

* Line coding

The digital data can be transmitted by various transmissions or line codes such as on-off, polar, bipolar and so on. This is called line coding.

1. Transmission bandwidth

For a given line-coding, the transmission bandwidth must be as small as possible.

2. Power efficiency

For a given bandwidth and a specified detection error probability, the transmitted power for a line code should be as small as possible.

3. Error detection and correction capability

It must be possible to detect and preferably correct detection errors.

4. Favourable power spectral density

It is desirable to have a zero power spectral density (PSD) at $\omega = 0$.

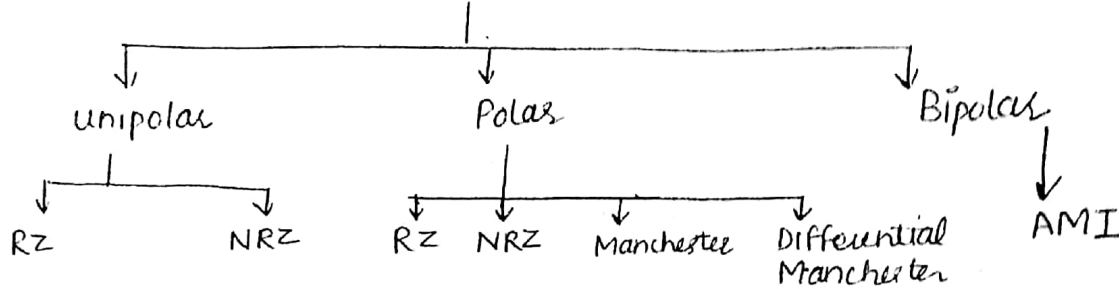
5. Adequate timing content

It must be possible to extract timing or clock information from the signal.

6. Transparency

It must be possible to transmit a digital signal correctly regardless the pattern of 1's and 0's.

* CLASSIFICATION OF LINE CODES



- ① unipolar : These have only one voltage level other than zero. Hence the encoded signal will have either $+A$ volts value or 0.
- ② Polar : These use 2 voltage levels other than zero such as $+A/2$ and $-A/2$ volts.
- ③ Bipolar codes : Bipolar coding uses three voltage levels positive, negative and zero which is similar to polar codes.

* Various line codes :

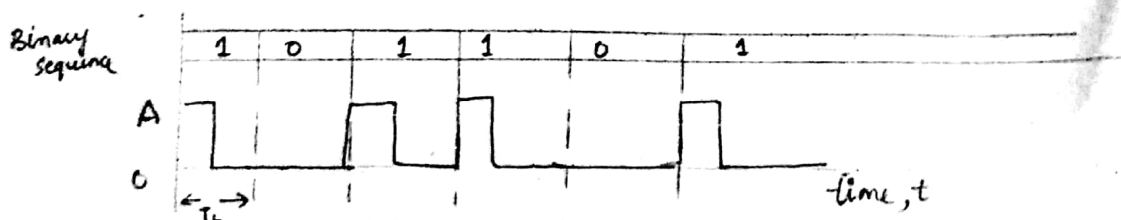
- (i) unipolar return to zero : zero value of waveform = '0'
 A volts value of waveform = 1

If symbol '1' is transmitted, then

$$x(t) = \begin{cases} A & \text{for } 0 \leq t < T_b/2 \\ 0 & \text{for } T_b/2 \leq t < T_b \end{cases}$$

and if symbol '0' is transmitted

$$x(t) = 0 \text{ for } 0 \leq t < T_b$$



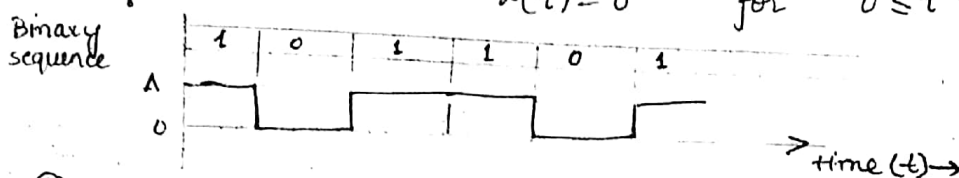
② unipolar NRZ

If symbol '1' is transmitted, we have

$$x(t) = A \quad \text{for } 0 \leq t < T_b$$

If symbol '0' is transmitted

$$x(t) = 0 \quad \text{for } 0 \leq t < T_b$$



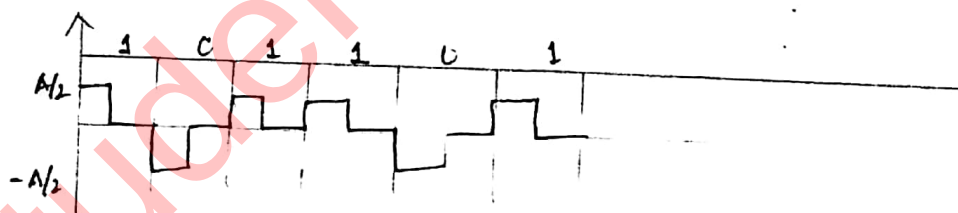
③ Polar RZ

If symbol '1' is transmitted, then

$$x(t) = \begin{cases} +A/2 & \text{for } 0 \leq t < T_b/2 \\ 0 & \text{for } T_b/2 \leq t < T_b \end{cases}$$

and if symbol '0' is transmitted, then

$$x(t) = \begin{cases} -A/2 & \text{for } 0 \leq t < T_b/2 \\ 0 & \text{for } T_b/2 \leq t < T_b \end{cases}$$



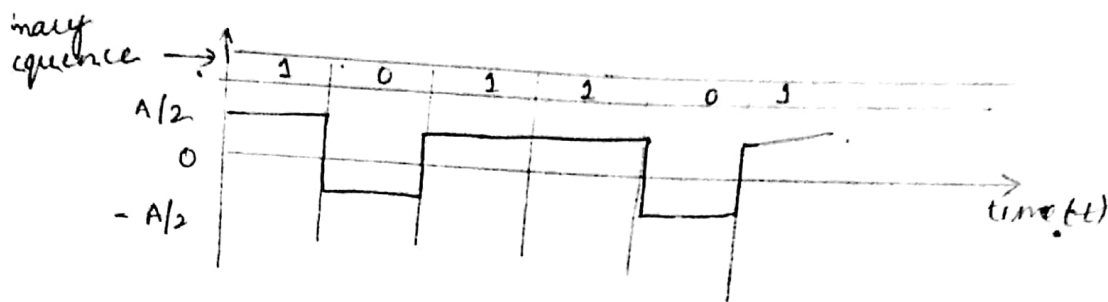
④ Polar NRZ

If symbol '1' is transmitted

$$x(t) = +A/2 \quad \text{for } 0 \leq t < T_b$$

If symbol '0' is transmitted

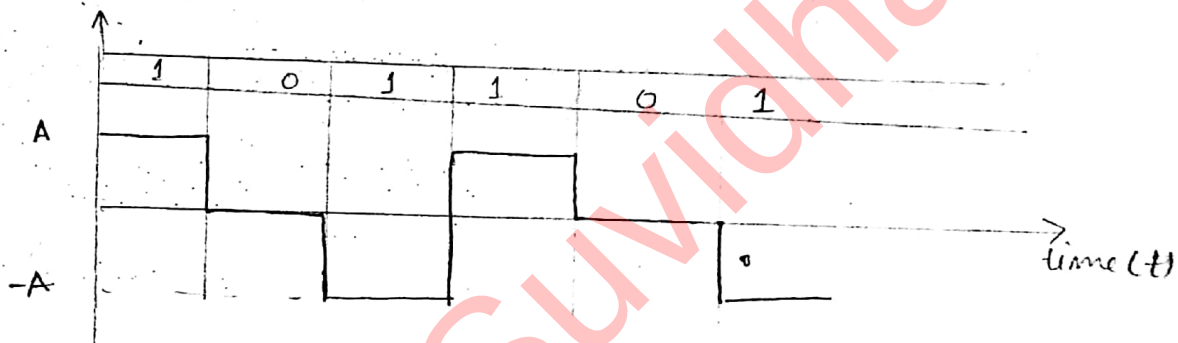
$$x(t) = -A/2 \quad \text{for } 0 \leq t < T_b$$



⑤ Bipolar NRZ [Alternate mark inversion (AMI)]

In this format, the successive '1's are represented by pulses with alternate polarity and 0's are represented by no pulse.

[Advantage: The ambiguities due to transmission sign inversion are eliminated]



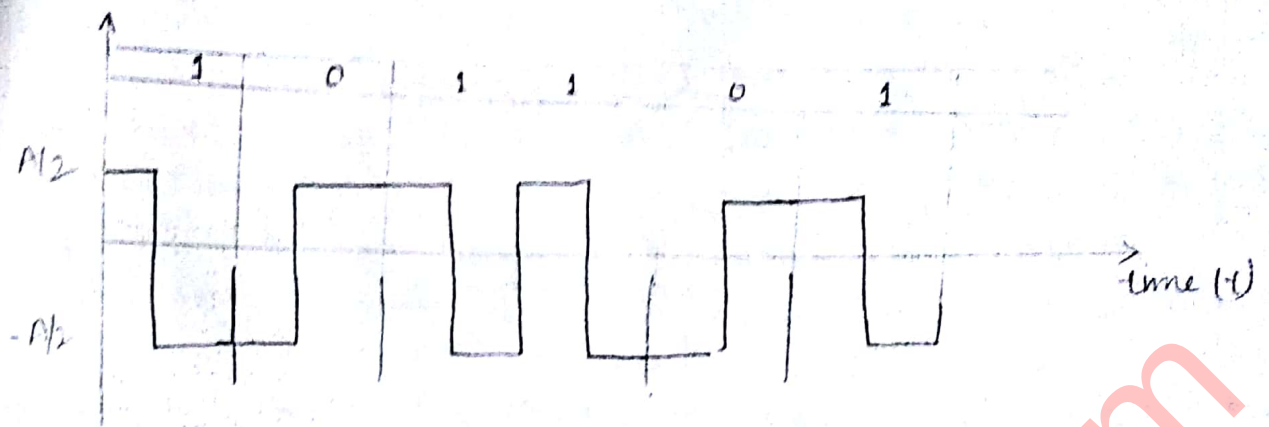
⑥ Split Phase Manchester Format (Polar type)

If symbol '1' is to be transmitted, then

$$x(t) = \begin{cases} A/2 & \text{for } 0 \leq t < T_b/2 \\ -A/2 & \text{for } T_b/2 \leq t < T_b \end{cases}$$

and if symbol '0' is transmitted, then

$$x(t) = \begin{cases} -A/2 & \text{for } 0 \leq t < T_b/2 \\ A/2 & \text{for } T_b/2 \leq t < T_b \end{cases}$$

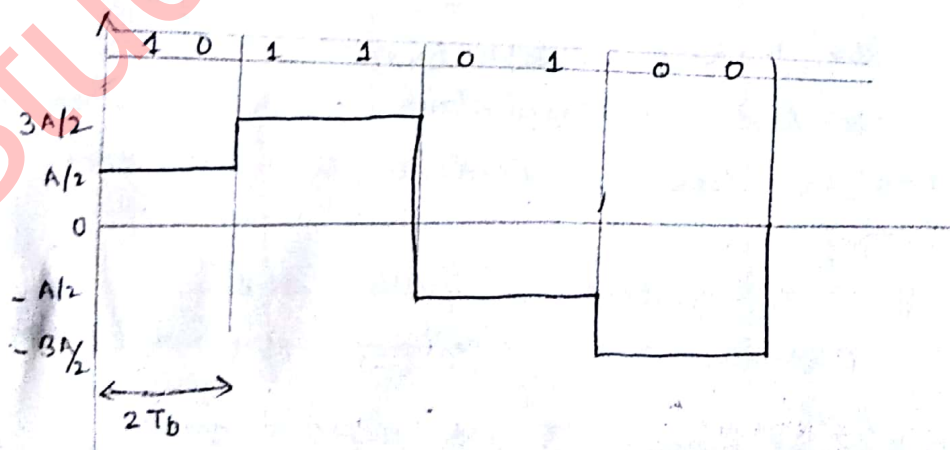


Advantage - It always has 0-dc value, regardless of the data sequence, but it has twice the bandwidth of the unipolar NRZ/polar NRZ because the pulses are half the width.

