

VIRTUAL WORK

It is an imaginary work done by the force system due to their corresponding infinite small displacement.

Principle of Virtual Work

If the system is in equilibrium, then total virtual work done by the force system due to corresponding virtual displacement are always zero.

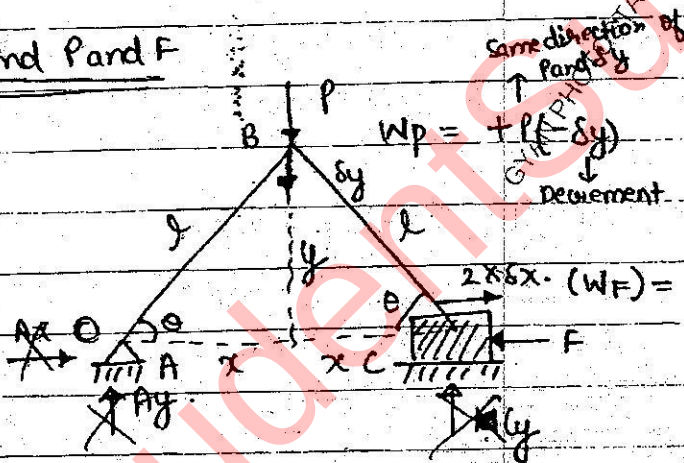
1) Frame Structure

→ Relation between external force - external force or external - internal force or external resistive reaction.

2) Portal Frame

External - External relation or External - Internal relation.

Find P and F



(Origin) - where the most unknown forces will be present and that are not required to be found.

→ Like reactions at A are not required.

→ Block resting on smooth surface

$$\text{Work} = F \times \text{distance}$$

distance (x) → Horizontal force →
Horizontal distance

Vertical Force →
Vertical distance

If Force and displacement in same direction

$$W_{VD} = (+) \text{Force} \times \text{displacement}$$

opp- direction $(W_{VD}) = (-) \text{Force} \times \text{Increment}$

(External-External relation)

$$\text{Total Virtual Work Done} = 0$$

$$T \cdot V \cdot W \cdot D = 0$$

$$+P(-\delta y) - F(+2\delta x) = 0$$

$$-P\delta y = 2F\delta x$$

$$-P(l\cos\theta) = 2F \times (-l\sin\theta)(\delta\theta)$$

$$P = 2F \tan\theta$$

$$\sin\theta = \frac{y}{l}$$

$$y = l\sin\theta$$

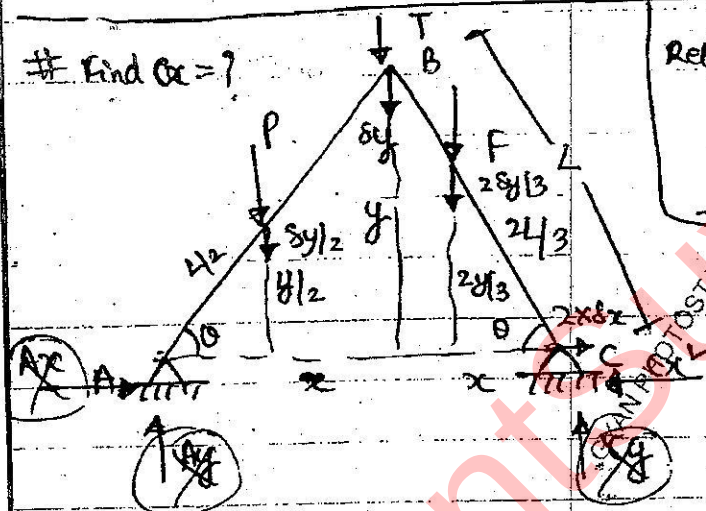
$$\delta y = l\cos\theta \delta\theta$$

$$\cos\theta = \frac{x}{l}$$

$$-\sin\theta \delta\theta = \frac{\delta x}{l}$$

$$\delta x = -l\sin\theta \delta\theta$$

Find $C_x = ?$



Relation b/w External reaction and external forces (C_x).
(Find horizontal component at hinge support C).

