- Invalid Argument called "Fallacy".

6. Predicate Logic : given P(x) < true / false

7. Properties of ∀, ∃ } Very Imp
8. Translation.

1.) Proposition :- A Logical Sentence called Proposition.

:- A logical sentence can be either true or false but not both.

:- Every logical sentence contains "is" necessarly.
Declaration (in english)

Ex:-

Today is Monday
This is white
Tomorrow is Sunday
} Not Logical sentences.

He is tall

This sentence is false
I always tell lies
} :- Not logical sentence (Proposition)

This sentence is true ✓ (Proposition)

(Negative things) not logical sentences

I don't exist × Not Proposition.

Question ?
Exclamation !
Command !

} → Not logical sentence.

what's your name ?
Good Day !
stand up !

x is even
x = 4
} → Not Proposition because x is unknown.

P(x) : x is odd } ✗

It's a predicate but not a logical sentence

:- for finite amount of time, we can decide.

On Sept. 27, It will rain } ✓

Next month 27, It will rain } ✗

⇒ $\vee, \wedge, \sim, \Rightarrow, \Leftrightarrow, \oplus, \uparrow, \downarrow$

P	q
0	0
0	1
1	0
1	1

P	~P
$x>0$	$x\le 0$

$P \wedge P' = 0$
$P \vee P' = 1$
} check

★ ⇒ complement of every, all is atleast

:→ Every Person tells truth →~ atleast one tell lies or not truth

p	q	$\sim p$	$\sim q$	$p \Rightarrow q$	$p \Leftrightarrow q$	$p \oplus q$	$p \uparrow q$	$p \downarrow q$	$p \vee q$	$p \wedge q$
0	0	1	1	1	1	0	1	1	0	0
0	1	1	0	1	0	1	1	0	1	0
1	0	0	1	0	0	1	1	0	1	0
1	1	0	0	1	1	0	0	0	1	1

$$p \oplus q = \sim(p \Leftrightarrow q)$$

$$p \Leftrightarrow q = \sim(p \oplus q)$$

$$p \uparrow q = \overline{p \wedge q}$$

⇒ Neither P NOR Q that means ⇒ 'NOR'

Both are false ⇒ NOR

atleast one false ⇒ NAND

⇒ Properties of OR, AND & NOT ($\vee, \wedge, \sim$) :-

i.e. (+, ·, ')

⇒ Laws of Boolean Algebra :- $(S, +, \cdot, ')$

:- 6 Axioms   5 theorms

Axioms :-

1. Closure
2. Commutative
3. Associative
4. Distributive
5. Identity
6. complement.

(1–3: Lattice)

theorms :-

1) Law of Absorption ✓ Lattice
2) Domination
3) Double compliment
4) De-morgan's
5) Idempotent ✓ Lattice

① Closure :-

$p, q \in S$ then $p + q \in S$

$p \vee q \in S$, $p \wedge q \in S$, $\sim p \in S$, $\sim q \in S$ or → $p + q \in S$, $p \cdot q \in S$, $p' \in S$, $q' \in S$ } Closure.

② Commutative :-

$p \vee q = q \vee p$, $p \wedge q = q \wedge p$ or $p + q = q + p$, $p \cdot q = q \cdot p$

③ Associative :-

$p + (q + r) = (p + q) + r$, $p \cdot (q \cdot r) = (p \cdot q) \cdot r$ or $p \vee (q \vee r) = (p \vee q) \vee r$, $p \wedge (q \wedge r) = (p \wedge q) \wedge r$

(4) Distributive :-

$$p\cdot(q+r) = pq + pr$$
$$p+(qr) = (p+q)\cdot(p+r)$$

$(R, +, \times, 5)$
$2+(3\times5)$ } Not boolean Algebra
↳ $(2+3)\times(2+5)$ → Not Correct

(5) ~~Idempo~~ Identity :-

$p+0 = p = 0+p$ ⎫ or $p\vee0 = p = 0\vee p$
$p\cdot1 = p = 1\cdot p$ ⎭ $p\wedge1 = p = 1\wedge p$

identity for '+' = 0 ⎫ or identity for 'V' = 0
for '·' = 1 ⎭ for '∧' = 1

*** Note :- Boolean Algebra must contain atleast two element (0, 1)

so smallest boolean algebra = with 2 element.