⑦ Polar Quaternary NRZ format

To reduce signalling rate '2', the message bits are grouped in blocks of two.

Message combination	$x(t)$
00	$-3A/2$
01	$-A/2$
10	$A/2$
11	$3A/2$



⑧ High density Bipolar (HDB) Signalling

In NRZ or RMI, the transmitted signal is equal to zero when a binary 0 is to be transmitted. This absence of signals can cause synchronisation problem at the receiver, if long sequence of 0's is transmitted.
 // soln.

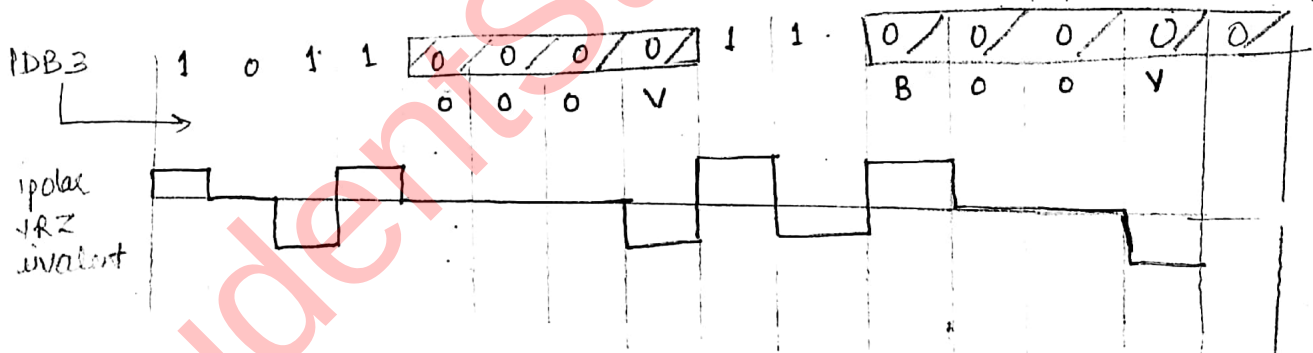
This can be solved by adding pulses when long strings of 0's exceeding a number n are being transmitted.

It is HDBN, here $N=1, 2, 3...$ (generally $N=3$)

In this when $(N+1)$ or minimum number of zeros occur, they are replaced by special binary sequence of $(N+1)$ length. These sequences contain some binary 1's which are necessary for synchronisation.

→ The $(N+1)$ long special sequences for HDB3 are 000V and 000B.

where B and V are both considered to be binary 1's.



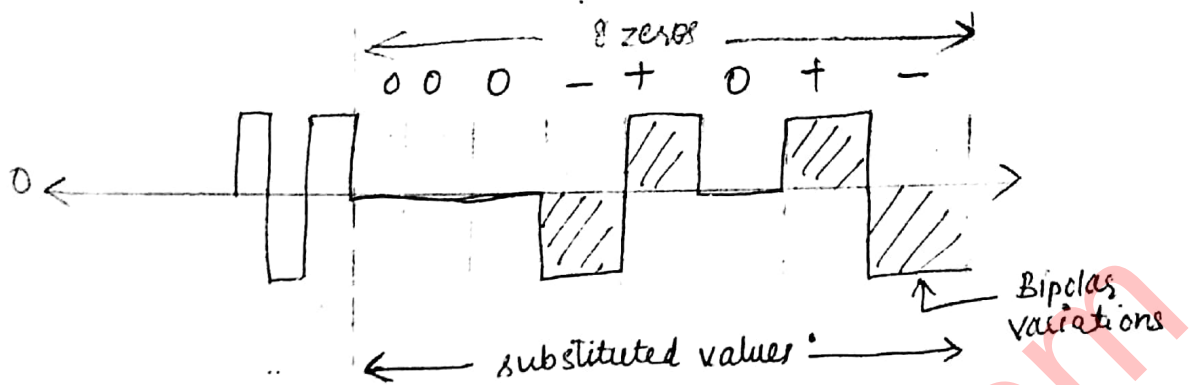
⑨ BZS (Bipolar N-zeroes substitution / suppression)

In order to have synchronisation between transmitter and receiver, the line code needs to cross the zero line frequently.

As per U.S. T1 standard, not more than 15 0's can be sent in succession to ensure proper synchronisation.

Example = B8ZS (Binary 8 zero suppression)

Whenever 8 successive 0's are detected, the implementation of this code will insert 8-bit special sequence.

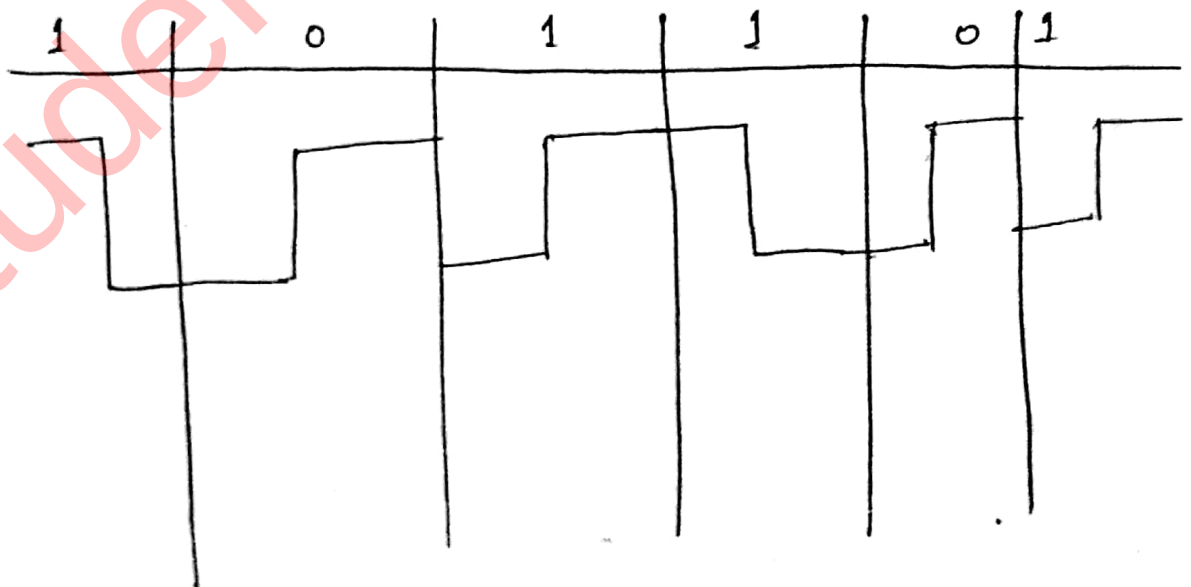


* Differential Manchester

If previous bit is zero transition in data otherwise same.

0 $\rightarrow$ +V to -V

1 $\rightarrow$ -V to +V



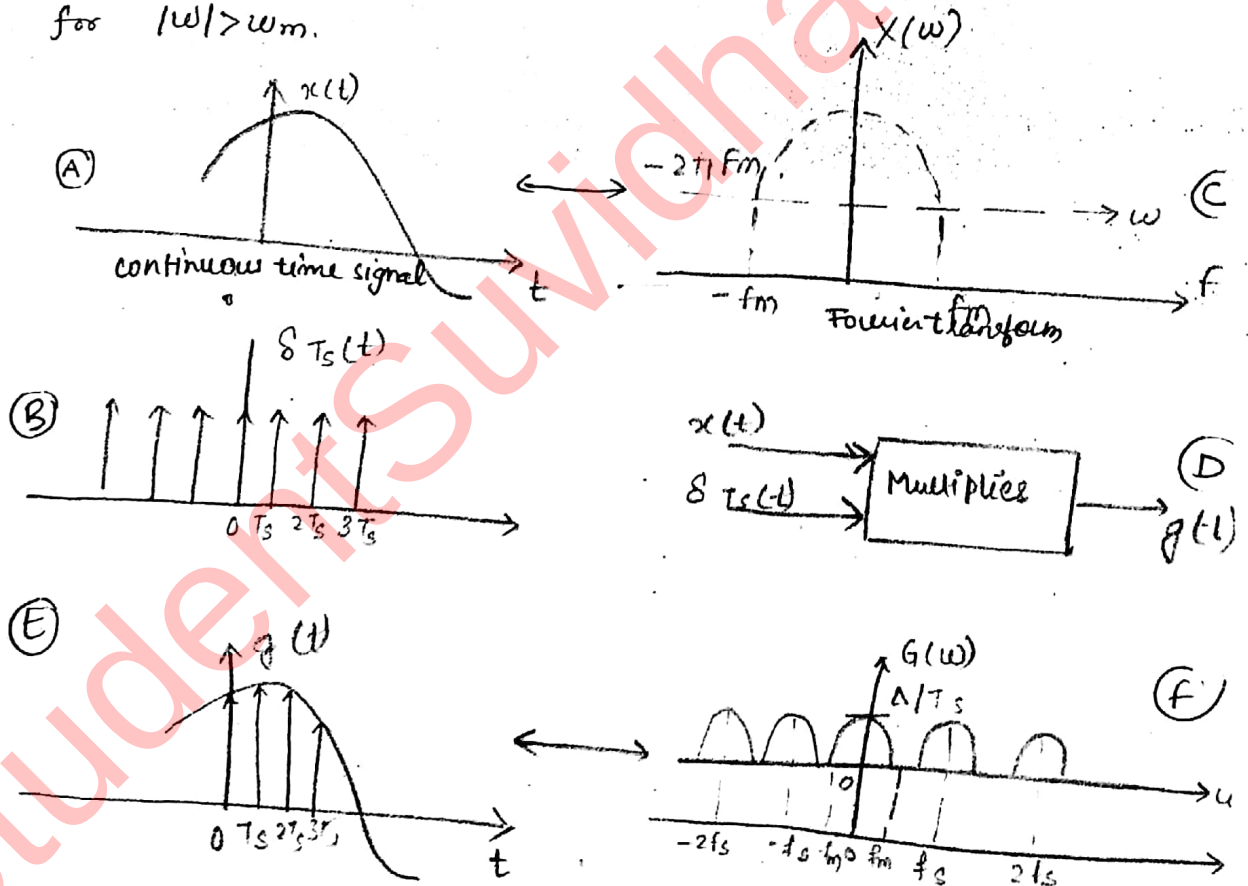
* Sampling Theorem

A continuous-time signal may be completely represented in its samples and recovered back if the sampling frequency is $f_s \geq 2f_m$.

