B.Sc. 4th Semester (Hons.) New Scheme
Examination, May–2016
PHYSICS
Paper–Phy–401
Mathematical Physics–IV

Time allowed : 3 hours] *[Maximum marks : 40*

Note : *Attempt **five** questions in all, selecting at least **two** questions from each unit.*

Unit–I

1. Evaluate by using Beta and Gamma functions :

(a) $\int_0^{\pi/2} \sin^2\theta \cos^4\theta \, d\theta$

(b) $\int_0^1 \frac{dx}{3\sqrt{1-x^3}}$

2. (a) Express $f(x)=7x^4+6x^3+3x^2+x-6$ in terms of Legendre polynomials.

(b) Prove that

$$(2n+1)\,P_n(x)=P'_{n+1}(x)-P'_{n-1}(x)$$

3. (a) Show that $\int x J_0^2(x)\,dx$

$$=\frac{1}{2}x^2[J_0^2(x)+J_1^2(x)]$$

(b) Find the solution of the differential equation (By Bessel's Method)

$xy'' + y' + y = 0$ using the substitution $z = 2\sqrt{x}$.

4. (a) Prove that

$$\int_{-\infty}^{\infty} \exp(-x^2)\, H_n(x)\, H_m(x)\, dx = 2^n\, n!\sqrt{\pi}$$

if $m = n$

(b) Prove that

$$H_n(x) = 2^n \left\{ \exp\left(-\frac{1}{4}\frac{d^2}{dx^2} \right) \right\} x^n.$$

Section–II

5. Solve the p.d.e

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

for which $u(0, t) = 0 = u(\ell, t) = 0$

$$\left(\frac{\partial u}{\partial t} \right)_{t=0} = u_0 \sin^3 \frac{\pi x}{\ell}$$

6. Solve $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ satisfying boundary condition $u(0, t) = u(\ell, t) = 0, t > 0$ and initial conditions : $u(x, 0) = \frac{100}{\ell} x$.

7. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, which satisfies the following boundary conditions $u(0, y) = u(a, y) = u(x, 0) = 0$ and $u(x, b) = \sin \frac{n\pi x}{a}$

8. Solve

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$