→ 2, 4, 8, ... in 2^n ← $(S, +, \cdot, \sim)$

6 is not a boolean algebra

$(S, +, \cdot, \sim) \rightarrow |S| = 2^n \quad n \geq 1$

⑥ Complement :-

$\forall p \; \exists 0 \quad p+0 = p = 0+p \}$ ✗

$\exists 0 \; \forall p \quad p+0 = p = 0+p$ ✓

$\forall p \; \exists p' \quad p+p' = 1$ $\}$ $1+0=1$

$\quad \& \; pp' = 0$ $\quad 1\cdot 0 = 0$

$\exists_1 \; \forall p \quad p\cdot 1 = p = 1\cdot p$ ✓

Negation :-

p	$\sim p$
is	is not
$=$	$\neq$
$<$	$\geq$
$>$	$\leq$
$p \vee q$	$p \downarrow q = \sim p \wedge \sim q$
$p \wedge q$	$p \uparrow q = \sim p \vee \sim q$
★ $p \Rightarrow q$ $\hookrightarrow p' \vee q$ or $p'+q$	$p \wedge \sim q$ or $p\cdot q'$

Ex :-

p : If it rains, I carry umbrella

~p : If. It rains and I didn't carry umberella

P	$\sim P$
$P \Leftrightarrow q$	$P \oplus q$
$P \oplus q$	$P \Leftrightarrow q$
$\forall x\ P(x)$	$\exists x \sim (P(x))$
$\exists x\ f(x)$	$\forall x \sim f(x)$
$\forall x \sim f(x)$	$\exists x\ f(x)$

Ex: P: L is reg $\Leftrightarrow$ No. of M-N equivalence is finite

$\oplus$ means A or B but not both

$\sim \forall x = \exists x$

$\sim \exists x = \forall x$

Ex

1) $\sim(\forall x\ (P(x) \wedge q(x)))$

$\exists x\ (\sim P(x) \vee \sim q(x))$

2) $\sim(\exists x\ (P(x) \Rightarrow q(x))$

$\forall x\ (P(x) \wedge \sim q(x))$

3) $\forall x\ \exists y\ \ x+y = y+x$

$\sim \Rightarrow \exists x\ \forall y\ \ x+y \neq y+x$

Contrapositive :-

$P \Rightarrow q = \sim p \Rightarrow \sim q$

★ ① Law of Absorption :-

$\forall x, y \quad x + (x \cdot y) = x$

$x \cdot (x + y) = x$

$A \cup (A \cap B) = A$

$A \cap (A \cup B) = A$

Ex:- $p + pqr = p$

Ex:- $p(qr's + q'rt + p)$

$= p$

★ :- $p + p'q = p + q$

:- $x(x' + y) = xy$

③ ★ Law of Domination :-

~~$p \vee (p \wedge q) = p$~~

~~$p \wedge (p \vee q) = p$~~

$\left.\begin{array}{l} p + 0 = p \\ p \cdot 1 = p \end{array}\right\}$ identity

0 is dominating. ← $p \cdot 0 = 0 = 0 \cdot p$

$\left.\begin{array}{l} p \cdot 0 = 0 = 0 \cdot p \\ p + 1 = 1 = 1 + p \end{array}\right\}$ → Domination

1 is dominating.

④ ★ Double complement :-

$\forall p \quad (p')' = p \quad \Big\} \quad \sim(\sim p) = p$

$(A^c)^c = A$

or

$p' = q \Leftrightarrow q' = p$

$(q')' = q$

3) ★ De-morgan's Law :-

$$(p+q)' = p' \cdot q'$$
$$(pq)' = p' + q'$$

4) ★ Idempotent :-

$$p + p = p$$
$$p \cdot p = p$$

:- $p^3 = p$

$3p = p$

Biggest polynomial in p is p only

Ex:-

$p^3 + 3p + p = p$ in boolean Algebra

Domination
Double compliment
De-morgan's

Not Lattice

:- 5 Properties for Lattice:

$$\left.\begin{array}{l}\text{Closure}\\ \text{Commutative}\\ \text{Associative}\\ \text{Idempotent}\\ \text{Absorption}\end{array}\right\} 5$$

Simplification :-

$$[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$$

$$((p'+q) \cdot (q'+r)) \Rightarrow (p'+r)$$

$$[(p'+q) \cdot (q'+r)]' + (p'+r)$$

$$= pq' + qr' + p' + r$$

$$= p' + q' + r + q$$

$$= 1 + r + p' = 1 \leftarrow \text{Tautology.}$$

Que: $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (r \Rightarrow p)$

$[(p'+q) \cdot (q'+r)] \Rightarrow (r'+p)$

$[(p'+q) \cdot (q'+r)]' + r'+p$

$pq' + qr' + r'+p$

$pq' + r' + p$

$p + r'$ ← Contingency

Que which one of the following is true:

I $\quad p \Rightarrow (q \Rightarrow r) \equiv (p \Rightarrow q) \Rightarrow r$

II $\quad p \oplus (q \oplus r) \equiv (p \oplus q) \oplus r$ ✓ $\quad$ Ex-OR follow associativity & commutativity.

