B.E.

Seventh Semester Examination, Dec.-2007

MECHANIC AL VIBRATION

Note : Attempt any five questions.

Q. 1. (a) Distinguish between periodic and a periodic motion.

Ans. Periodic and a periodic motion : The motion which respects itself after an regular equal interval of time is known as periodic motion. The equal interval is called time period. If we consider a motion of the type $x_1 = A_1 \sin \omega t$, here ω is the natural frequency and the motion will be repeated after $2\pi/\omega$. The harmonic motion is are one of the form of periodic motion. The harmonic motion is repeated in terms of circular sine and cosine functions. All harmonic motions are periodic in nature but vice-versa is not always true. In the equation $x_1 = A_1 \sin \omega t$, x_1 is the displacement and A_1 the amplitude.

Q. 1. (b) A body is subjected to the two harmonic motions as $x_1 = 15 \sin\left(\omega t + \frac{\pi}{6}\right)$ and $x_2 = 8 \cos\left(\omega t + \frac{\pi}{3}\right)$ what extra motion should be given to the body to bring it to the static equilibrium.

Ans.

$$x_1 = 15 \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$x_2 = 8 \cos\left(\omega t + \frac{\pi}{3}\right)$$

$$x = x_1 + x_2$$

$$A \sin(\omega t + \alpha) = 15 \sin\left(\omega t + \frac{\pi}{6}\right) + 8 \cos\left(\omega t + \frac{\pi}{3}\right)$$

$$= 15[\sin \omega t + \cos 30° + \cos \omega t \sin 30°] + 8[\cos \omega t \cos 60° - \sin \omega t \sin 60°]$$

$$= 13 \sin \omega t + 7.5 \cos \omega t + 4 \cos \omega t - 6.928 \sin \omega t$$

$$= 6.071 \sin \omega t + 11.5 \cos \omega t$$

$$A \sin \omega t \cos \alpha + A \cos \omega t \sin \alpha = 6.071 \sin \omega t + 11.5 \cos \omega t$$

$$A \cos \alpha = 6.071$$

$$A \sin \alpha = 11.5$$

$$A = \sqrt{(6.071)^2 + (11.5)^2} = 13.004 \text{ units}$$

$$\tan\alpha = \frac{11.5}{6.071} = 1.894 \Rightarrow \alpha = 62.169°$$

The resultant equation is given by

$$x = 13.004\sin(\omega t + 62.169°)$$

To bring it in static equilibrium, the resultant should act opposite to it.

Q. 1. (c) A force $P_o \sin\omega t$ acts on a displacement $x_o \sin\left(\omega t - \frac{\pi}{6}\right)$ what is the work done during (a) first second (b) the first 1/40 second. Where, $P_o = 25\,N$; $x_0 = 0.5m$ and $\omega = 10\pi\ rad/sec.$

Ans.

$$w = \text{Work done} = \int_0^{t_1} F\frac{dx}{dt}dt$$

$$= P_o x_0 \omega \int_0^{t_1} \sin\omega t \cos(\omega t - 30°)\,dt$$

$$= P_o x_0 \omega \int_0^{t_1} \frac{1}{2}[\sin(2\omega t - 30°) + \sin 30°]\,dt$$

$$= \frac{P_o x_0 \omega}{2}\left[\frac{\cos(2\omega t - 30°)}{2\omega} + \frac{1}{2}t\right]_0^{t_1}$$

Substituting for F_o, x_o & ω, we have

$$\omega = \frac{25 \times 0.05 \times 10\pi}{2}\left[\left(\frac{\cos 2\pi t_1 - 30°}{2\pi}\right) + \frac{\cos 30°}{2\pi} + \frac{1}{2}t_1\right]$$

(i) When $t_1 = 1 sec.$

$$\omega = 19.637[0.0133 + 0.0137 + 0.5] = 10.343\,Nm$$

(ii) When $t = \frac{1}{40}$ sec.