Here f_s is sampling frequency & f_m is the maximum frequency present in the signal.

Proof:

Let us consider a continuous time signal $x(t)$ whose spectrum is bandlimited to f_m Hz. Hence there is no frequency component beyond f_m Hz. Therefore, $X(\omega) = 0$ for $|\omega| > \omega_m$.



Sampling of $x(t)$ at a rate of f_s Hz may be achieved by multiplying $x(t)$ by an impulse train $\delta_{T_s}(t)$. The impulse train $\delta_{T_s}(t)$ consist of unit impulses repeating periodically every T_s seconds.

The resulting or sampled signal may be written as:-

$$g(t) = x(t) \cdot \delta_{Ts}(t) \quad \text{--- (1)}$$

The trigonometric Fourier series expansion of impulse train $\delta_{Ts}(t)$ is

$$\delta_{Ts}(t) = \frac{1}{T_s} [1 + 2\cos\omega_s t + 2\cos 2\omega_s t + 2\cos 3\omega_s t + \dots] \quad \text{--- (2)}$$

$$\text{where } \omega_s = \frac{2\pi}{T_s} = 2\pi f_s$$

The sampled signal is

$$g(t) = \frac{1}{T_s} [x(t) + 2x(t)\cos\omega_s t + 2x(t)\cos 2\omega_s t + \dots] \quad \text{--- (3)}$$

$$\text{But } x(t) \longleftrightarrow X(\omega)$$

$$2x(t)\cos\omega_s t \longleftrightarrow [X(\omega - \omega_s) + X(\omega + \omega_s)]$$

$$2x(t)\cos 2\omega_s t \longleftrightarrow [X(\omega - 2\omega_s) + X(\omega + 2\omega_s)]$$

Hence by taking Fourier transform of $g(t)$

$$G(\omega) = \frac{1}{T_s} [X(\omega) + X(\omega - \omega_s) + X(\omega + \omega_s) + \dots]$$

$$= \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(\omega - n\omega_s) \quad \text{--- (4)}$$

The spectrum $G(\omega)$ consist of $X(\omega)$ repeating periodically with period $\omega_s = \frac{2\pi}{T_s}$ rad/sec

Reconstruction

Now if we want to reconstruct $x(t)$ from $g(t)$, we must be able to recover $X(\omega)$ from $G(\omega)$. This is possible only if there is no overlap between successive cycles of $G(\omega)$.

Hence

$$f_s > 2 f_m$$

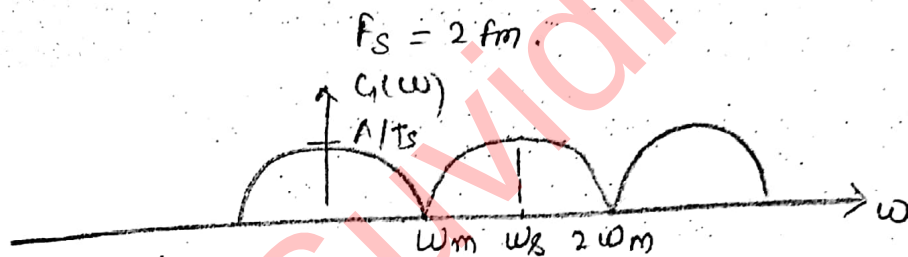
But the sampling interval is $T_s = \frac{1}{f_s}$

$$T_s < \frac{1}{2 f_m}$$

* Nyquist Rate

When sampling rate becomes exactly equal to $2 f_m$ samples per second, then it is called Nyquist rate.

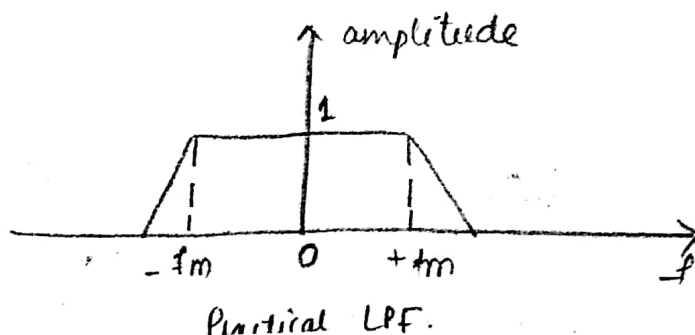
It is also called minimum sampling rate.

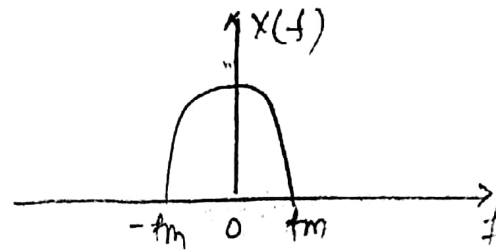


Que: $x(t) = 3 \cos(50\pi t) + 10 \sin(300\pi t) - \cos(100\pi t)$
calculate Nyquist rate

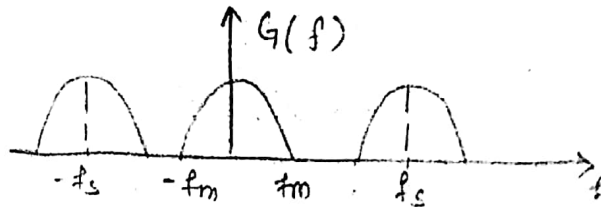
* Reconstruction Filter

It is used to recover original signal from its samples. This is also known as Interpolation filter.

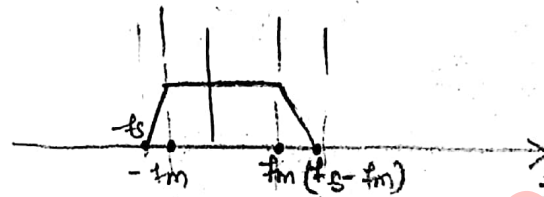




← Spectrum of original signal



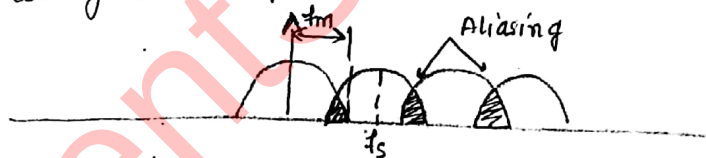
← spectrum of sampled signal



← amplitude Response of LPF

* Effect of under Sampling (Aliasing)

When a continuous time signal is sampled at a rate lower than Nyquist rate ($f_s < 2f_m$), then the successive cycles of the spectrum $G(\omega)$ of the sampled signal $g(t)$ overlap with each other.



This is a phenomenon in which a high frequency component in the frequency spectrum of the signal takes identity of lower-frequency component in the spectrum of sampled signal.

Tutorial sheet

Que-1) $x(t) = 3 \cos(500\pi t) + 10 \sin(3000\pi t) - \cos(1000\pi t)$
3000 ans calculate Nyquist rate

Que-2) find Nyquist rate & Nyquist interval for
1400 ans $x(t) = \frac{1}{2\pi} \cos(4000\pi t) \cos(1000\pi t)$

unipolar
RZ

unipolar
NRZ

islar
RZ

islar
NRZ

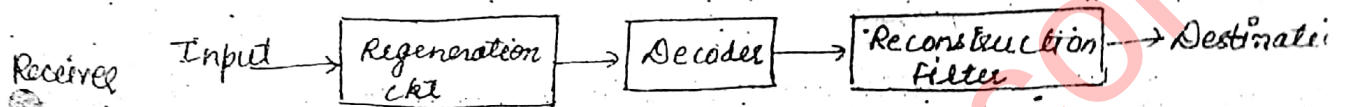
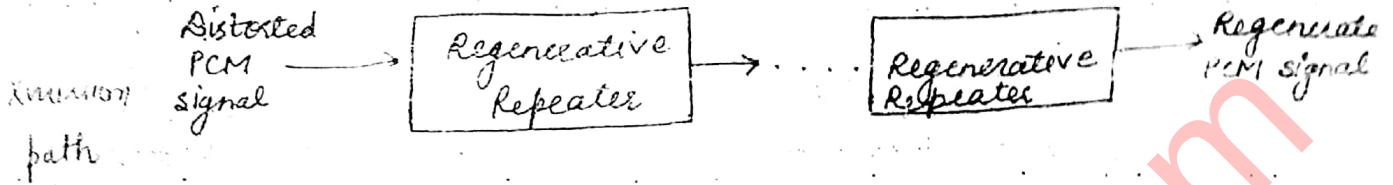
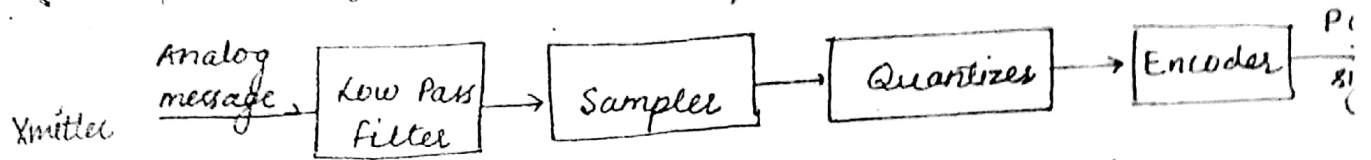
Bipolar
NRZ

Split
phase
Manchester

KK.
Polar quaternary
NRZ

Pulse code modulation (PCM)

It is digital pulse modulation technique



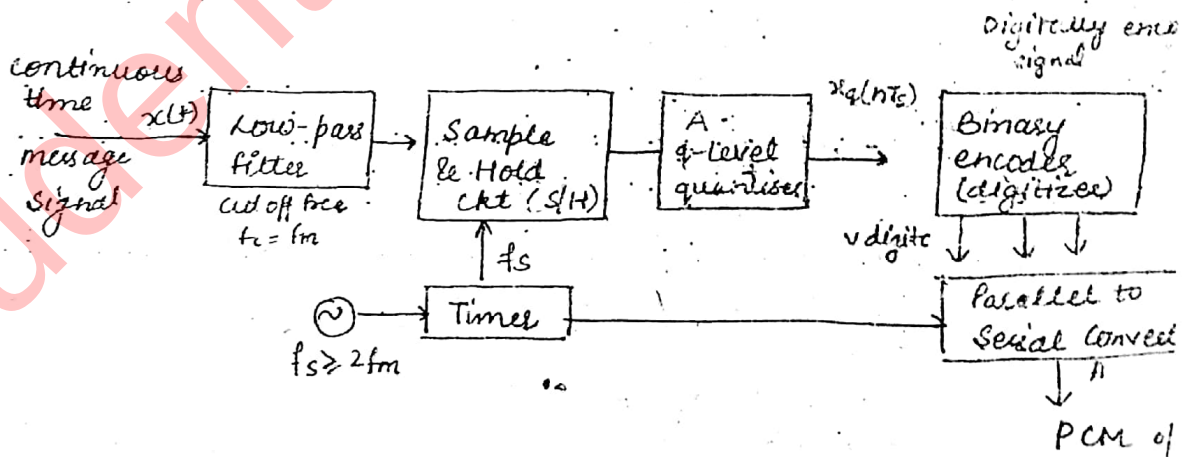
→ The quantizing and encoding operations are performed in same ckt called ADC.

