

Digital circuits

combination ckt

Sequential ckt

⇒ Present O/p is only depend on present I/p.

⇒ Present o/p
{

 Present I/p
 Previous O/p

⇒ No feedback

⇒ feedback.

⇒ No memory

⇒ Memory.

⇒ e.g. Half Adder (HA)

⇒ e.g. FlipFlop (FF)

FA

Register

MUX

Counter.

DEMUX

COMBINATIONAL CIRCUIT

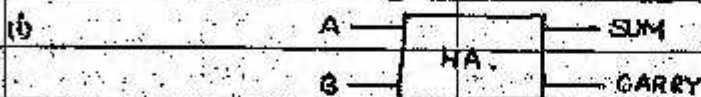
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Procedure to Design :-

- (i) Identify I/p and o/p.
- (ii) Construct truth table
- (iii) Write logical expression in SOP or POS form.
- (iv) Minimize logical expression if possible
- (v) Implement logic circuits.

(A) HALF ADDER (HA) :-



(ii) Truth table :-

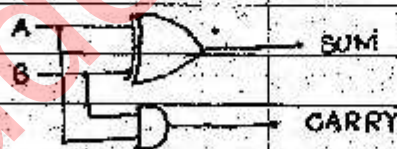
A	B	SUM	CARRY
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

(iii) logical expression :-

$$\text{SUM} = \bar{A}B + A\bar{B} = A \oplus B$$

$$\text{CARRY} = AB$$

(iv) Implement :-

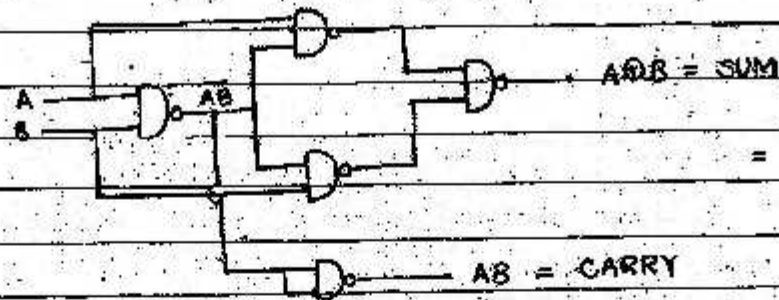


Important Ques :-

- (i) logical Expression for SUM: $A \oplus B$ CARRY: AB
- (ii) Min. no. of NAND Gate : 5
- (iii) Min no. of NOR Gate : 5
- (iv) no. of MUX : 3
- (v) no. of DECODER: 1. 2x4 decoder and 1 OR Gate

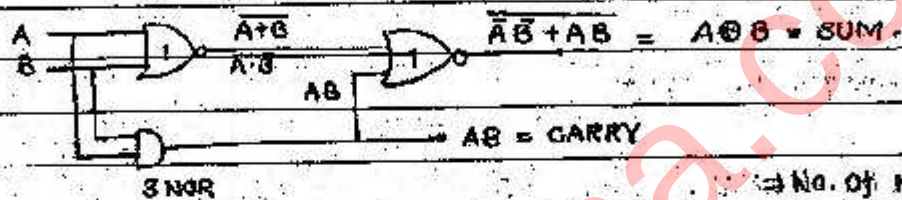
PAGE

HA using NAND gate :-



= no. of gate = 5.

HA using NOR gate :-



= No. of NOR gate 5

Total no. of gate = 2+3 = 5.

(B) HALF SUBTRACTOR :-



ii) Truth table :-

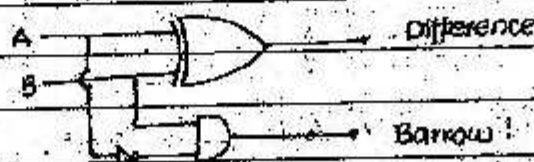
A	B	Diff	Borrow
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

iii) logical expression :-

Difference = $\bar{A}B + A\bar{B}$

BORROW = $\bar{A}B$

iv) Implement :-





Note :- (i) if control = 0. then $A \oplus 0 = A$
then $Y = AB$ and ckt is HA.

(ii) if control = 1 then, $A \oplus 1 = \bar{A}$
then $Y = \bar{A}B$ and ckt is HS.