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$$\omega = 19.637[0.0139 + 0.0137 + 0.0125] = 0.789\text{Nm}$$

Q. 2. (a) A mass of 1 kg is attached to a spring having a stiffness of 3920 N/m. The mass slides on a horizontal surface, the coefficient of friction between the mass & surface being 0.15. Determine the frequency of vibrations of the system and the amplitude after one cycle of initial amplitude is 0.25cm. Determine the rest position.

Ans.

$$m = 1\text{kg}, \ \mu = 0.15$$

$$K = 3920 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{3920}{1}}$$

$$= 62.6 \text{ rad/sec.}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{62.6}{2\pi} = 9.97\text{Hz}$$

$$\text{Force} = \mu mg$$

$$= 0.15 \times 1 \times 9.8$$

$$= 1.47\text{N}$$

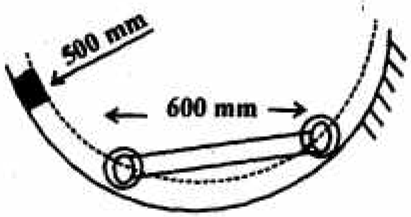
Reduction in amplitude/cycle

$$= \frac{4F}{K} = \frac{4 \times 1.47}{3920} = 0.0015\text{m}$$

$$\text{Finally the amplitude} = 0.0025 - 0.0015$$

$$= 0.001\text{m}$$

Q. 2. (b) A bar 600mm long rolls on wheels of negligible weight on a circular path with radius 500 mm as shown in fig. II (b). Determine the nat., freq., of oscillation for the bar if, it moves in the vertical plane when it is displaced slightly from its equilibrium position.



Ans. Translational distance $= (R - r)\sin\theta = (R - r)\theta$

$\therefore$ Translational velocity $= \frac{d}{dt}[R-r]\theta = (R-r)\dot{\theta}$

$\therefore$ $R\theta = r\phi$

Relative rotational (angular displacement) of the cylinder

$$= (\phi - \theta)$$

so, rotational velocity of the cylinder $= (\dot{\phi} - \dot{\theta})$

$$K.E. = (K.E.)_{translational} + (K.E.)_{rotational}$$

$$= \frac{1}{2}M(R-r)^2\dot{\theta}^2 + \frac{1}{2}I_0(\dot{\phi}-\dot{\theta})^2$$

Where, I_0 = Mass moment of inertia $= \frac{1}{2}Mr^2$

Potential energy $= Mg(R-r)(1-\cos\theta)$

Now, the total energy K.E.+P.E.= constant

$$= \frac{1}{2}M(R-r)^2\dot{\theta}^2 + \frac{1}{2}I_0(\dot{\phi}^2-\dot{\theta})^2 + Mg(R-r)(1-\cos\theta) =$$

constant

Replacing $\dot{\phi} = \frac{R}{r}(\dot{\theta})$, we get

$$\frac{1}{2}M(R-r)^2\dot{\theta}^2 + \frac{1}{4}M(R-1)^2\dot{\theta}^2 + Mg(R-r)(1-\cos\theta) = \text{constant}$$

$$\frac{1}{2}M(R-r)^2\dot{\theta}^2 + \frac{1}{4}M(R-1)^2\dot{\theta}^2 + Mg(R-r)(1-\cos\theta) = \text{constant}$$

$$\frac{1}{3}M(R-r)^2\dot{\theta}^2 + Mg(R-r)(1-\cos\theta) = \text{constant}$$

Differentiating the above equation w.r.t. time, we get

$$\frac{3}{4}(R-r)^2 2\dot{\theta}\ddot{\theta} + (R-r)g(\sin\theta).\dot{\theta} = 0$$

$$\frac{3}{2}(R-r)\ddot{\theta}+g\theta=0$$

$$\ddot{\theta}+\frac{2g}{3(R-r)}\theta=0$$

$$\omega_n=\sqrt{\frac{2g}{3(R-r)}}=\sqrt{\frac{2\times 9.81}{3\times(500-300)}}=0.1808\text{ rad / sec.}$$