→ At receiver, it is called DAC.

→ PAM, PWM, PPM are analog pulse modulation, while PCM is digital pulse modulation.

PCM generator / Transmitter

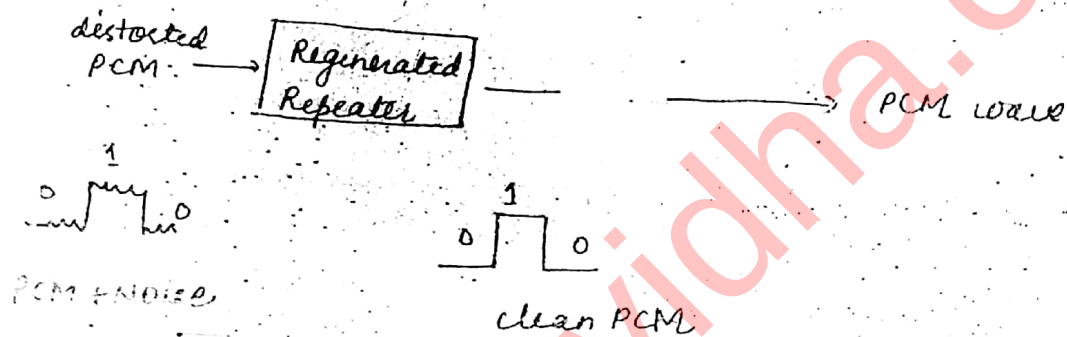
→ Signal is firstly passed through filter to limit components till f_m Hz.



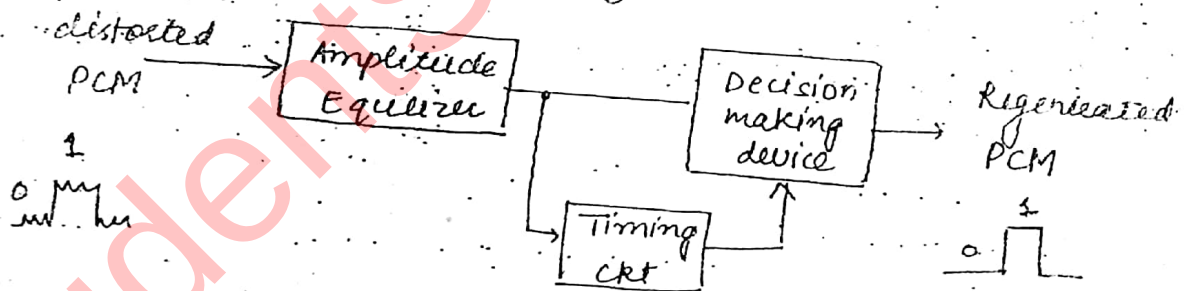
→ Sampling rate is selected above nyquist rate to avoid aliasing.

- The output of sample & hold ckt is $x(nT_s)$.
- A q -level quantiser compares input $x(nT_s)$ with its fixed digital level.
- It then assigns any one of the digital level to $x(nT_s)$ which results in minimum distortion. This is called Quantisation error.
- Hence it is called $\hat{x}_q(nT_s)$. It is then given to binary encoder. This encoder converts input signal to $'v'$ digits binary word. This is called digitalizer.

II Transmission Path



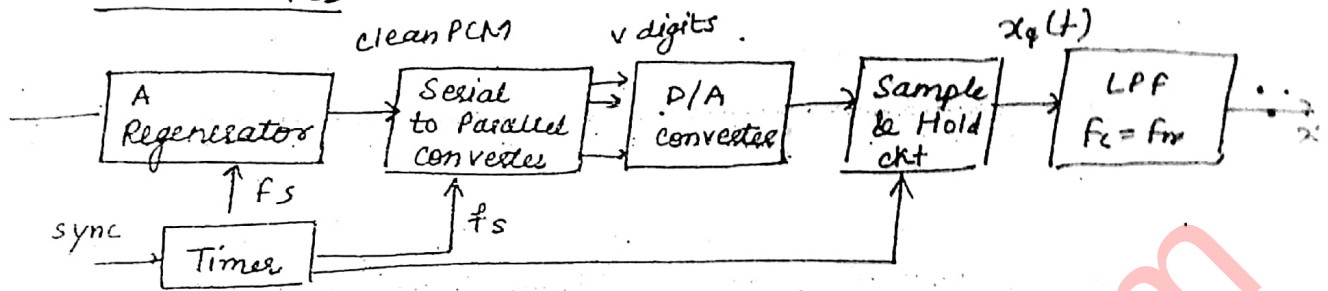
- The regenerative repeater consist of equalization, timing & decision making ckt.



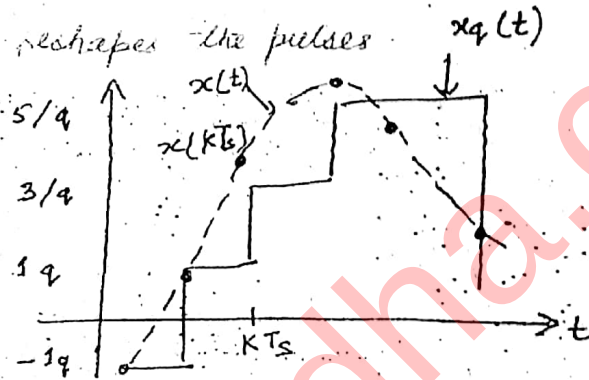
- The amplitude equalizer shapes the distorted PCM so as to compensate for the amplitude & phase distortions.
- The timing ckt produces a periodic pulse train which is derived from input PCM pulses. This is then applied to the decision making device.
- The decision device uses the pulse train for sampling the equalized PCM pulses.

A decision is made by comparing equalized PCM with the reference level, called decision threshold.

* PCM Receiver



→ Regenerator reshapes the pulses



* Quantizer

✓ A q -level quantizer compares discrete level input $x(n)$ with its fixed digital levels. It assigns any one of the digital level to $x(n)$ with its fixed digital level which results in minimum distortion or error. This is called quantisation error. The output is hence called

$x_q(nT_s)$. In this, the total amplitude range which the signal may occupy is divided into a number of standard levels → classification of Quantisation (based on step size)

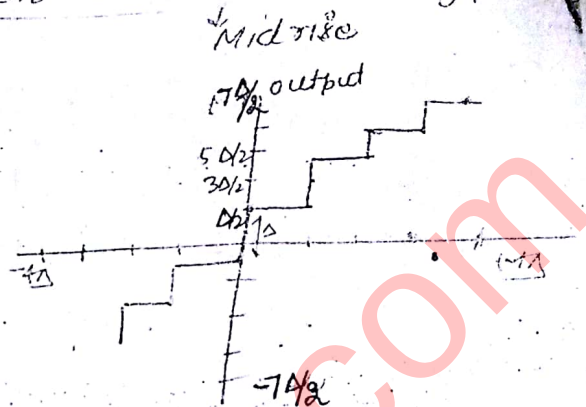
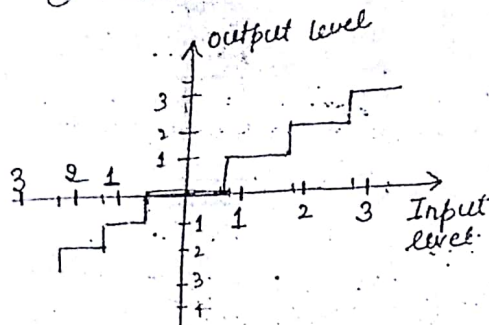
uniform

Non-uniform

1. uniform: In this the 'step size' remains same throughout the input range.
2. Non-uniform: In this the step size varies according to the input signal value.

(constant step size)
 → In uniform Quantisation we have two types

① Midtread



→ In this the origin lies in the middle of a tread of staircase like graph.

→ Origin is in the middle of rising part of a staircase graph.

→ Both are symmetric about the origin.

* Working:-

→ Midrise type quantisation used.

Let us assume that the input varies from -4Δ to 4Δ . Δ is step size.

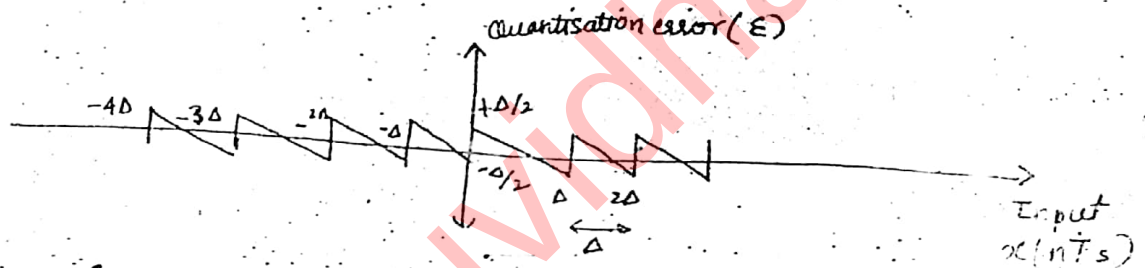
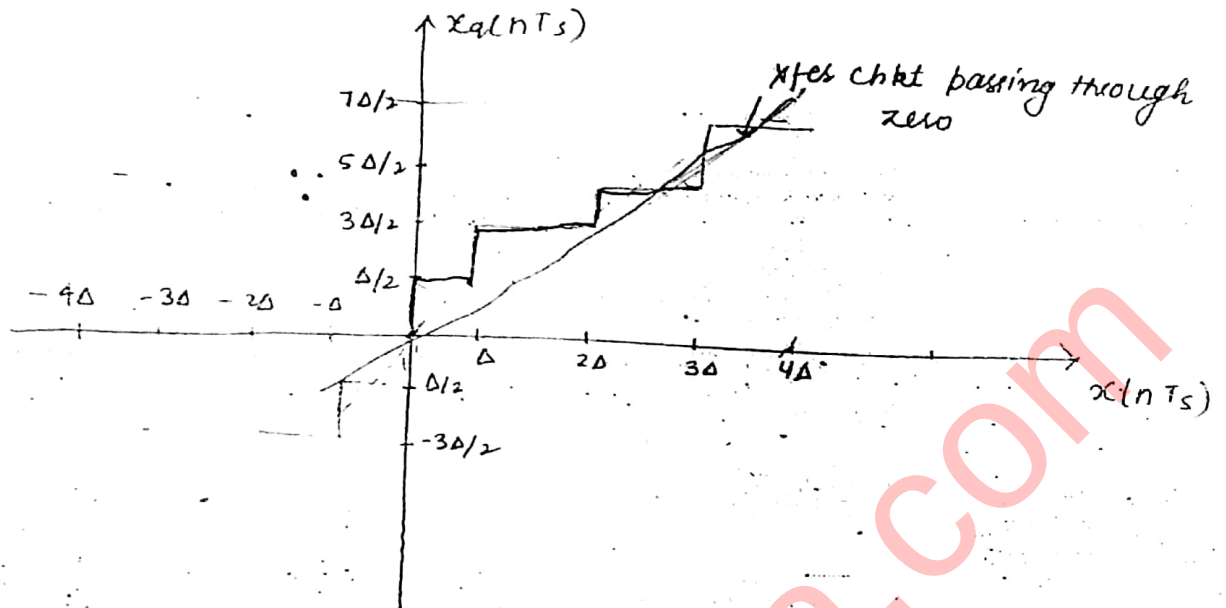
→ Input $x(nT_s)$ can take any value between -4 to 4 . The fixed digital levels are available at $\pm \Delta/2$, $\pm 3\Delta/2$ and $\pm 5\Delta/2$. These levels are available due to its characteristics.