Important Ques :-

No. of NAND gate = 5

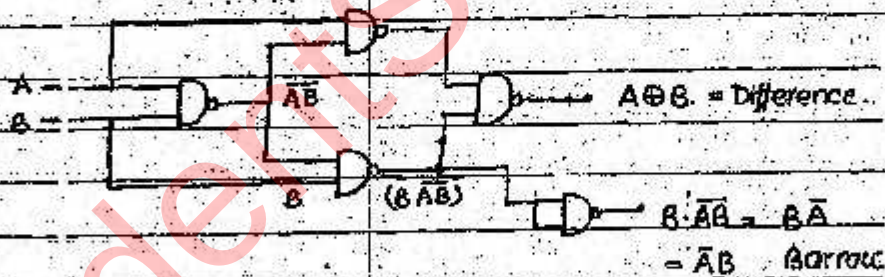
No. of NOR gate = 5

No. of MUX :- 3. (2x1 MUX)

No. of DECODER :- 1 (2x4) Decoder and 1 OR gate.

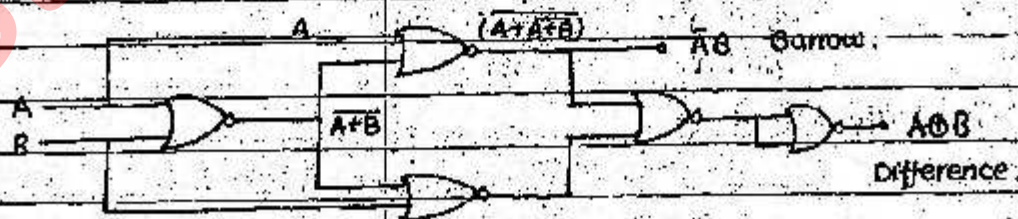
logical expression for Difference = $A\bar{B} + \bar{A}B$, Borrow = $\bar{A}B$

HS using NAND gate :-



No. of NAND gate = 5

HS using NOR gate :-

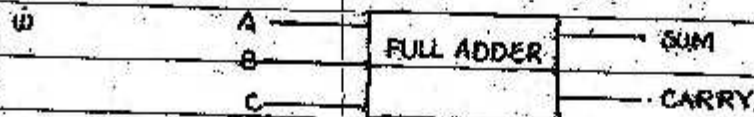


Now, $A + \bar{A}B = \bar{A}(\bar{A} + B) = \bar{A}(\bar{A} + \bar{B}) = \bar{A}B$

⇒ No. of NOR gate = 5

PAGE

(C) FULL ADDER :-



(i) Truth table:-

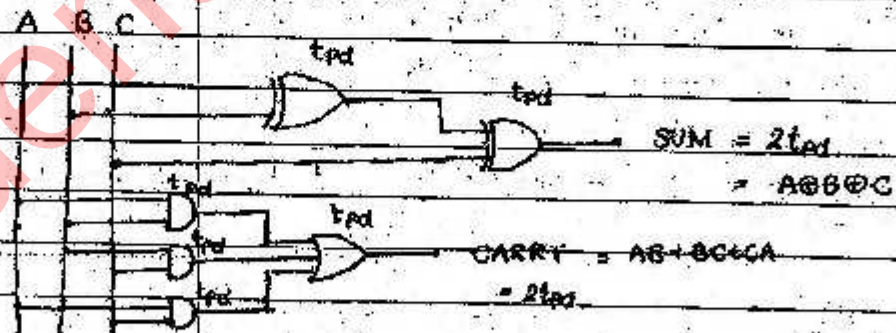
A	B	C	SUM	CARRY
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

(ii) logical expression :-

$$\begin{aligned} \text{SUM} &= \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC = A \oplus B \oplus C \\ &= \sum m(1, 2, 4, 7) \end{aligned}$$

$$\begin{aligned} \text{CARRY} &= \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC \\ &= AB + BC + AC \end{aligned}$$

⇒ The truth table of carry shows the majority of 1's function.
 $= \sum m(3, 5, 6, 7)$

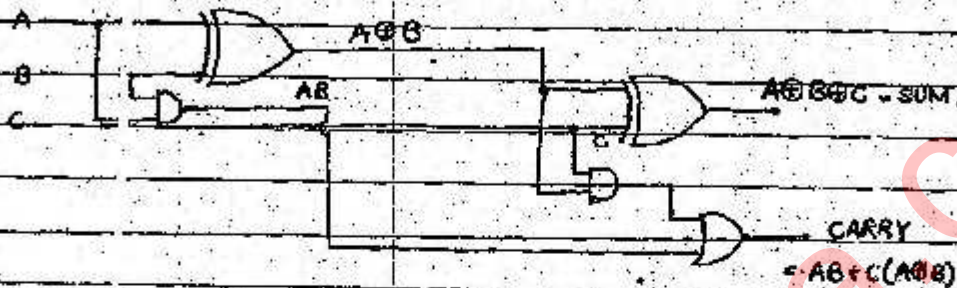


⇒ In full adder each logic gate have propagation delay of t_{pd} to provide sum or carry O/P, it requires to $2t_{pd}$ delay.