$p \Rightarrow (q \Rightarrow r) \equiv (p \Rightarrow q) \Rightarrow r$

III $\quad p \Rightarrow (q \vee r) \equiv (p \Rightarrow q) \vee (p \Rightarrow r)$

LHS:

$\underline{p' + q + r}$ $\qquad$ $p'+q + p'+r$

$\underline{p'+q+r}$.

IV $\quad (p \vee q) \Rightarrow r \equiv (p \Rightarrow r) \vee (q \Rightarrow r)$ $\quad$ Not equal

$(p+q)'+r$ $\qquad$ $\underline{p'+r + q'+r}$

$\underline{p'q' + r}$

Qu: V $A \cup (B - C) = (A \cup B) - (A \cup C)$

$\cancel{a + bc'}$

$a + bc'$ $(a+b) - (A+C)$

$(a+b) \cdot (a+c)'$

$(a+b)\ (\bar{a} \cdot \bar{c})$

$a\bar{c} + bc'$

$b - c = b'c$ ✗

bc'

$a - b = ab'$

$a \cap b'$

** Note :-

$\Rightarrow \{\vee, \wedge, \Leftrightarrow, \oplus\}$ (4) $\Rightarrow$ Commutative & Associative

$\Rightarrow \{\Rightarrow\}$ $\Rightarrow$ Neither associative Nor commuitative

implication

Ex:-

$p \Rightarrow (q \Rightarrow r) \neq (p \Rightarrow q) \Rightarrow r$

$p \Rightarrow (q' + r)$ $(p' + q)' + r$

$p' + q' + r \neq pq' + r$

$\Rightarrow \{\uparrow, \downarrow\} \Rightarrow$ Commutative but not associative

NAND NOR

Ex:- $(p \uparrow q) \equiv (q \uparrow p)$ ✓

$p \uparrow (q \uparrow r) \neq (p \uparrow q) \uparrow r$

$\Rightarrow$ No such operator exist which is associative but not commutative.

★ functionally Complete :-

(i) Functionally complete (FC)

(ii) Minimal functionally complete (MFC)

(iii) Smallest minimal functionally complete (SMFC)

⇒ FC : i) $\{\vee, \wedge, \sim\}$ ✓ If any set contains all these three is F.C. or either $\{\wedge, \sim\}$ or $\{\vee, \sim\}$ is F.C.

ii) $\{\vee, \sim\}$ ✓

$\{\wedge, \vee\}$ ✗ Not F.C.

iii) $\{\wedge, \sim\}$ ✓

★ (iv) $\{\Rightarrow, \sim\} = \{\vee, \sim\}$ ✓ F.C.

⇒ MFC : i) $\{\vee, \wedge, \sim\}$ ✗ Not MFC.

⇒ No subset of set must be F.C. for being M.F.C

i) $\{\vee, \sim\}$ ✓

ii) $\{\wedge, \sim\}$ ✓

iii) $\{\uparrow\}$ ✓

iv) $\{\downarrow\}$ ✓

v) $\{\Rightarrow, \sim\}$ ✓ M.F.C

} M.F.C.

By NAND :-

$\sim P = P \uparrow P$

$P \cdot q = (P \uparrow q) \uparrow (P \uparrow q)$

$P + q = (P \uparrow P) \uparrow (q \uparrow q)$

★ $P' \Rightarrow q \equiv P + q$

$\{\Rightarrow\}$ ✗ Not MFC. Not F.C.

SMFC :- smallest functionally complete.

i) $\{\uparrow\}$ ✓ NAND

ii) $\{\downarrow\}$ ✓ NOR

T → Tautology

C → Contradiction

CT → Contingency

Note :-

$\star$ unsatisfiable ⇒ Contradiction (C)

satisfiable ⇒ Contingency or Tautology

CT or T

** Notes :-

$\star$ Atleast one row is '1' & atleast one row of table is '0' ⇒ (CT)

$\star$ Atleast one row is '1' ⇒ (satisfiable) ✓ i.e. T or C.T.