$$T \cdot V \cdot W \cdot D = 0$$

$$+T(-\delta y) + F(-2\delta y/3)$$

$$+P(-\delta y/2) - C_x(+2\delta x) = 0$$

$$-\left[\frac{P}{2} + T + \frac{2F}{3}\right] l\cos\theta \delta\theta = 2C_x \times (-l\sin\theta \delta\theta)$$

$$C_x = \frac{(3P + 6T + 4F)\cot\theta}{12}$$

Relation b/w external and Internal force

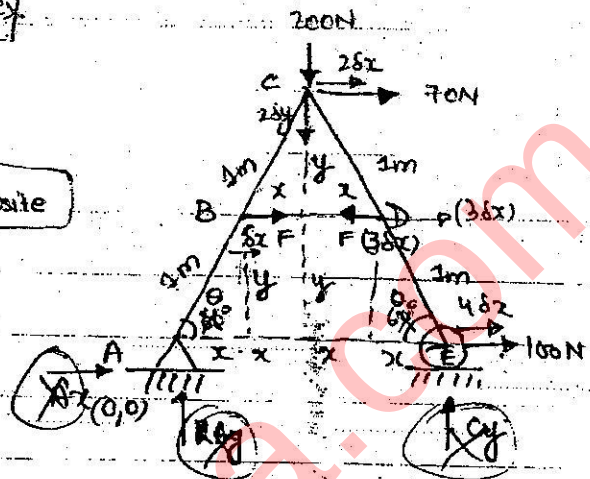
Find tension in Member BD.

Soln

Ans

Replace member BD by two equal and opposite tensile forces. Let it be F.

$$\delta W = 0.$$



$$+200(-\delta y) + (70)(+\delta x) + F(+\delta x) - F(+\delta x) + 100(+\delta x) = 0.$$

$$200(-2)(\cos \theta) \delta \theta + 540(-\sin \theta)(\delta \theta) = 2F \times (-\sin \theta)(\delta \theta).$$

$$\sin \theta = \frac{y}{r}$$

$$(\cos \theta) \delta \theta = \delta y$$

$$\cos \theta = \frac{x}{r}$$

$$-\sin \theta \delta \theta = \delta x$$

$$F = \frac{400 \cos \theta + 540 \sin \theta}{2 \sin \theta}$$

Find P in terms of W.

$$\sin \theta = \frac{y}{r}$$

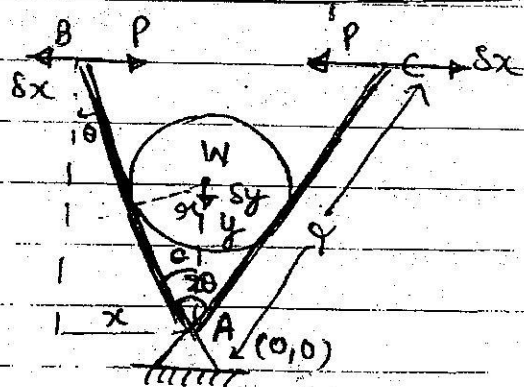
$$y = r \cos \theta$$

$$\delta y = -r \cos \theta \delta \theta$$

$$\sin \theta = \frac{x}{r}$$

$$x = r \sin \theta$$

$$\delta x = r \cos \theta \delta \theta$$



$$+W(-\delta y) - P(+\delta x) \times 2 = 0.$$

$$-W(-r \cos \theta \delta \theta) = 2P \times (r \sin \theta \delta \theta)$$

$$P = \frac{W(\cos \theta)}{2 \sin \theta}$$

→ Moment gives (Angular displacement)

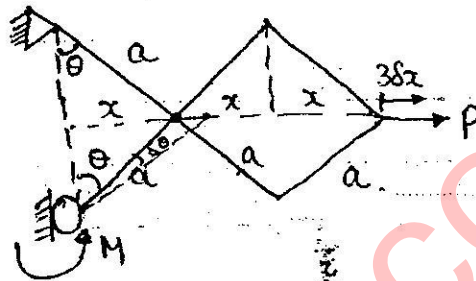
Find M in terms of P .