Now, $CARRY_{SUM} = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$
 $= AB(C + \bar{C}) + C(\bar{A}B + AB)$
 $= AB + C(A \oplus B)$ — (ii)

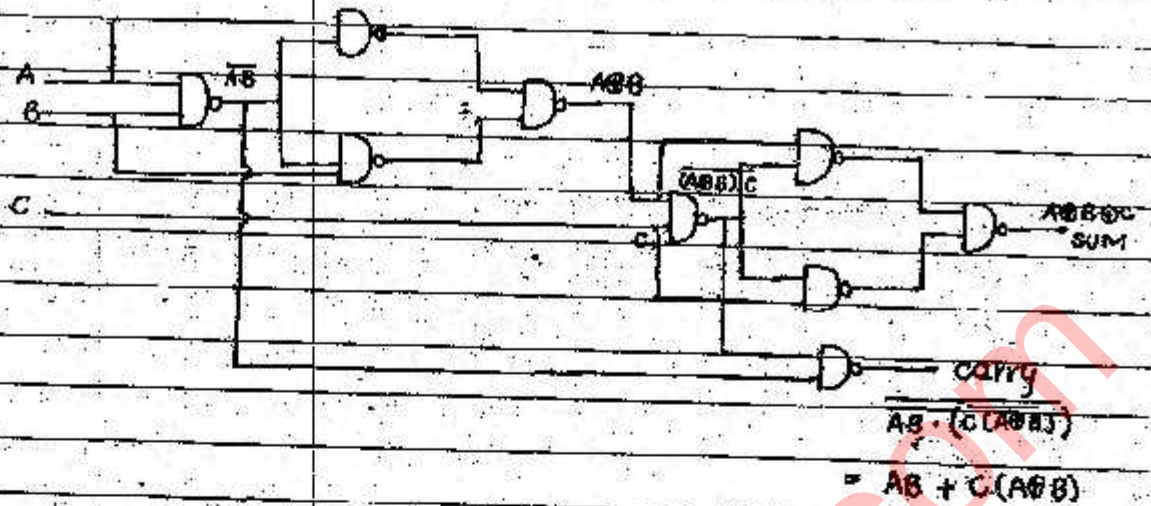
$CARRY = AB + C(A \oplus B)$

Implementation:-

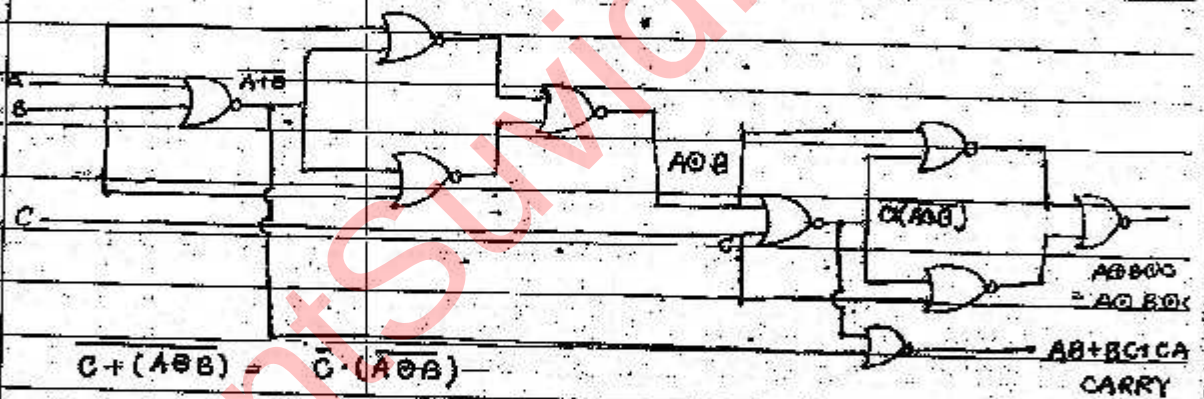


Important ques:-

- (i) logical expression for $SUM = A \oplus B \oplus C$ $CARRY = AB + BC + AC$
- (ii) No. of HA and OR gate = 2HA, 1-OR
- (iii) Min. no. of NAND = 9
- (iv) min. no. of NOR = 9
- (v) No. of MUX = 4
- (vi) No. of DECODER = 1, (3x8) Decoder and 2-OR gate

(i) Implementation of Full adder using NAND gate:-(ii) Implementation of Full adder using NOR gate:-

Since $A \oplus B \oplus C = A \oplus B \oplus C$. The the o.kt is same only
NAND is replaced by NOR.