Q. 3. (a) A mass (rotor) of 5 kg is mounted midway on a 1 cm dia shaft supported at the ends of the two bearings. The bearing span is 25 cm. Because of certain manufacturing inaccuracies, the CG of the disc is 0.025 mm away from the G.C. of the rotor. If the system rotates at 2500 rpm. Find the amplitude of steady state vibration and dynamic force transmitted to the bearing if (a) shaft is horizontal (b) shaft is vertical.

$$\mathbf{E=1.96\times 10^{11}\,N/m^2.}$$

Ans. Assuming simply supported beam, we get

$$\delta=\frac{\omega l^3}{48EI}=\frac{mgl^3}{48EI}\quad\text{or}\quad\frac{mg}{\delta}=\frac{48EI}{l^3}$$

$$K=\frac{48EI}{l^3}=\frac{48\times 1.96\times 10^{11}\times\frac{\pi}{64}\times(0.01)^4}{(0.25)^3}=295680\text{ N/m}$$

$$\omega_n=\sqrt{\frac{k}{m}}=\sqrt{\frac{295680}{5}}=243.178\text{ rad / sec.}$$

$$\omega=\frac{2\pi N}{60}=\frac{2\pi\times 2500}{60}=261.90\text{ rad / sec.}$$

$$y=\frac{\omega^2 l}{\omega_m^2-\omega^2}=\frac{(261.9)^2\times 0.025\times 10^{-3}}{(243.178)^2-(261.9)^2}=\frac{1.714}{-9456}=-1.812\times 10^{-4}$$

Load on each bearing $$=\frac{kg}{2}=-\left\{\frac{295680\times 1.812\times 10^{-4}}{2}\right\}$$

$$=-26.788\text{ N}$$

Q. 3. (b) A seismic instrument with a natural frequency of 6 Hz is used to measure the vibration of a machine running at 120 rpm. The instrument gives the reading for the relative displacement of the

seismic mass as 0.05mm. Determine the amplitude of displacement, velocity and acceleration of vibrating machine. Neglect damping.

Ans. Here, $F_n = 6\,Hz$

$N = 120$ rpm

Reading of instrument

$Z = 0.05$ mm

The controlling equation is

$$\frac{Z}{B} = \frac{r^2}{\sqrt{\left(1-r^2\right)^2 + (2\zeta r)^2}}$$

$$= \frac{r^2}{\left(1-r^2\right)} \text{ (when } \xi = 0.0\text{)}$$

As $$F = \frac{N}{60} = \frac{120}{60} = 2Hz$$

$$r = \frac{2}{6} = 0.33$$

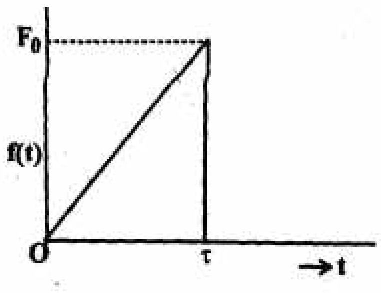
Using the above equation, we get

$$\frac{0.05}{B} = \frac{(0.33)^2}{\left(1-(0.33)^2\right)}$$

$$\frac{0.05}{B} = 0.122$$

$$B = 0.409 \text{ mm}$$

Q. 4. (a) An undamped spring mass system is subjected to a saw-tooth pulse shown in fig. 4(a). Obtain response equation.



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Ans. The equation of motion can be written as

$$m\ddot{x} + kx = F[u(t) - u(t-T)]$$

Applying Laplace transform to the differential equation,

$$L[m\ddot{x}] = m[S^2X(s) - Sx_0 - \dot{x}_0]$$

$$L[kx] = kX(s)$$

$$LF[u(t)] = \frac{F}{S}$$

$$LF[u(t-T)] = \frac{Fe^{-ST}}{S}$$

Initial conditions : $x(0) = 0$, $\dot{x}(0) = 0$

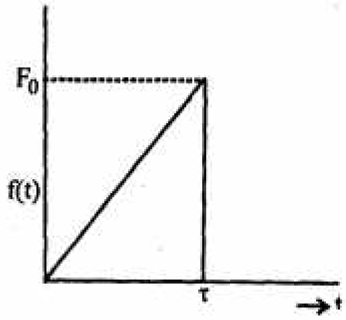
Substitution the values in equation (1), we get