→ If $x(nT_s) = 4\Delta$, then $x_q(nT_s) = \frac{7}{2}\Delta$
 If $x(nT_s) = -4\Delta$, then $x_q(nT_s) = -\frac{7}{2}\Delta$.

→ maximum quantisation error would be $\pm \Delta/2$

$$E = x_q(nT_s) - x(nT_s)$$

If $x(nT_s) = 0$, quantizer will assign any one of the nearest values either $\Delta/2$ or $-\Delta/2$.



maximum quantisation error = $|\Delta/2|$

$$\begin{aligned} \rightarrow \text{Error with input } E &= x_q(nT_s) - x(nT_s) \\ &= \Delta/2 - 0 \\ &= \Delta/2 \end{aligned}$$

$$\rightarrow \text{For } \Delta < x(nT_s) < 2\Delta, x_q(nT_s) = 3\Delta/2$$

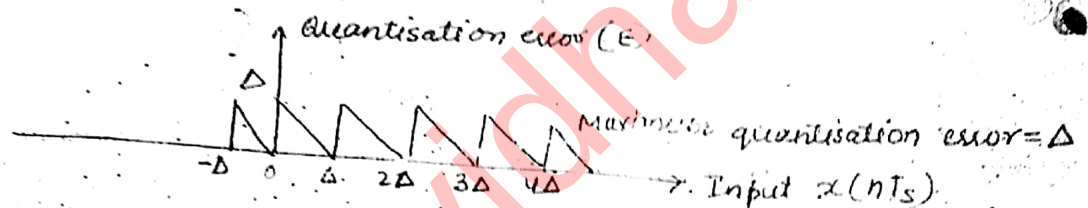
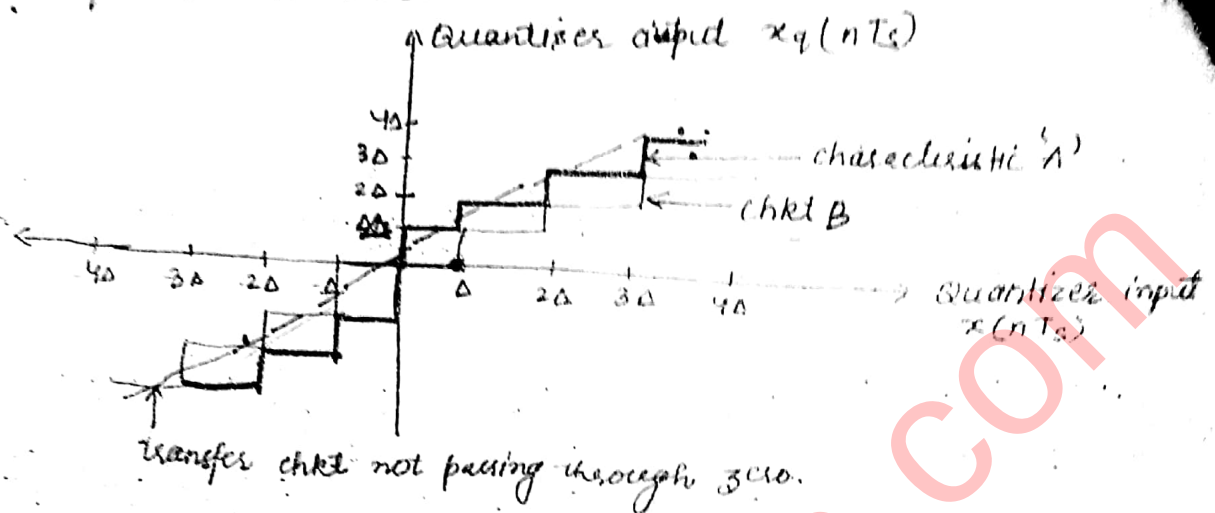
$$-\Delta < x(nT_s) < -2\Delta, x_q(nT_s) = -3\Delta/2$$

$$\text{So max } E = \Delta/2$$

$$E_{\max} = |\Delta/2|$$

$\rightarrow$ To minimize this error Non-uniform quantisation is used

→ The digital level at $+\Delta/2, +3\Delta/2$ is taken as coding the quantisation error.



$$\begin{aligned} \text{If } 0 < x(nT_s) < \Delta & \quad x_q(nT_s) = \Delta \\ 2\Delta < x(nT_s) < 3\Delta & \quad x_q(nT_s) = 3\Delta \end{aligned}$$

→ Quantization Noise/Error in PCM

The quantisation error is given by :-

$$* E = x_q(nT_s) - x(nT_s)$$

Let us assume that the input $x(nT_s)$ has continuous amplitude in the range $-x_{max}$ to $+x_{max}$

→ So when input is 4Δ , output is $+7/2\Delta$, for -4Δ input output is $-7/2\Delta$.

$$\text{So } +x_{max} = 7/2\Delta \quad \text{and} \quad -x_{max} = -7/2\Delta$$

$$\begin{aligned} * \text{Total amplitude Range} &= x_{max} - (-x_{max}) \\ &= 2x_{max} \end{aligned}$$

→ If total amplitude range is divided into q levels of quantiser, then the step size ' Δ '

$$* \quad \text{Step size} = \Delta = \frac{x_{\max} - (-x_{\max})}{q} = \frac{2x_{\max}}{q}$$

* S/N Ratio for Linear Quantization

$$\rightarrow \quad S/N = \frac{\text{Normalised signal power}}{\text{Normalised noise power}}$$

$$\text{But } N = \frac{\Delta^2}{12}$$

$$\text{So } \frac{S}{N} = \frac{S}{(\Delta^2/12)} \quad \text{--- (1)}$$

→ The number of bits ' v ' and quantisation level are related as

$$q = 2^v \quad \text{--- (2)}$$

→ Let us assume that the output has continuous amp. So the total amplitude range is

$$= x_{\max} - (-x_{\max})$$

$$= 2x_{\max} \quad \text{--- (3)}$$

→ Step size will be

$$\Delta = \frac{2x_{\max}}{2^v} \quad \text{--- (4)}$$

$$\text{So } \Delta = \frac{2x_{\max}}{2^v} \quad \text{--- (5)}$$

→ Putting this value in eqⁿ 1

$$\frac{S}{N} = \frac{\text{Normalised signal power}}{\left(\frac{2x_{\max}}{2^v}\right)^2 \frac{1}{12}}$$

$$\frac{S}{N} = \frac{P}{\frac{4x_{\max}^2}{2^{2v}} \cdot \frac{1}{12}} = \frac{3P}{x_{\max}^2} \cdot 2^{2v}$$

$$\boxed{\frac{S}{N} = \frac{3P \cdot 2^{2v}}{x_{\max}^2}}$$

→ S/N ratio of a quantizer increases exponentially with increasing bits per sample

Case If $x_{\max} = 1$ ($x(t)$ is normalised)

$$\frac{S}{N} = 3P \cdot 2^{2v}$$

If destination signal power is also normalised

$$P \leq 1$$

then

$$\boxed{\frac{S}{N} \leq 3 \times 2^{2v}}$$

Taking in decibels

$$\frac{S}{N} \text{ dB} = 10 \log_{10} \left(\frac{S}{N} \right) \leq 10 \log_{10} [3 \times 2^{2v}]$$

$$\boxed{\frac{S}{N} \text{ dB} \leq (4.8 + 6v) \text{ dB}}$$

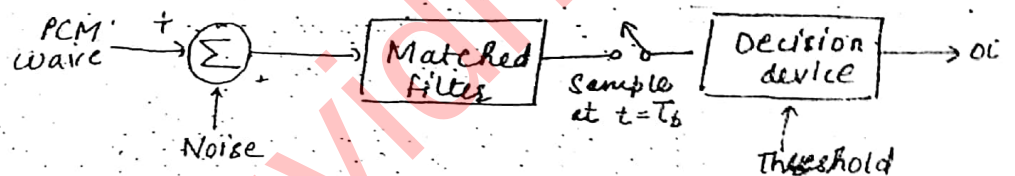
Performance analysis of PCM: Probability of error.
 Here we need to consider two major sources of

- (1) channel Noise / Decoding Noise
- (2) Quantisation Noise (produced at transmitter)

(1) channel Noise - It is the introduction of transmission errors at the receiver when PCM signal is received. Due to this noise the receiver will make a mistake about whether the signal is 0 or 1.

(2) Probability of Error (P_e): It gives the probability that the symbol at receiver output differs from that transmitted.

↓
 To obtain this we use Matched filter and assume that noise is AWGN (additive white gaussian noise).



→ It is
$$P_e = \frac{1}{2} \operatorname{erfc} \left[\frac{1}{2} \sqrt{\frac{E_{\max}}{N_0}} \right]$$

$E_{\max}$ = Peak signal energy

N_0 = Noise spectral density

→ Put $E_{\max} = P_{\max} T_b$

where $P_{\max}$ = max peak signal power

T_b = bit duration

$$P_e = \frac{1}{2} \operatorname{erfc} \left[\frac{1}{2} \sqrt{\frac{P_{\max} \times T_b}{N_0}} \right]$$

$N_0 T_b$ = average noise power contained in BW

By Shannon Hartley theorem $C = B \log_2 [1 + S/N]$

Ques - $f_m = 3\text{kHz}$, M-level PCM having Quantisation level 16. What can be max no. of bits/sample & min sampling Rate
 $q = 2^v$, $f_s \geq 2f_m$

Ques - Given an audio signal consisting of sinusoidal tone
 $x(t) = 3 \cos(500\pi t)$

- (i) Determine the S/N ratio, when quantiser uses 10 bit PCM.
- (ii) How many bits of quantisation are needed to achieve a S/N Ratio of 40 dB atleast.

$$\textcircled{1} (S/N)_{dB} = 4.8 + 6v$$

$$v = 10$$

$$S/N = 4.8 + 6 \times 10 = 64.8 \text{ dB}$$

$$\textcircled{2} 4.8 + 6v \geq 40$$

$$v \geq 7 \text{ bits}$$

✓ Non-uniform Quantisation.

(5)

→ Here the step size changes with change in the amplitude of the input. For weak signal step size is small, therefore the quantisation noise reduces. This is achieved through COMPANDING.

* COMPANDING:

→ This is non-uniform quantisation. It is implemented to improve S/N ratio.

$$N_q = \frac{\Delta^2}{12} \quad \text{Quantisation noise.}$$

→ If step size is fixed, then noise power is also fixed. But the signal power is not constant. It is proportional to the square of signal amplitude. ($S \propto A^2$)

→ Hence S is small for weak signal; but N_q is const. Hence S/N ratio is very poor. This affects the quality of the signal. The remedy is to use companding.