and,

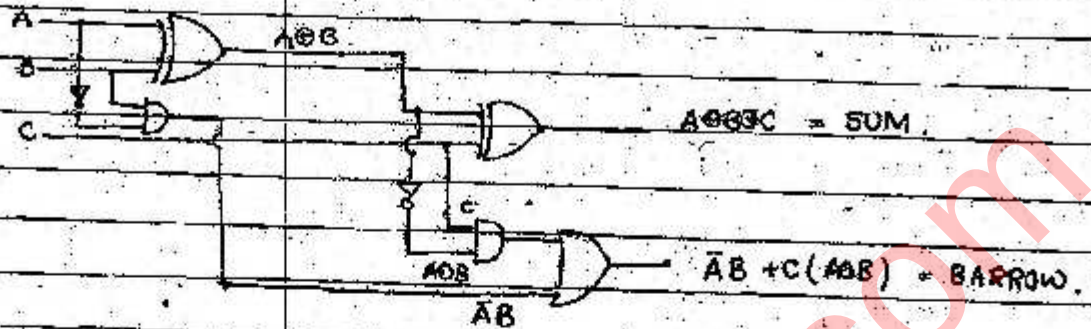
$$\begin{aligned}
 \overline{A+B} + \overline{C} \cdot (\overline{A \oplus B}) &= (\overline{A+B}) \cdot \overline{C} \cdot (\overline{A \oplus B}) \\
 &= (A+B) (C + \overline{A \oplus B}) \\
 &= AC + A\overline{A}\overline{B} + AAB + BC + B\overline{A}\overline{B} + BAB \\
 &= AC + AB + BC + AB \\
 &= C(A+B) + AB \\
 \text{CARRY} &= AB + BC + CA
 \end{aligned}$$

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Borrow expression :-

$$\begin{aligned} & \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC \\ &= \bar{A}B(C+\bar{C}) + C(\bar{A}\bar{B}+AB) \\ &= \bar{A}B + C(A\oplus B) \end{aligned}$$



⇒ Full subtractor will be implemented with 2- HS and 1 OR gate.

Important ques:-

no. of NAND gate = 9

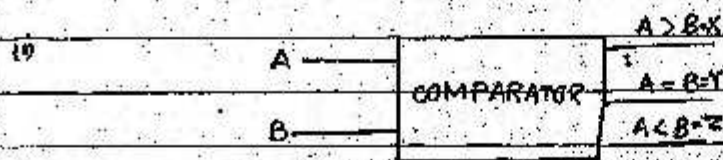
no. of NOR gate = 9

logical expression for Difference = $A \oplus B \oplus C$

logical expression for Borrow = $\bar{A}B + \bar{A}C + BC$ or, $\bar{A}B + C(A \oplus B)$

no. of MUX :-

no. of Decoder :- 1 (3x8) Decoder and 2 OR gate

COMPARATOR :-**(ii) Truth table :-**

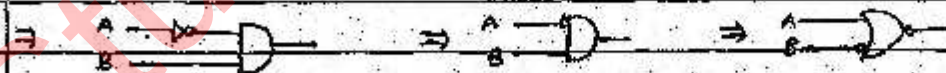
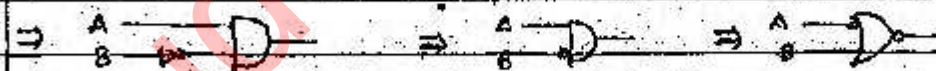
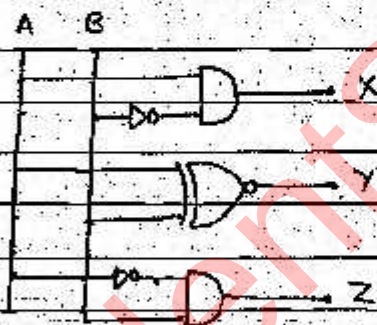
A	B	X	Y	Z
0	0	0	1	0
0	1	0	0	1
1	0	1	0	0
1	1	0	1	0

(iii) Expression :-

$$X = A\bar{B}$$

$$Y = A \oplus B = \bar{A}\bar{B} + AB$$

$$Z = \bar{A}B$$

(iv) Implementation :-

⇒ Note for equality condition $A \oplus B$ condition holds.

⇒ If A_3, A_2, A_1, A_0 are equal to B_3, B_2, B_1, B_0

Then the equality condition is

$$(A_3 \oplus B_3) (A_2 \oplus B_2) (A_1 \oplus B_1) (A_0 \oplus B_0)$$

PARALLEL ADDER :-

There are three type of adder

1. Serial adder (we write with sequential gkt)
2. Parallel adder.
3. look ahead carry adder.

⇒ In serial adder only one Full adder (FA) is used to add group of bits.