$$ms^2X(s) + KX(s) = \frac{F}{s} - \frac{Fe^{-ST}}{s}$$

$$= \frac{F}{s}\left(1 - e^{-ST}\right)$$

$$\left(ms^2 + k\right)X(s) = \frac{F}{s}\left(1 - e^{-ST}\right)$$

$$X(s) = \frac{F}{s\left(1 - e^{-ST}\right) / ms^2 + k} \quad X(s) = \frac{F}{s\left(1 - e^{-ST}\right) / \left(ms^2 + k\right)}$$



Defining $\omega_n^2 = \frac{k}{m}$ in the above equation,

$$X(s) = \frac{F}{m}\left[\frac{1-e^{-ST}}{s\left(s^2 + w_n\right)}\right]$$

From the table of Laplace transforms, the inverse,

$$L^{-1}\left[\frac{1}{s\left(s^2 + w_n^2\right)}\right] = \frac{1}{w_n^2}\left(1 - \cos w_n t\right)$$

And, $$L^{-1}\left[\frac{e^{-ST}}{s\left(s^2 + w_n^2\right)}\right] = \frac{1}{w_n^2}\left[1 - \cos w_n (t-T)\right] u(t-T)$$

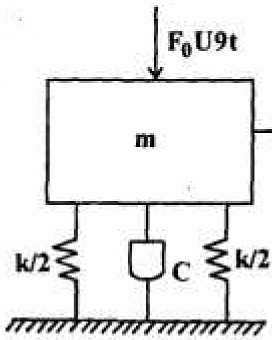
For, $0 < t < T$, the solution is

$$x(t) = \frac{F}{mw_n^2}\left(1 - \cos w_n t\right)$$

And for $t > T$, the solution is given by

$$x(t) = \frac{F}{mw_n^2}\left[\left(1 - \cos w_n t\right) - 1 - \cos w_n t(t-T)\right]$$

Q. 4. (b) A spring-mass system is shown in fig. 4(b). If the system is initially released and a step-function excitation is applied to the mass, find the response of the system .



Ans. When the container hits the hard floor, its velocity is

$$\dot{x} = \sqrt{2gh}$$

The equation of motion when the container is in contact with the hard floor

$$m\ddot{x} + kx = 0$$

The initial conditions are

$$x(0) = 0$$

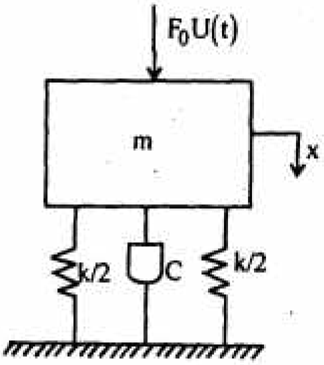
$$\dot{x}(0) = \sqrt{2gh}$$

Applying the Laplace transform to the differential equation of motion

$$L[m\ddot{x} + kx] = 0$$

$$m\left[s^2X(s) - sx_0 - \dot{x}_0\right] + KX(s) = 0$$

$$\left[s^2X(s) - \sqrt{2gh}\right]6 + \frac{k}{m}X(s) = 0$$



$$\left[s^2X(s) - \sqrt{2gh}\right] + w_n^2X(s) = 0$$

$$X(s) = \frac{\sqrt{2gh}}{s^2 + w_n^2}$$

Taking the Laplace transform, inverse

$$x(t) = L^{-1}X(s)$$

$$= L^{-1}\frac{\sqrt{2gh}}{w_n}\cdot\frac{1.w_n}{s^2+w_n^2}$$

$$= \sqrt{2gh}L^{-1}\frac{w_n}{s^2+w_n^2}$$

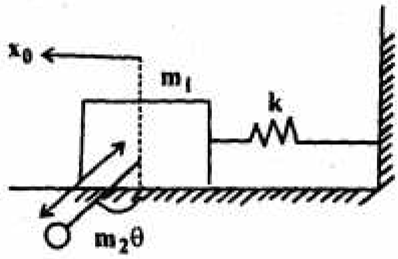
$$x(t) = \frac{\sqrt{2gh}}{w_n}\sin w_n t$$