Companding = Compressing + Expanding

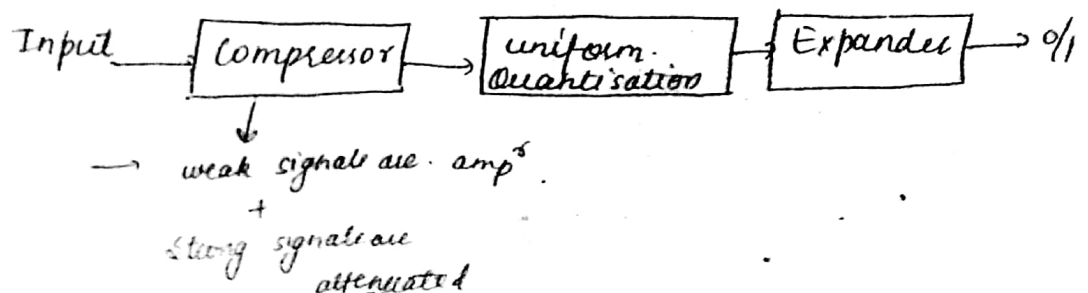
→ Generally it is difficult to implement non-uniform quantisation because it is not known in advance the changes in signal level.

Hence

*

Weak signals are amplified & strong signals are attenuated before applying them to uniform quantiser. This is called compression.

At receiver exactly opposite is followed which is called Expansion.



COMPANDER CHKT |
compressor chkt

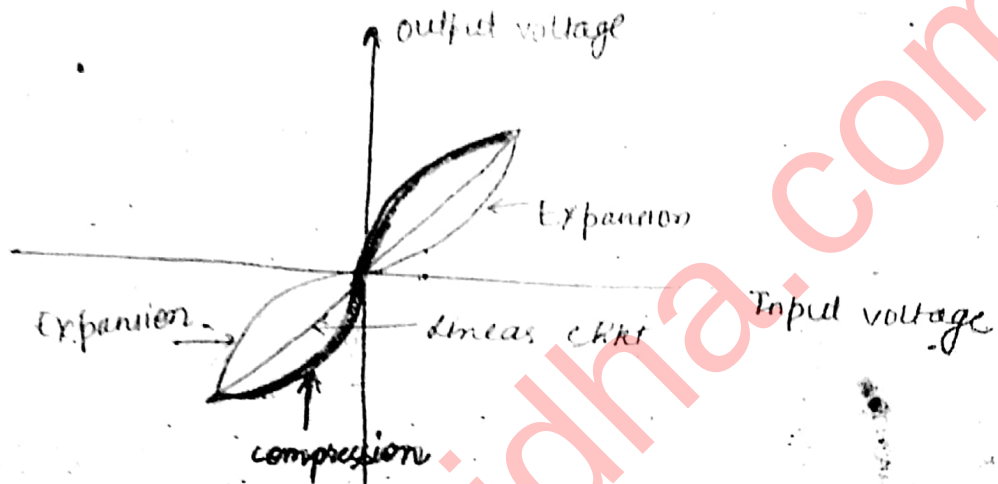
↳ for +ve input

→ Expander

Compressor
o/p

gain for
high

compressor
o/p



→ due to inverse nature of compressor and expander the overall chkt of compander is straight line
 → This indicates that all the boosted signals are brought back to their original amplitudes

* Different types of compressor chkt

→ Ideally we need linear compressor chkt for small amplitudes of the input signal & logarithmic chkt elsewhere. It can be done by two methods

↳ μ-law companding

↳ A-law companding

μ -law companding (midread)

Here the compressor chkt is continuous. It is app. linear for smaller values of input levels and logarithmic for high input levels.

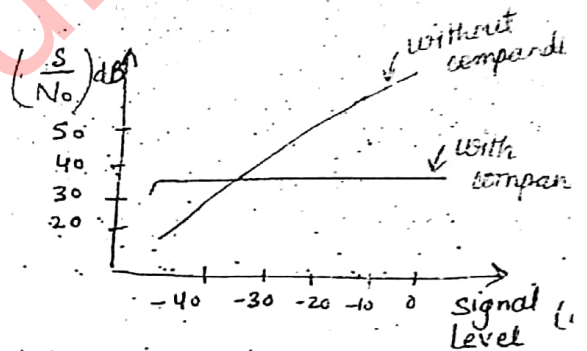
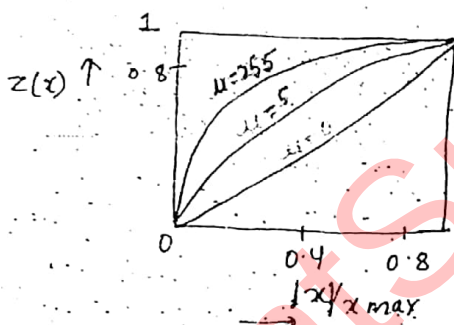
→ The μ -law chkt is mathematically expressed as:

$$z(x) = \frac{(\text{sgn } x) \ln(1 + \mu|x|/x_{\max})}{\ln(1 + \mu)}$$

where $0 \leq |x|/x_{\max} \leq 1$

- $z(x)$ is output and x is input to the compressor.
 - $|x|/x_{\max}$ represents normalized value of input w.r.t.
 - $\text{sgn}(x)$ represents ± 1 (ve and +ve values of s/p re of
 - The chkt are diff for different value of μ .
- Practically used value for $\mu = 255$

If $\mu = 0$, uniform quantisation



PCM performance with μ -law companding

Applications

- used in speech & music signals.
- used in PCM telephone system in US.

A-law companding (midrise)

→ In this, the compressor chkt is piecewise linear, made of linear segment for low level input and logarithmic segment for high level inputs

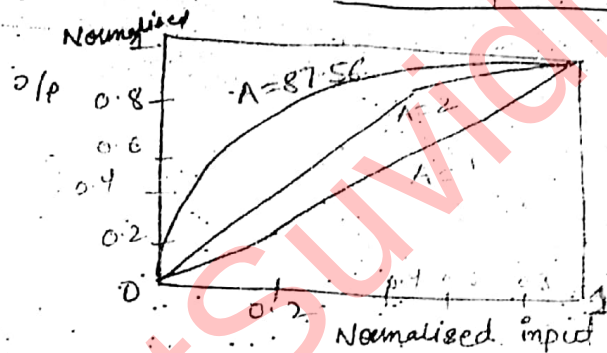
→ for different value of A , the output is

$$\frac{x(x)}{x_{\max}} = \begin{cases} \frac{A|x|/x_{\max}}{1 + \log_e A} & \text{for } 0 \leq \frac{|x|}{x_{\max}} \leq \frac{1}{A} \\ \frac{1 + \log_e [A|x|/x_{\max}]}{1 + \log_e A} & \text{for } \frac{1}{A} \leq \frac{|x|}{x_{\max}} \leq 1 \end{cases}$$

→ $A=1$, chkt is linear means uniform quantisation

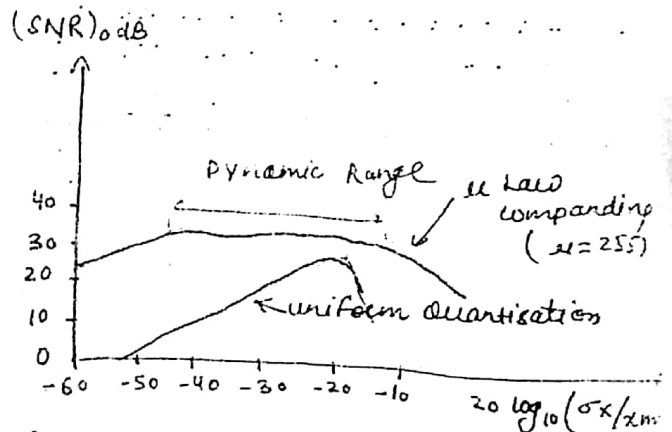
The practically used value is 87.56

Application used in PCM telephone system in Europe



* Effects of companding

→ The performance of uniform quantizers is highly dependent on the input, whereas the μ -law compander has a dynamic range of 30dB from (-15 dB to +15 dB)



→ Dynamic Range is defined as the range of input over which output SNR remains within 3dB of the max value of 35 dB

Dynamic Range is defined as the ratio of the possible magnitude to the smallest possible magnitude of signal

$$DR = \frac{V_{max}}{V_{min}}$$

- If Law has midlevel at origin
A law has midlevel at origin
(A law has no zero value)



Delta Modulation

✓ In PCM transmits all the bits which are used to code a sample. Hence, signaling rate & transmission channel bandwidth are large in PCM.



✓ To overcome this problem - Delta Modulation is used.

✓ Delta Modulation transmits only one bit per sample.

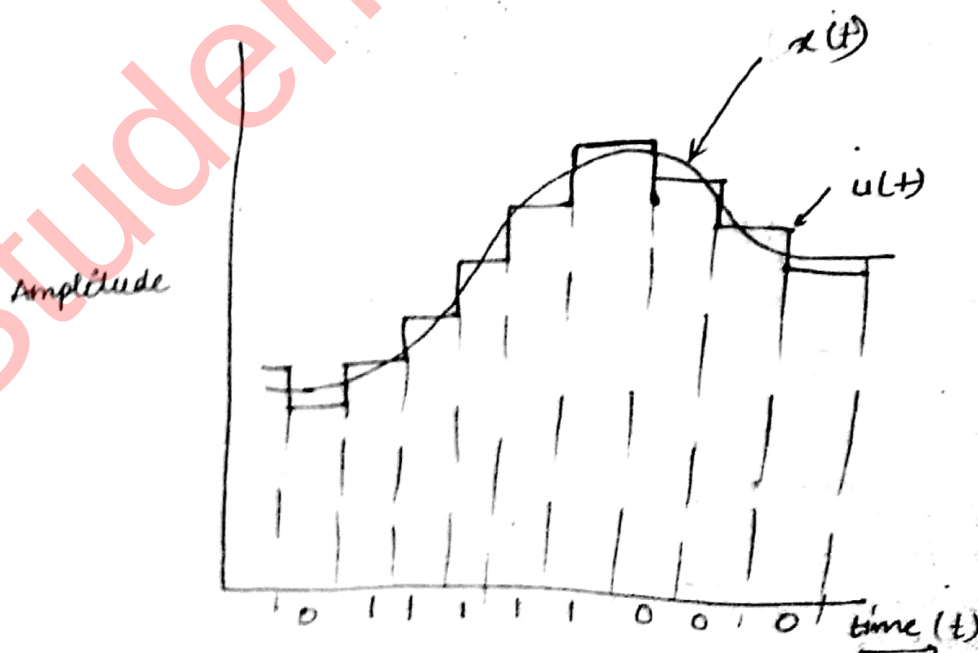
✓ In this the present sample value is compared with previous sample value & its result whether the amplitude is increased or decreased is transmitted.

✓ Input signal is approximated to step signal by delta modulator. The step size is fixed.

✓ The step size difference between input signal & staircase approximated signal is confined to 2 levels $+\Delta$ & $-\Delta$.

✓ If difference is +ve then approximated signal is increased by one step Δ . If the difference is -ve, the signal is decreased by Δ .

✓ When step is reduced '0' is transmitted and if step is increased '1' is transmitted.