⇒ It is slowest adder.

Parallel adder :-

⇒ For 4 bit adder

:- 3 FA and 1 HA required.

:- or 4 FA is required.

FA PA FA HA

1	1	0	1
1	0	1	1
1	1	1	1

1 1 0 0 0

⇒ Parallel adder is used to add group of bits.

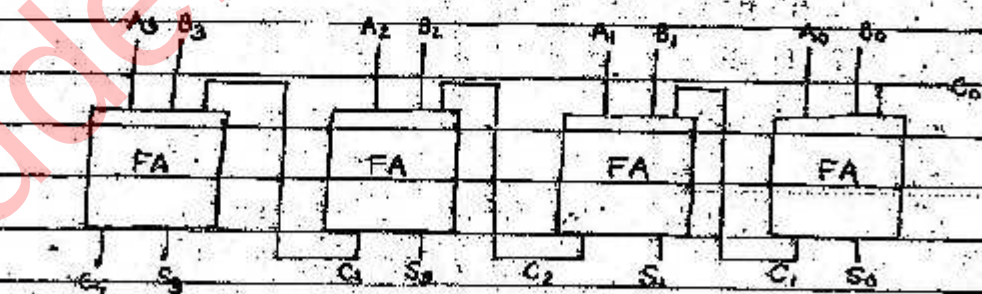
⇒ To add two N bit no. it requires (N-1) Full adder and 1 Half adder; or,

N Full adder; or,

(2N-1) Half adder and (N-1) OR gates required.

Note,

Diagram of Parallel adder:-



A ₃	A ₂	A ₁	A ₀		
1	1	0	1		
1	0	1	1		
B ₃	B ₂	B ₁	B ₀		

C ₄	S ₃	S ₂	S ₁	S ₀
CARRY	SUM			

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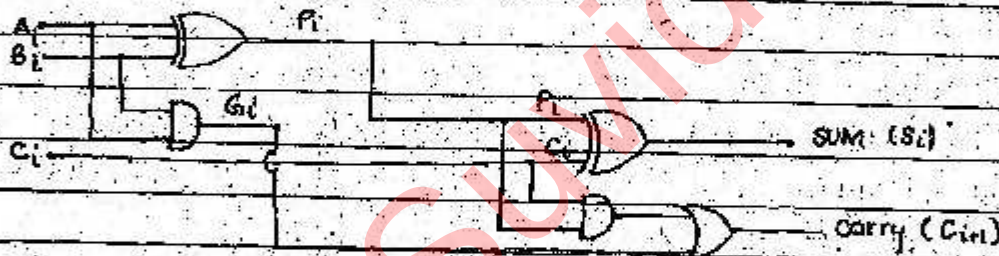
PAGE

- ⇒ Parallel adder is also called Ripple carry adder.
- ⇒ Propagation delay from I/p array to O/p array. Hence it is also known as Ripple carry adder.
- ⇒ In parallel adder each FA will provide 2 logic gate delay. In n bit parallel adder provide total delay of.

$$T_{\text{delay}} = 2nt_{\text{pd}}$$

LOOK AHEAD CARRY CIRCUIT :-

- ⇒ Disadvantage of parallel adder is carry propagation delay present.
- ⇒ As no. of bit increases - speed of operation reduced.
- ⇒ To avoid this look ahead carry adder is used.



P_i = Propagation

G_i = Generation term

$$P_i = A_i \oplus B_i$$

$$G_i = A_i \odot B_i = A_i B_i$$

$$S_i = P_i \oplus C_i$$

$$C_{i+1} = P_i C_i + G_i$$

For four (4) bit look ahead carry adder :-

$$\begin{array}{cccc} \text{I/p.} & A_3 & A_2 & A_1 & A_0 \\ & B_3 & B_2 & B_1 & B_0 \end{array}$$