$$\sin \theta = \frac{x}{a} \quad x = a \sin \theta$$

$$\delta x = a \cos \theta \delta \theta$$

$$-M(\delta \theta) + P(+3\delta x) = 0$$

$$M = 3Pa \cos \theta$$



Find spring force and corresponding angle θ , if unstretched length of the spring

$$\delta U = 0 \quad +P(\delta y) - F(\delta y) = 0$$

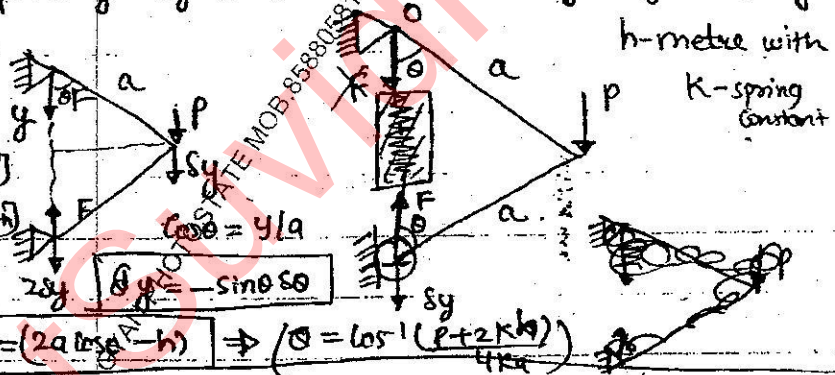
$$\text{spring force } F = \frac{P}{2}$$

$$F = k\delta = k[\text{stretched} - \text{unstretched}]$$

$$= k[\text{Force length} - \text{Length of spring in fig}]$$

$$F = k[2y - h]$$

$$\frac{P}{2k} = (2a \cos \theta - h) \Rightarrow \theta = \cos^{-1} \left(\frac{P + 2kh}{4ka} \right)$$



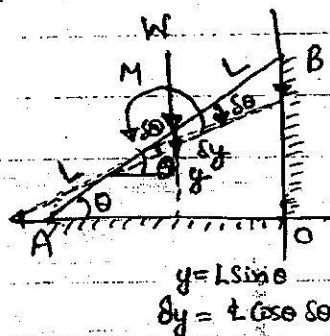
Find M in terms of W .

$$-M(-\delta \theta) + W(-\delta y) = 0$$

$$+M\delta \theta = W\delta y$$

$$M\delta \theta = W(L \cos \theta \delta \theta)$$

$$M = W \cdot L \cos \theta$$



so and
in same
direction
∴ downward
in A.
∴ $(-\delta \theta)$

Find M in terms of F .

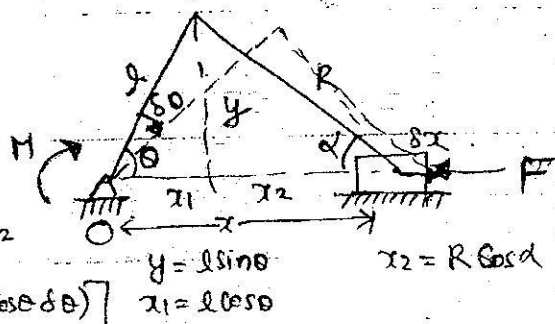
$$x_2^2 + y^2 = R^2$$

$$x_2^2 = R^2 - y^2$$

$$x_2 = \sqrt{R^2 - L^2 \sin^2 \theta} \Rightarrow x = \sqrt{R^2 - L^2 \sin^2 \theta}$$

$$x = x_2 + x_1 = L \cos \theta + \sqrt{R^2 - L^2 \sin^2 \theta}$$

$$\delta x = -L \sin \theta \delta \theta + \frac{1}{2} (R^2 - L^2 \sin^2 \theta)^{-1/2} (-2L^2 \sin \theta \cos \theta \delta \theta)$$



$$\delta x = -L \left[\sin \theta + \frac{L \sin 2\theta}{2\sqrt{R^2 - L^2 \sin^2 \theta}} \right] \delta \theta$$