→ Error b/w sampled value of $x(t)$ and last approximated sample $\hat{x}(nT_s)$ is

$$\checkmark \quad e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

↓
error at present sample

$x(nT_s)$ = sampled signal of $x(t)$

$\hat{x}(nT_s)$ = last sample approximation

→ $\checkmark$ Let $u(nT_s) \rightarrow$ present sample approximation of staircase output

then

$$\checkmark \quad u[(n-1)T_s] = \hat{x}(nT_s)$$

→ Let a quantity $b(nT_s)$ is

$$\checkmark \quad b(nT_s) = \Delta \cdot \text{sgn}[e(nT_s)]$$

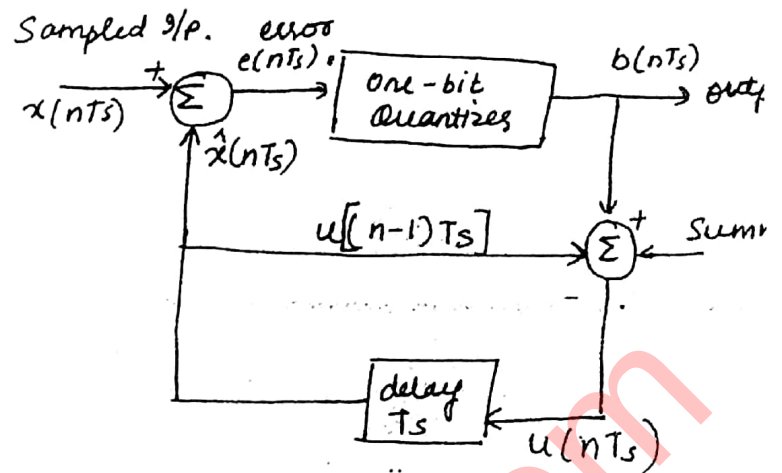
So depending on sign of error $e(nT_s)$, Δ is decided

$$\checkmark \quad b(nT_s) = \begin{cases} +\Delta & \text{if } x(nT_s) \geq \hat{x}(nT_s) \\ -\Delta & \text{if } x(nT_s) < \hat{x}(nT_s) \end{cases}$$

→ also if $b(nT_s) = +\Delta$ then binary '1' is transmitted
if $b(nT_s) = -\Delta$ then binary 0 is transmitted

T_s = sampling interval

Transmitter part



→ The present sample approximation is

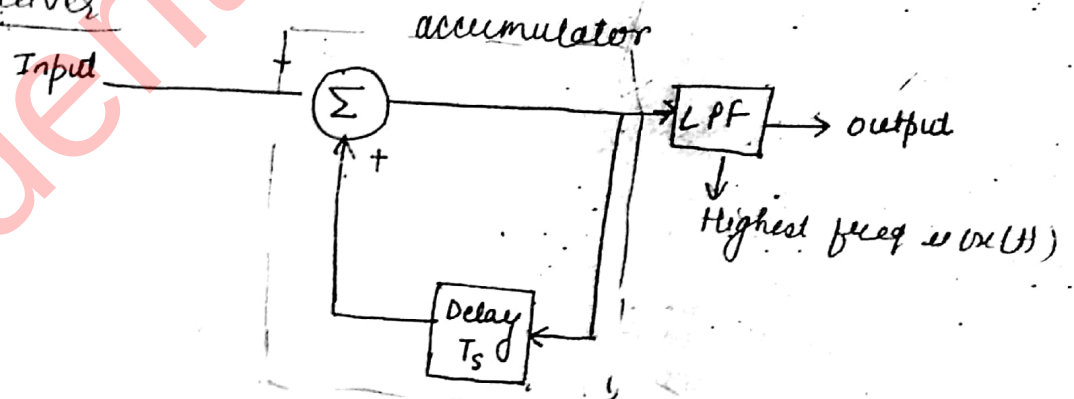
$$u(nT_s) = u(nT_s - T_s) \pm [\pm \Delta]$$

$$u(nT_s) = u[(n-1)T_s] + b(nT_s)$$

→ Depending upon sign of $e(nT_s)$, Quantiser generates an o/p of $+\Delta$ or $-\Delta$

↓
+1 0

Receiver



→ Accumulator generates staircase accumulated signal & it is delayed by one sampling period T_s . It is then added to input signal.

If input is binary '1' then add $+\Delta$ step to its previous output

If input = 0, then Δ is subtracted from signal.

→ Advantages of delta Modulation:-

- 1) As only one bit per sample is transmitted, therefore signalling rate or transmission channel BW is quite small.
- 2) Transmitter & receiver implementation is very much simple.

→ Disadvantages

- 1) Slope overload distortion
- 2) Granular or idle noise

✓ ① Slope overload distortion

↳ It is caused due to large dynamic range of the input signal.

- Sometimes the rate of rise of input signal $x(t)$ is so high that the staircase signal cannot approximate it, so step size Δ becomes too small for $u(t)$ to follow the steep segment of $x(t)$.

- Hence there is large error b/w staircase approximated signal & the original signal $x(t)$.

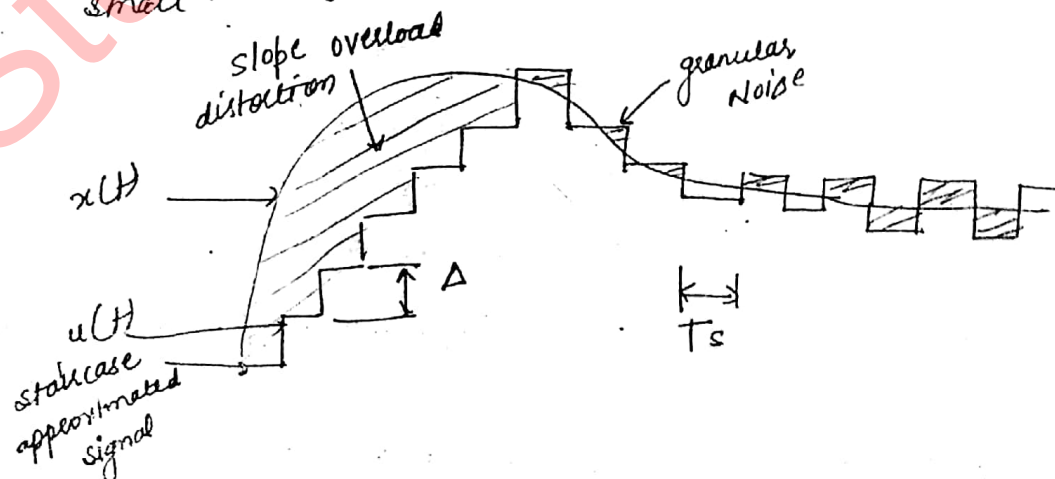
- This error is called slope overload distortion.

- So to reduce this, the step size must be increased when slope of signal $x(t)$ is high.

② Granular Noise

↳ It occurs when the step size is too large compared to small variations in input signal.

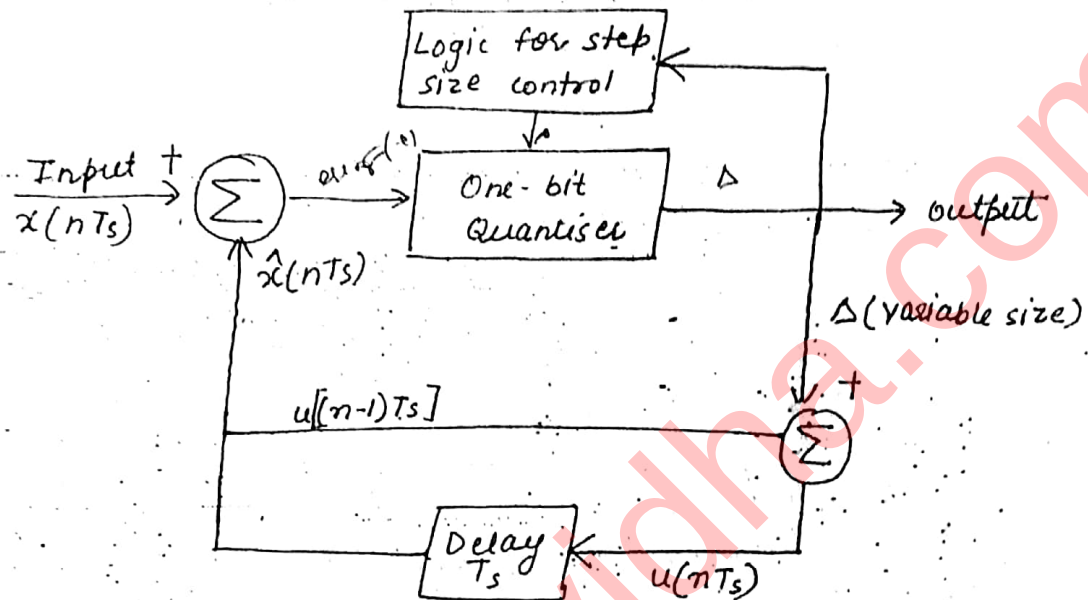
- So for small variation in input signal, the staircase signal is changed by large amount (Δ) because of large step size. So the error b/w input & approximated signal is called granular noise. The soln is to make step size small.



Adaptive delta Modulation

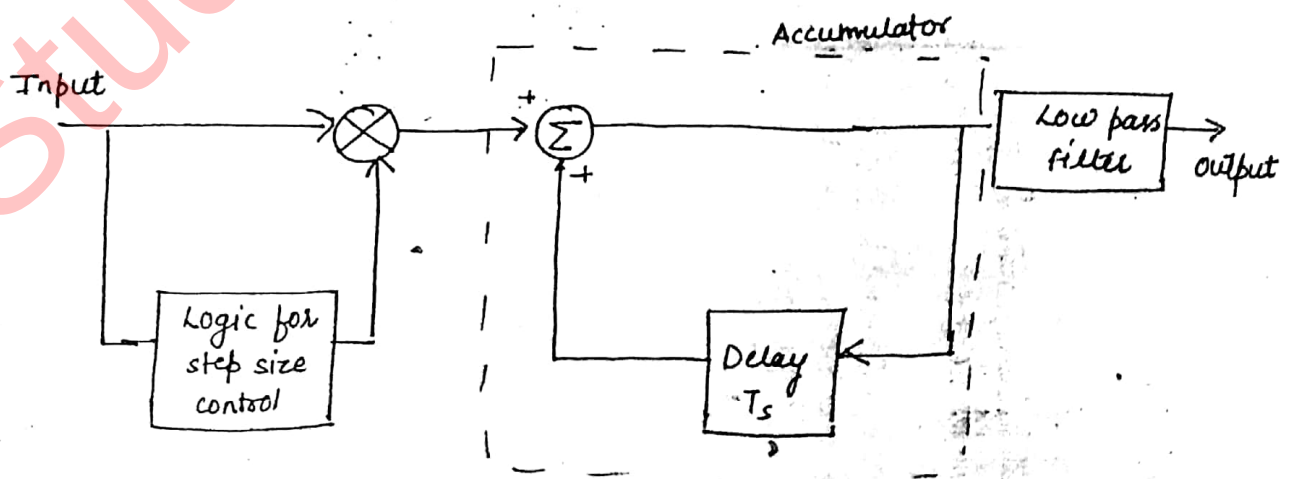
→ To overcome quantisation errors due to slope overload distortion and granular noise, the step size (Δ) is made adaptive to variations in the input signal $x(t)$.