$$\text{Then } S_0 = A_0 \oplus B_0$$

$$P_1 = A_1 \oplus B_1$$

$$P_2 = A_2 \oplus B_2$$

$$G_0 = A_0 B_0$$

$$S_0 = P_0 \oplus G_0$$

$$G_1 = A_1 B_1$$

$$S_1 = P_1 \oplus G_1$$

$$G_2 = A_2 B_2$$

$$S_2 = P_2 \oplus G_2$$

$$G_3 = A_3 B_3$$

$$S_3 = P_3 \oplus G_3$$

$C_{i+1} = P_i C_i + G_i$

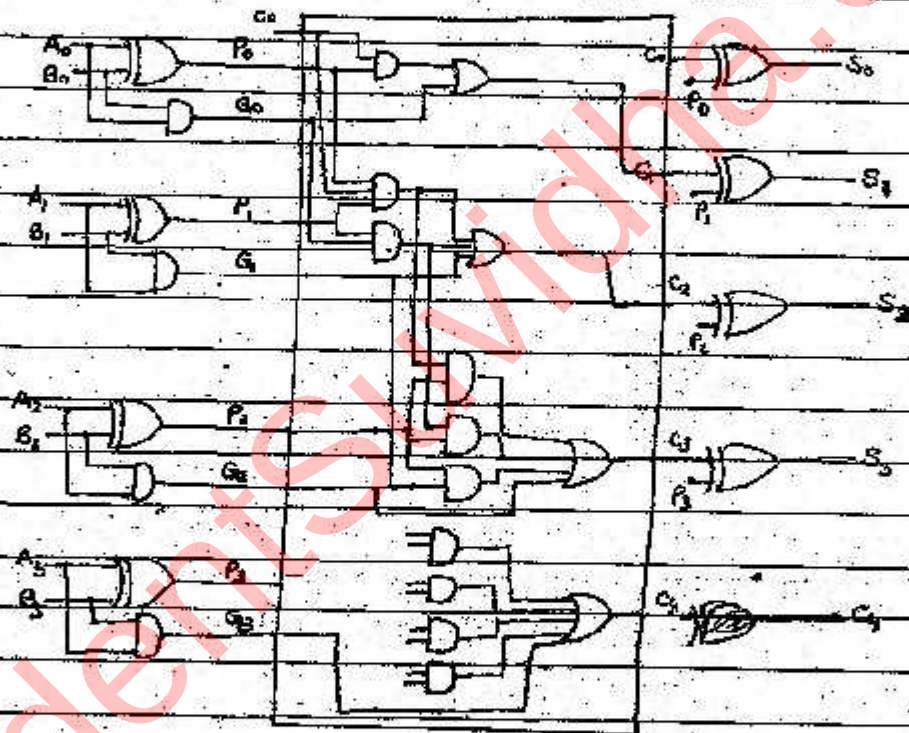
look ahead carry generator expression.

$$C_1 = P_0 C_0 + G_0$$

$$C_2 = P_1 C_1 + G_1 = P_1 (P_0 C_0 + G_0) + G_1 = P_1 P_0 C_0 + P_1 G_0 + G_1$$

$$C_3 = P_2 C_2 + G_2 = P_2 P_1 P_0 C_0 + P_2 P_1 G_0 + P_2 G_1 + G_2$$

$$C_4 = P_3 C_3 + G_3 = P_3 P_2 P_1 P_0 C_0 + P_3 P_2 P_1 G_0 + P_3 P_2 G_1 + P_3 G_2 + G_3$$



$$\Rightarrow \text{Total no. of AND gate inside} = 1+2+3+4 = \frac{n(n+1)}{2} = \frac{4 \times 5}{2} = 10$$

$$\text{no. of AND Gate} = \frac{n(n+1)}{2}$$

$$\text{no. of OR Gate} = n$$

$$\Rightarrow \text{Total propagation delay} = 2t_{pd}$$

$\Rightarrow$ This is faster than parallel adder.