Rotating M, we get:-

$$+M(-\delta \theta) - F(+\delta x) = 0$$

$$-M \delta \theta = F \delta x$$

$$-M(\delta \theta) = -L \left[\sin \theta + \frac{L \sin 2\theta}{2\sqrt{R^2 - L^2 \sin^2 \theta}} \right] \delta \theta$$

$$M = L \left[\frac{\sin \theta + \frac{L \sin 2\theta}{2\sqrt{R^2 - L^2 \sin^2 \theta}}}{1} \right]$$

Find Force F.

$$x = R \cos \theta$$

$$\delta x = R(-\sin \theta) \delta \theta$$

$$y = L \sin \theta$$

$$\delta y = L \cos \theta \delta \theta$$

$$+10(+\delta y) + 12(+2\delta y) + 14(+2\delta y) - 8(3\delta y)$$

$$- F(-\delta x) \times 2 = 0$$

$$(10 + 24 + 28 - 24) \delta y + F(\delta x) \times 2 = 0$$

$$-8\delta y + 2F\delta x = 0 \quad 38\delta y + F(2\delta x) = 0$$

$$F\delta x = 43\delta y$$

$$F = \frac{38}{2}$$

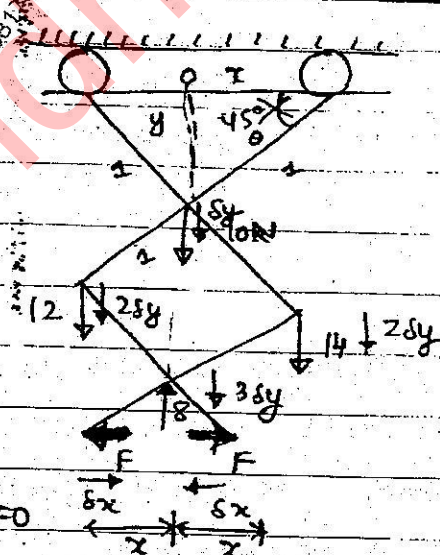
$$F = 19 \text{ N}$$

$$F = 19 \text{ N}$$

$$F = 43 / (16 \cos \theta) \times 2$$

$$-L \sin \theta \delta \theta$$

$$F = 43 / 10$$



Here it was tough to know

origin, so we can take the centre of two rolls to be origin and then distances to be calc.

PORTAL FRAME

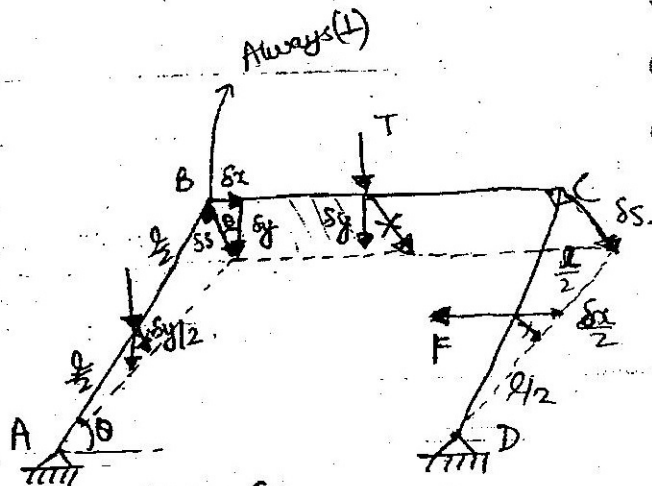
Find relation between P , T and F .

$$+ T(\delta y) + P(\delta y/2) - F(\frac{\delta x}{2}) = 0$$

$$(T + \frac{P}{2}) \delta y = F(\frac{\delta x}{2})$$

$$\boxed{(2T + P) \cot \theta = F}$$

(No increment or decrement to be taken here. as y is not decreasing or increasing)



$$\sin \theta = \frac{\delta x}{\delta s}$$

$$\boxed{\delta x = \delta s \sin \theta}$$

$$\cos \theta = \frac{\delta y}{\delta s}$$

$$\delta y = \delta s \cos \theta$$