Maximum acceleration of the mass

$$x_{max.} = \frac{\sqrt{2gh}}{w_n}.w_n^2 = w_n\sqrt{2gh}.$$

Q. 5. (a) Find the frequencies of the system shown in fig. 5(a).

$k = 90$ N/m; $l = 0.25$m; $m_1 = 2$kg, $m_2 = m_1/4$



Ans. Here $k = 90$ N/m, $l = 0.25$ m

$$m_1 = 2\text{kg},\ m_2 = \frac{m_1}{4} = 0.5\text{kg}$$

Resolving the forces vertically for m_2

$$m_2 g = T\cos\theta$$

When P is very small $\sin\theta \simeq \theta$ and $\cos\theta = 1$

So, horizontal displacement $= x + \rho\theta$

and acceleration $= \ddot{x} + \rho\ddot{\theta}$

Horizontal force $m_2(\ddot{x} + \rho\ddot{\theta}) = -T\theta$

So, $m_{2\theta} = T$ and $m_2(\ddot{x} + \rho\ddot{\theta}) = -T\theta$

$$m_2(\ddot{x} + \rho\ddot{\theta}) + T\theta = 0$$

$$m_2(\ddot{x} + \rho\ddot{\theta}) + m_{2g}\theta = 0\text{, put } T = m_2 g$$

$$(\ddot{x} + \rho\ddot{\theta}) + \rho\theta = 0$$

$$l\ddot{\theta} + g\theta = -\ddot{x}$$

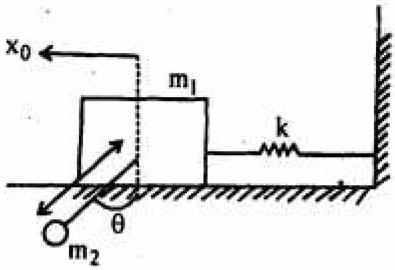
Consider the force for mass m_1. All forces are acting horizontally

$$m_1\ddot{x} = -k_x + T\sin\theta$$

$$= -kx + T\theta$$

$$m_1\ddot{x} + kx - T\theta - 9$$

Putting $T = m_{2g}$



$$m_1\ddot{x} + kx = m_2\theta = 0$$

$$m_1\ddot{x} + kx = m_{2g}\theta$$

Let us assume the solution of the form

$x = A\sin wt$ and $P = \phi\sin wt$

Substituting these solutions in the above two equation, we get

$$-lw^2\phi + g\phi - w^2 A = 0$$

$$\&, -m_1 w^2 A + KA - m_2 g\phi = 0$$

The frequency equation can be written as

$$\left(-lw^2+g\right)\left(k-m_1w^2\right)-w^2m_2g=0$$

$$-klw^2+m_1lw^4+gk-m_1gw^2-w^2m_2g=0$$

$$w^4=\frac{(kl+m_1g+m_2g)w^2}{m_1l}+\frac{gk}{m_1l}=0$$

So, $$w^2=\frac{(m_1-m_2)g+kl\pm\sqrt{[(m_1+m_2)g+kl]^2-4ml_1lkg}}{2m_1l}$$

Substituting the numerical values in the above equation,

$$w^2=\frac{(2+0.50)81+90\times0.25\pm\sqrt{[(2+0.5)9.81+90\times2.5]^2-4\times2\times0.25\times90\times9.81}}{2\times2\times0.25}$$