→ If input is varying slowly, the step size is reduced.
If the segment is steep the step size is increased.



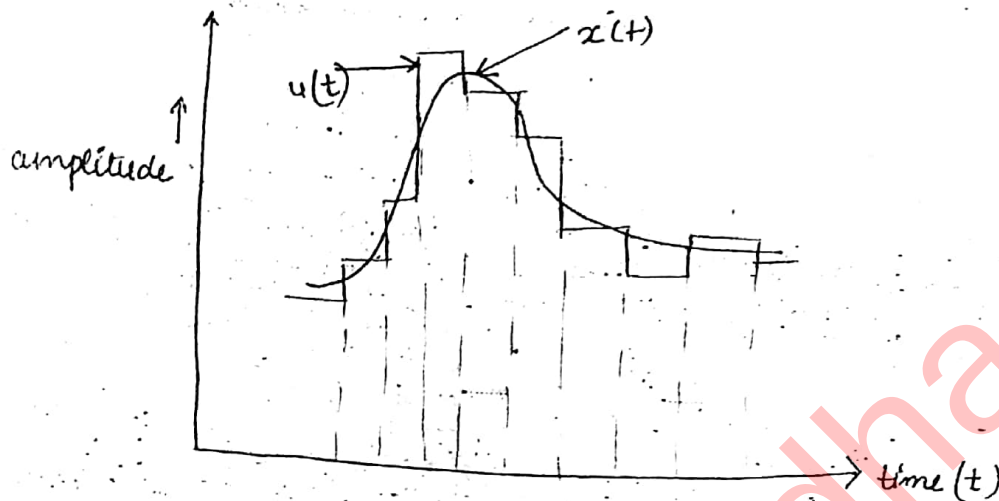
The logic for step size control is added to the diagram.
The step size increases or decreases according to the specified rule depending on one-bit quantiser output.

ex- If o/p of Quantiser is high - step size doubled for next sample.
" " " low - step size may be reduced



→ In this step size is produced from each incoming bit.

The previous input and present input decides the step size. It is then applied to accumulator that builds up staircase waveform. The LPF then smoothes the staircase.



Advantages

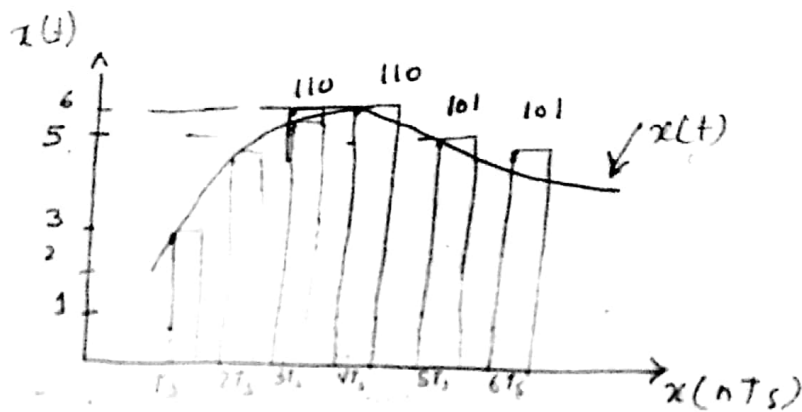
- (1) S/N ratio becomes better.
- (2) Due to variable step size, dynamic range of ADM is wider than simple DM.
- (3) utilization of BW is better than delta modulation.

Differential Pulse Code Modulation :

→ It may be observed that the samples of a signal are highly correlated with each other. This is due to the fact that any signal does not change fast. So the value from present sample to next sample does not differ by large amount. So the adjacent samples carry same info with little difference.

↓

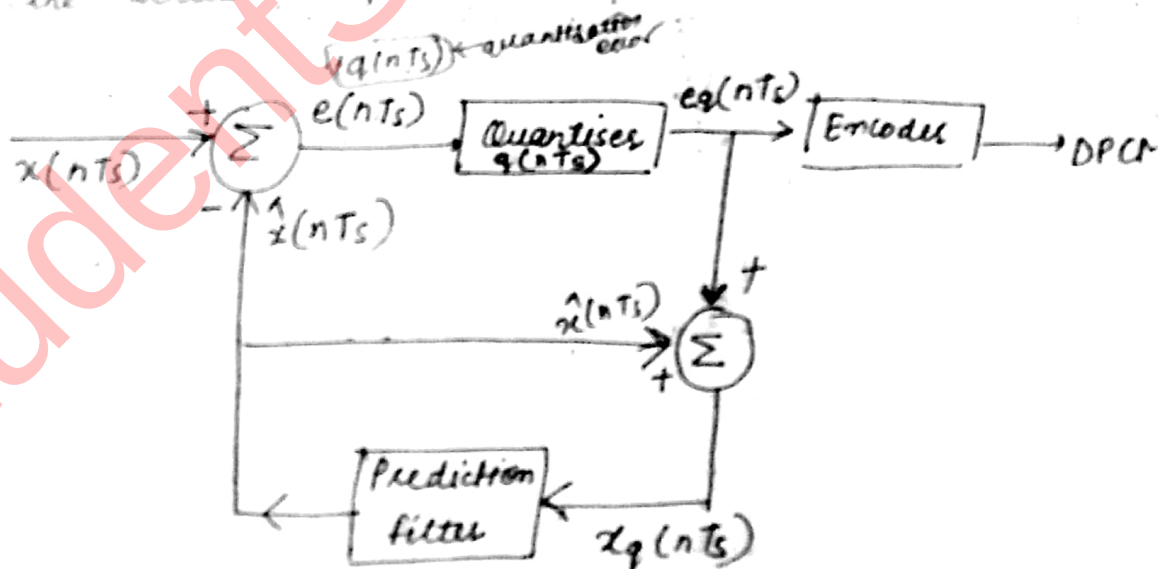
So when these samples are encoded, the encoded signals contains some redundant info.



ex

- $x(t)$ is sampled by flat-top sampling at intervals
- Samples are encoded using 3 bits (7 levels) PCM
- After quantisation some values overlap, this info can be carried by a single sample. But the info is carried by 3 samples, hence the value is redundant.
- So, if this redundancy is reduced, then overall bit rate will decrease to no. of bits required to transmit one sample will also be decreased.
- This is called DPCM (differential PCM)

It works on principle of ~~prediction~~. The value of present sample is predicted from the past samples. The prediction is not exact but it is very close to the actual sample value.



- The sampled signal is $x(nTs)$, and predicted signal is $\hat{x}(nTs)$. The comparator finds out the difference b/w

actual sample value $x(nT_s)$ & predicted sample $\hat{x}(nT_s)$. This is called prediction error $e(nT_s)$

$$* \quad e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

→ The predicted value is produced by using prediction filter $\hat{x}(nT_s)$

→ The quantiser output signal gap $e_q(nT_s)$ and the previous prediction is added & given as input to the prediction filter. This signal is called $x_q(nT_s)$

↓ This makes prediction more close to actual sampled signal

↓ This $e_q(nT_s)$ is very small & can be encoded by using small number of bits

$$\text{Quantiser o/p} = e_q(nT_s) = e(nT_s) + q(nT_s)$$

↑ Quantisation error

$$x_q(nT_s) = \hat{x}(nT_s) + e_q(nT_s) \quad - (2)$$

Putting value (1) in (2)

$$x_q(nT_s) = \hat{x}(nT_s) + e(nT_s) + q(nT_s) \quad - (3)$$

$$\text{But error} = e_q(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

$$e(nT_s) + \hat{x}(nT_s) = x(nT_s) \quad - (4)$$

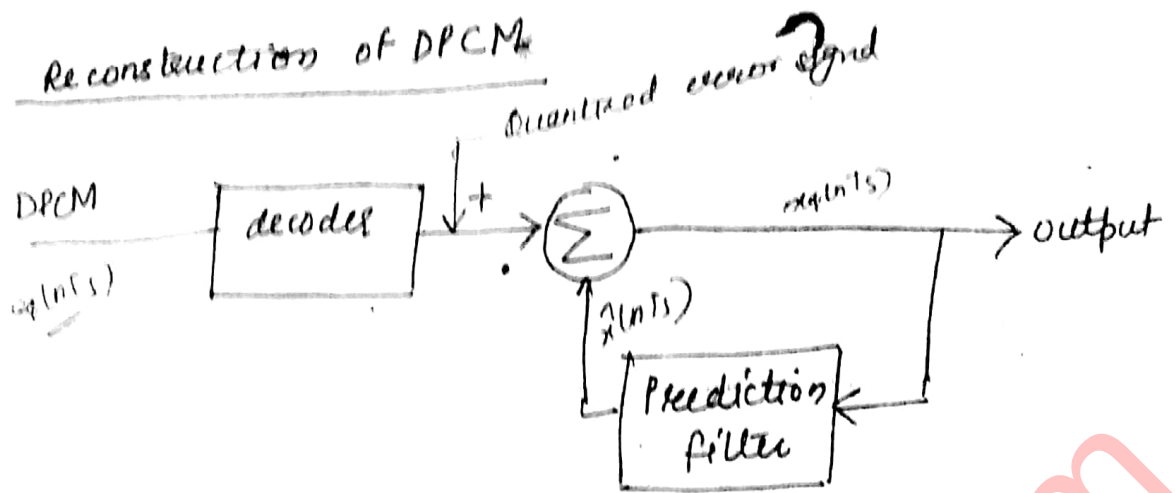
So final value

$$x_q(nT_s) = x(nT_s) + q(nT_s)$$

Hence the quantised version of signal $x_q(nT_s)$ is sum of original signal & quantisation error.

Hence the eqn does not depend on prediction filter chkt.

Reconstruction of DPCM



- The decoder reconstructs the quantised error signal from incoming binary signal.
- The prediction filter output & quantised error signal are summed up to give quantised version of original signal.
- Hence the signal at receiver differs from actual signal by quantisation error $q(nT_s)$.

S/N Ratio for DPCM

$$SNR = \frac{\text{Mean square value of signal}}{\text{Mean square value of quantisation noise}}$$

$$= \frac{\sigma_x^2}{\sigma_q^2}$$

σ_x^2 = variance of original input signal

σ_q^2 = variance of quantisation noise.

$$SNR = \frac{\sigma_x^2}{\sigma_e^2} \times \frac{\sigma_e^2}{\sigma_q^2} \quad \text{where } \sigma_e^2 = \text{variance of prediction error}$$

$$SNR = G_p (SNR)_p \quad \leftarrow \text{Prediction error to quantisation noise}$$

ASK

↳ Carrier - analog
↳ Info - digital
↳ o/p - analog.

* Duo Binary Pulse

let us take b_k as input

amplitude +1 = 1 symbol

amplitude -1 = 0 symbol

⇓

duo binary encoder

⇓

3 level output (c_k)

where c_k is sum of present binary digit b_k and its previous value

$$c_k = +2 \rightarrow b_k = b_{k-1} = 1$$

$$0 \rightarrow b_k \neq b_{k-1}$$

$$-2 \rightarrow b_k = b_{k-1} = 0$$