$\star$ Every Row is '1' ⇒ (T) tautology

$\star$ Every Row is '0' ⇒ (C) contradiction

$\star$ Atleast one row is '0' ⇒ C or C.T. ✓ No name

~~(unsatisfiable)~~

$\star$ ≠ 0 ⇒ satisfiable i.e (T or C.T.)

(Not zero)

Note :-

$T' = C$

$C' = T$

$CT' = CT$

* Complement of tautology = Contradiction

* " Contradiction = Tautology

" ~~Contradictio~~

* Contingency = Contingency.

* Compliment of unsatisfiable = Tautalogy

Compliment of satisfiable = **C** or CT

Note :-

$T \Rightarrow SAT$ ← one way ← (reverse not true)

Every tautalogy is satisfiable.

$CT \Rightarrow SAT$

Every contingency is satisfiable (reverse not true)

$SAT \not\Rightarrow T$ or CT

$C \Leftrightarrow unSAT$

* P & P' are SAT $\Rightarrow$ P is **C.T.**

(CT) (T)

P is SAT ⇒ P is T ✗

may be T or C.T.

Normal forms :–

-: PDNF

-: PCNF

Ex:- $P \Leftrightarrow (q \Rightarrow r')$

PDNF :– (Min term summation) Principle disjunctive Normal form or

PCNF :– (Max term product)

Principle Conjunctive Normal form
Or
Principle canonical NF.

Minterm summation (PDNF) :- $pq'r + p'qr + p'q'r + p'q'r'$

* PDNF of contradiction contain '0' minterms.
* PDNF of Tautology contain 2^n minterms.

Maxterm Product (PCNF) :- $(p'+q'+r) \cdot (p+q+r) \cdot (p+q+r')$

* PCNF of contradiction contain 2^n maxterm
* PCNF of tautology contain 0 maxterm

Que:- $p \Leftrightarrow (q \Rightarrow r')$

$\Rightarrow 1 \to 1\,0 \to 0$

$1\,1\,0$

p	q	r	$q \Rightarrow r'$	$p \Leftrightarrow (q \Rightarrow r')$
0	0	0	1	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	0	0

PdNF

PCNF

$$PDNF \Rightarrow \boxed{p'qr + pq'r' + pq'r + pqr'}$$

$$PCNF \Rightarrow \boxed{(p+q+r)\cdot(p+q+r')\cdot(p+q'+r)\cdot(p'+q'+r')}$$

Precedence order :-

$$\boxed{() \geq \sim \geq \wedge \geq \vee \geq \Rightarrow \geq \Leftrightarrow}$$

Biconditional least Precedence order

Properties of PDNF & PCNF :-

★ PDNF & PCNF are unique for given Expression.

★ $x \equiv y$ iff $PDNF(x) \equiv PDNF(y)$

or iff $PCNF(x) \equiv PCNF(y)$

★ No of (PDNF) when no. of variables $= n$

when $n(PDNF) = x$

$n(PCNF) = 2^n - x$

★ x minterm in PDNF with variables = n

No. of Maxterm in PCNF = $2^n - x$

★ $m + n = 2^x$

↳ $\log_2(m+n) = x$

★ with n variables :-

? Minterms :- 2^n

? Maxterm :- 2^n

:- No. of Boolean function :- 2^{2^n}

:- How many distinct PCNF possible with n variables = 2^{2^n}

:- Distinct PDNF possible with n variables = 2^{2^n}

Two choices, either ~~to~~ take the maxterm or minterm or not so they are 2^{2^n}.

:- If the Question in DNF format:

$pq + rq' + pr + p\bar{q}r'$ then

don't ~~a~~ try to minimize, check whether all minterms are present or not

If all 8 (for n = 3) are present then It's a tautology, o/w not.

Date: 26/07/2017

* In n variables, How many of them

$$\begin{cases} \to T \to 1 \\ \to C \to 1 \\ \to CT \to 2^{2^n}-2 \\ \to SAT \to 2^{2^n}-1 \end{cases}$$