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FULL SUBTRACTOR :-



iv) Truth table :-

A	B	C	Diff (A-B-C)	BARROW
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

v) Logic expression :-

$$\text{Diff} := \sum m(1, 2, 4, 7)$$

$$= A \oplus B \oplus C$$

$$\text{BARROW} = \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + ABC$$

$$= BC + \bar{A}(\bar{B}C + B\bar{C})$$

$$= BC + \bar{A}(B \oplus C)$$

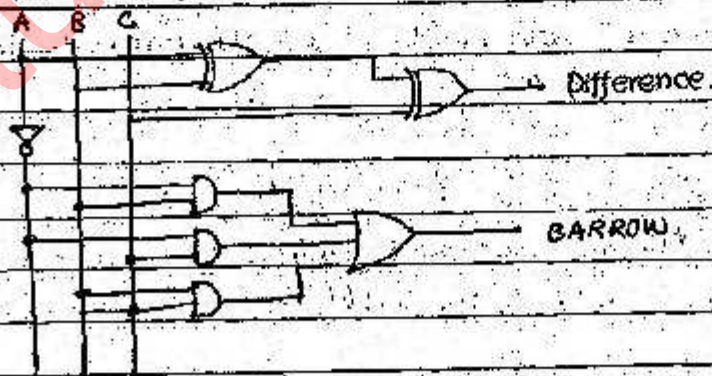
add $\bar{A}BC$ two more :-

$$= \bar{A}B(\bar{C} + C) + (\bar{A} + A)BC + C\bar{A}(B + \bar{B})$$

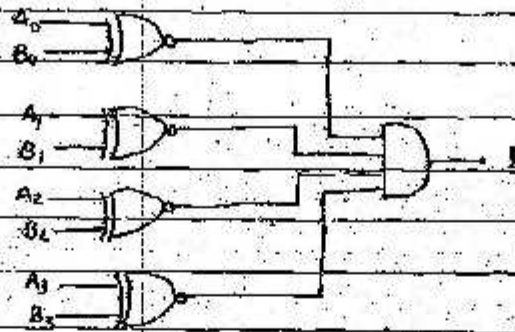
$$= \bar{A}B + \bar{A}C + BC$$

$$= \sum m(1, 2, 3, 7)$$

vi) Implementation :-



Then the ckt is :-



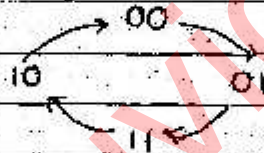
K-MAP :-

⇒ It is used when o/p is 0, 1, and X (don't care)

⇒ In K-MAP gray code representation is used

⇒ K-map is graphical representation

⇒ Gray Code representation



⇒ each successive term is changed by only one bit.

Two variable - A	MSB \ LSB B	
	0	1
0	0	1
1	2	3
	00	01
	10	11

(For Two variable)

For Three variable :-

MSB A \ LSB B	C			
	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	8	9	11	10
10	12	13	15	14

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PAGE

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four variable:-

DATE

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

⇒ Minimize:-

Q:- $f(A, B) = \sum m(0, 2, 3)$

Sol:-

A \ B	0	1
0	1	1
1	1	1

$f(A, B) = A + \bar{B}$ (+ is put due to SOP form)

Q:- $f(A, B) = \sum m(1, 2, 3)$

Sol:-

A \ B	0	1
0	0	1
1	1	1

$f(A, B) = A + B$

Q:- $f(A, B) = \sum m(0, 1, 2, 3)$

Sol:- $f(A, B) = 1$

⇒ In k-map if all are one means the function is 1.

Q:- $f(A, B) = \sum m(1, 3) + \sum d(2)$

Sol:-

A \ B	0	1
0	0	1
1	1	1

$f(A, B) = B$ (no need of any gate)

Alternate

PAGE

Q: $f(A, B) = \sum m(0, 3) + \sum d(2)$

Sol:

A \ B	0	1
0	1	0
1	0	1

$f(A, B) = A + B$

Q: $f(A, B) = \sum (0, 3) + \sum d(2, 1)$

Sol:

A \ B	0	1
0	1	0
1	0	1

$f(A, B) = 1$

- ⇒ In SOP form if all are 1's means o/p is 1.
 ⇒ All are don't care means don't care.

Three variable :-

Q: $f(A, B, C) = \sum m(1, 3, 5, 7)$

Sol:

A \ BC	00	01	11	10
0	0	1	1	0
1	0	1	1	1

$f(A, B, C) = C$

Q: $f(A, B, C) = \sum m(0, 1, 3, 6)$

Sol:

A \ BC	00	01	11	10
0	1	1	0	0
1	0	0	0	1

$f(A, B, C) = \bar{A}\bar{B} + \bar{A}C + AB\bar{C}$

Q: $f(A, B, C) = \sum m(1, 3, 6, 7)$

Sol:

A \ BC	00	01	11	10
0	0	1	1	0
1	0	0	1	1

⇒ if we take BC then it is redundant term and it must be removed.

Procedure :-

- (i) Octets
- (ii) Quads
- (iii) Pairs
- (iv) Single term
- (v) Remove redundant

Priority.
↓

Q:- $f(A, B, C) = \sum m(0, 1, 2, 4, 7)$

Sol:-

A \ BC	00	01	10	11
0	1	1	1	0
1	1	0	1	1

$$= \bar{A}\bar{B} + \bar{A}\bar{C} + \bar{B}\bar{C} + ABC$$

Q:- $f(A, B, C) = \sum m(0, 1, 5, 6, 7)$

Sol:-

A \ BC	00	01	10	11
0	1	1	0	0
1	0	1	1	1

$$\begin{aligned} f(A, B, C) &= \bar{A}\bar{B} + \bar{B}\bar{C} + AB \\ &= \bar{A}\bar{B} + AB + AC \end{aligned} \quad \text{two sol:}$$

⇒ K-map provide minimize expression but not necessarily unique. i.e. two sol^s also.

