
    - Select (10 φ)
- B
  - 20
    - Select (5 φ)

$$ {}^{20}C_{10} * {}^{20}C_5 $$

- Interview
  - 10 B
  - 15 G
  - 5
    - 2B
    - 3G

fixed

$$ {}^{10}C_2 * {}^{15}C_3 $$

- 52 cards
  - 4 king
  - 4 Queen
  - 5
    - = 3K
    - = 2 φ

$$ {}^4C_3 \times {}^4C_2 $$

or

- 52 cards
  - 4 king
  - 4 Queen
  - 5
    - = 2K
    - = 2 φ
    - 1 other card

52 − 8 = 44

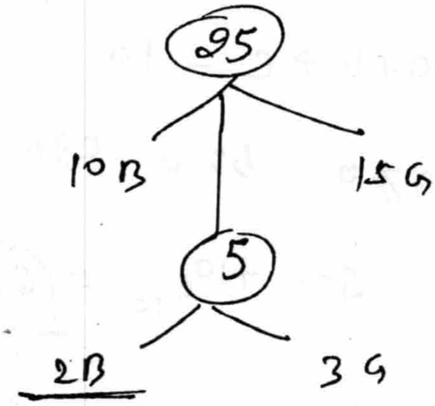
k, φ

$$ {}^4C_2 \times {}^4C_2 \times {}^{44}C_1 $$

eg:-

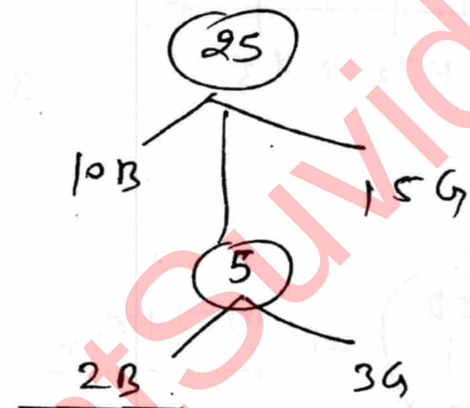
- 25
  - 10 B
  - 15 G
  - 10
    - ≥ 3 B: 4, 5, 6, 7, 8, 9, 10
    - ≤ 6 G: 6, 5, 4, 3, 2, 1

:- 1 person always included



$^{9}C_1 \times {}^{15}C_3$

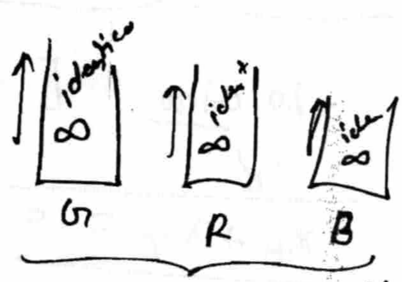
:- 1 person always is excluded



$^{9}C_2 \times {}^{15}C_3$

2.) $^{n-1+r}C_r$:-

1) Choclate Problem :-



select 10 choclates, Choclate heaven

chocolates jars are with unlimited chocs

$x_G + x_R + x_B = 10$

10 + 0 + 0 = 10

0 10 + 0 = 10

9 1 0 = 10

8 1 1 = 10 etc

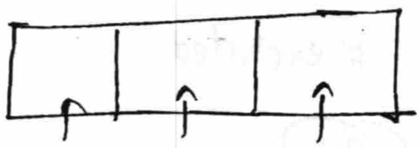
ii) Non - Negative Integral Solution :-

$$a + b + c = 10$$

$a \geq 0 \quad b \geq 0 \quad c \geq 0$ } Non-Neg Numbers

$$3-1+10C_{10} = 66$$

iii) Indentical Ball in Box Problem :-
(Distributed)



. 10 ideal balls

$x_1 + x_2 + x_3$

$\therefore 3-1+10C_{10} = 66$

Distribution

D — D

D — ID

ID — D $\Rightarrow$ $n-1+rC_r$

$\hookrightarrow$ identical object to Different object (Distinct)

4) outcome If Identical Coins & Dice :-

10 coins [for identical coins order not matters]

$$x_H + x_T = 10 \Rightarrow 2-1+10C_{10}$$

Dice :- 10 dice

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 10$$

$$6-1+10C_{10} \Rightarrow 15C_{10}$$