* For n variables for a given assignment the value of p, q, r

p = 1, q = 0, r = 1

# of minterms will be evaluate to T ⇒ 1

only $pq'r$

# of minterms will be evaluate to F ⇒ 2^n-1

For n variables, given assignment the Truth value

# of maxterms will be evaluate to T ⇒ 2^n-1

# of maxterms will be evaluate to F ⇒ 1.

implication & Biconditional :-

$p \Rightarrow q$

* p implies q
* If p, then q
* If p, q
* q, If p

$\Leftarrow$

(after If (p) $\Rightarrow$)

whatever comes after If (P) $\Rightarrow$

* q follows from p $\quad p \Rightarrow q$
* from p, q follows $\quad p \Rightarrow q$

from $\Rightarrow$ Left

$\Rightarrow$ follows Right side

* only If $\Rightarrow$
* If $\Leftarrow$

x only If y

$x \Rightarrow y$

x If y

$y \Rightarrow x$

* p is sufficient for q

sufficient $\Rightarrow$

$p \Rightarrow q$

$p \Leftrightarrow q$

* p biconditional q
* p iff q
* p if and only If q

p If q and p only If q

$(p \Rightarrow q) \wedge (q \Rightarrow p)$

i.e. $p \Leftrightarrow q$

* p is necessary and sufficient for q.

$(q \Rightarrow p) \wedge (p \Rightarrow q)$

* p is necessary but not sufficient for q.

p is necessary for q but and p is not sufficient for q.

$(q \Rightarrow p) \wedge \neg (p \Rightarrow q)$

$p \not\Leftrightarrow q$

$(q' + p) \wedge (pq')$

$= pq' + pq'$

$= pq' \Rightarrow p \wedge \neg q$

* q is necessary for p

$\Leftarrow$ necessary

$p \Rightarrow q$

(sufficient, & necessary in between the sentence). Put

Ex:- for q, p is necessary,

Rewrite:- p is necessary for q

$q \Rightarrow p$

Note:-

$p \Rightarrow q$

$q \Rightarrow p$

$\sim p \Rightarrow \sim q$ converse

$\sim q \Rightarrow \sim p$ convers

Note :-

but	and
when	If
whenever	If
unless	OR
* nevertheless	AND
neither p nor q	$\neg p \vee \neg q \equiv p \downarrow q$
either p or q	$p \vee q$

Ex:- p unless q $\Rightarrow$ p or q

$p \vee q$

Que:- you can not ride the roller coaster unless you are more than 4 ft height.

p: you can ride RC

q you > 4 ft

p' unless q

$p' \vee q$ i.e. $p \Rightarrow q$

Que:- You can ride the roller coaster if you are > 4 ft tall unless you are < 5 yrs age..

$(p \text{ if } q)$ unless r

$q \Rightarrow p$ unless r

$q \Rightarrow p \vee r$ or $(\sim p \Rightarrow \sim q) \vee r$

$q' + p + r$

$\neg q \vee p \vee r$

$p \vee q' \vee r$

$q \Rightarrow p \vee r$

Note :-

:- Direct $p \Rightarrow q$

:- Converse $q \Rightarrow p$

:- Inverse $\sim p \Rightarrow \sim q$

:- Contrapositive $\sim q \Rightarrow \sim p$

Ex:- I stay only if you go
p — I stay; q — you go

Converse ?

Direct :- $p \Rightarrow q$

Converse :- $q \Rightarrow p$

Ex 2:-

If $p = 0$ & $q = 0$ then $pq = 0$

Contrapositive:

i.e. $\sim q \Rightarrow \sim p$

so:- ~~pq~~ If $pq \neq 0$ then $p \neq 0$ or $q \neq 0$

*Note: for $(p \Rightarrow q) = 1$

By truth table

$p \Rightarrow q = 0$ only

$p = 1$ & $q = 0$

so

If $p = 1$, then q must be 1 for $p \Rightarrow q = 1$

so $1 \Rightarrow 1$

$1 \Rightarrow 1$

If p is $0 \Rightarrow 0,1$ (q)

p $0,1 \Leftarrow 1$ (q)

$0 \Leftarrow 0$

:-

$(p \vee q) \Rightarrow p$ ✗

$p \wedge q \Rightarrow p$ ✓

Que:-

$\forall x\,(P(x) \wedge Q(x)) \Leftrightarrow (\forall x\, P(x)) \wedge (\forall x\, Q(x))$

$\underset{1}{}$ $(x_1, x_2 \ldots x_n)$ $\qquad \underset{1}{} \qquad \underset{1}{}$

for making it true

$P(x) = 1$, $Q(x) = 1$ so it's true

:- $\forall x\,(P(x) \vee Q(x)) \Rightarrow (\forall x\, P(x)) \vee (\forall x\, Q(x))$ ✗ Not true

$\forall x\,(P(x) \vee Q(x)) \Leftarrow (\forall x\, P(x)) \vee (\forall x\, Q(x))$ ✓ True