Q:- $f(A, B, C) = \sum m(0, 1, 2, 5, 7) + \sum d(3, 6)$

Sol:-

A \ BC	00	01	10	11
0	1	1	0	1
1	0	1	1	1

$$f(A, B, C) = \bar{A} + C$$

Q:- $f(A, B, C) = \sum m(0, 6, 7) + \sum d(3, 5)$

Sol.

A \ BC	BC	BC	BC	BC
A	1	1	X	1
A		X	1	1

$$f(A, B, C) = \bar{A}\bar{B} + AB$$

Q. $f(A, B, C) = \sum m(0, 1, 6, 7) + \sum d(3, 4, 5)$

Sol.

A \ BC	BC	BC	BC	BC
A	1	1	*	*
A	*	*	1	1

$$f(A, B, C) = \bar{B} + AB$$

Four Variable :-

Q. $f(A, B, C, D) = \sum m(0, 1, 3, 5, 7, 8, 9, 11, 13, 15)$

Sol.

AB \ CD	CD	CD	CD	CD
AB	1	1	1	
AB	1	1	1	1
AB		1	1	1
AB	1	1	1	1

$$f(A, B, C, D) = D + \bar{B}\bar{C}$$

Q. $f(A, B, C, D) = \sum m(0, 1, 4, 5, 8, 9, 13, 15)$

Sol.

AB \ CD	CD	CD	CD	CD
AB	1	1		
AB	1	1		
AB		1	1	
AB	1	1	1	

$$f(A, B, C, D) = \bar{A}\bar{C} + \bar{B}\bar{C} + ABD$$

Q. $f(A, B, C, D) = \sum m(0, 2, 8, 10, 14) + \sum d(5, 15)$

Q:-

Sol:-

	AB	CD	AB	CD	AB	CD
AB	1				1	
AB		*				
AB			*			
AB	1					

$$f(A, B, C, D) = \bar{B}\bar{D} + AC\bar{D}$$

POS (Product of Sum) :-

Simplify :-

Q:-

Q:- $f(A, B) = \pi M(0, 2, 3)$

Sol:-

A \ B	B, 0	B, 1
0, A	0	
1, A	0	0

$$f(A, B) = B \cdot \bar{A}$$

Q:- $f(A, B) = \pi M(0, 3) + \pi d(1)$

Sol:-

A \ B	B	$\bar{B}$
A	0	X
$\bar{A}$		0

$$f(A, B) = A + \bar{B} = A\bar{B}$$

Q:- $f(A, B, C) = \pi M(0, 1, 3, 5, 7)$

Sol:-

A \ B+C	B+C	$B+\bar{C}$	$\bar{B}+\bar{C}$	$\bar{B}+C$
A 0	1	1	1	
A 1		1	1	

$$\begin{aligned} f(A, B, C) &= \bar{C} + (A+B) \\ &= \bar{C} (A+B) \end{aligned}$$

Q:- for the K-map minimize POS expression is

A \ BC	BC	B+C	B+C	B+C	B+C
A	0	X	X	1	
A	1	1	0	X	

$$f(A, B, C) = (B+C)(B+C)$$

⇒ The two function are same if the position of 1's are 0's are same in K-map. and if the 1's place 0 are placed and at 0's place 1's are placed then the function is complement to each other.

⇒ Problem - 26 - Page - 13

Q:- $W = R + \bar{P}A + \bar{R}S$

$X = P\bar{A}\bar{R}\bar{S} + \bar{P}\bar{A}\bar{R}\bar{S} + P\bar{A}\bar{R}\bar{S}$

PA \ RS	RS	RS	RS	RS
PA		1	1	1
PA	1	1	1	1
PA		1	1	1
PA		1	1	1

PA \ RS	RS	RS	RS	RS
PA	1			
PA				
PA	1			
PA	1			

$Y = RS + PR + \bar{P}\bar{A} + \bar{P}\bar{A}$

$Z = R + S + \bar{P}\bar{A} + \bar{P}\bar{A}$

PA \ RS	RS	RS	RS	RS
PA	0	0	1	0
PA	1	1	1	1
PA	1	1	0	0
PA	0	0	0	0

PA \ RS	RS	RS	RS	RS
PA	0	1	1	1
PA	1	1	1	1
PA	0	1	0	0
PA	0	1	1	1

⇒ Then $W = Z = \bar{X}$

NOTE $\Rightarrow$ If Truth table is :-

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	0

(The method for writing the expression from Truth table)

Then,

$$\begin{aligned} Y &= \bar{A}\bar{B} * 0 + \bar{A}B * 1 + A\bar{B} * 0 + AB * 0 \\ &= \bar{A}B + 0 \\ &= B(\bar{A} + 0) \\ &= B(\bar{A}) \end{aligned}$$