$$=24.5+22.5\pm\sqrt{(24.5+22.5)^2-1764}$$

$$=47\pm\sqrt{2209+1764}$$

$$=47\pm21.095$$

$$w_1=8.25\,\text{rad/sec.}$$

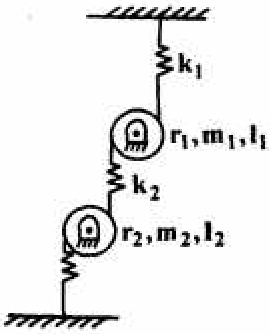
$$w_2=5.08\,\text{rad/sec.}$$

Q. 5. (b) Find nat. frequencies of the system shown in fig. (b),

$$\mathbf{K_1=40\times10_3N/m\ ;\ K_2=50\times10_3N/m\ ;}$$

$$\mathbf{K^3=60\times10_3N/m}$$

$$\mathbf{m_1=10kg\ ;\ m_2=12kg\ ;\ r_1=.1m\ ;\ r_2=.11m}$$



Ans. The torque equation is $\Sigma T = I\ddot{\theta}$

The equations of motion can be written as,

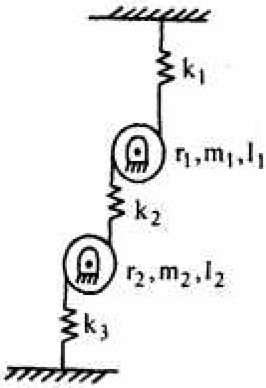
$$I_1\ddot{\theta}_1 = -K_1(r_1\theta_1)r_1 - K_2(r_1\theta_1 - r_2\theta_2)r_1$$

$$I_1\ddot{\theta}_1 + \left(K_1r_1^2 + K_2r_1^2\theta_1 - K_2r_1r_2\theta_2\right) = 0$$

Similarly

$$I_2\ddot{\theta}_2 = -K_3(r_2\theta_2)r_2 - K_2(r_2\theta_2 - r_1\theta_1)r_2 = 0$$

$$I_2\ddot{\theta}_2 + \left(K_3r_2^2 + K_2r_2^2\right)P_2 - K_2r_1r_2\theta_1 = 0$$



Assuming that the solution of the form

$$\theta_1 = \phi_1 \sin wt\ ,\ \ddot{\theta}_1 = -w^2\phi_1 \sin wt$$

$$\theta_2 = \phi_2 \sin wt,\ \ddot{\theta}_2 = -w^2\phi_2 \sin wt$$

Substituting these values in above equation, we get

$$= I_1 w^2 \phi_1 + \left(K_1 r_1^2 + K_2 r_1^2\right)\phi_1 - K_2 r_1 r_2 \phi_2 = 0$$

$$= I_2 w^2 \phi_2 + \left(K_3 r_2^2 + K_2 r_2^2\right)\phi_2 - K_2 r_1 r_2 \phi_1 = 0$$

$$\frac{\phi_1}{\phi_2} = \frac{K_1 r_1 r_2}{K_2 r_1^2 + K_2 r_1^2 - I_1 w^2} = \frac{K_3 r_2^2 + K_2 r_2^2 - I_2 w^2}{K_2 r_1 r_2}$$

$$\left(K_3 r_2^2 + K_2 r_2^2 - I_2 w^2\right)\left(K_1 r_1^2 + K_2 r_1^2 - I_1 w^2\right) - K_2^2 r_1^2 r_2^2 = 0$$

$$K_1 K_3 r_1^2 r_2^2 + K_2 K_3 r_1^2 r_2^2 - I_1 w^2 K_3 r_2^2 + K_1 K_2 r_1^2 + K_2^2 r_1^2 r_2^2$$

$$-w^2 I_1 K_2 r_2^2 - I_2 w^2 K_1 r_1^2 - I_2 w^2 K_2 r_1^4 = 0$$

$$-K_2^2 r_1^2 r_2^2 = 0$$

$$w^4 I_1 I_2 - w^2\left(I_1 K_3 r_2^2 + I_1 K_2 r_2^2 + I_2 K_1 r_1^2 + I_2 K_2 r_1^2\right)$$

$$+K_1 K_3 r_1^2 r_2^2 + K_2 K_3 r_1^2 r_2^2 + K_1 K_2 r_1^2 r_2^2 = 0$$

$$w^4 - w^3\left(\frac{K_3 r_2^2}{I_2} + \frac{K_2 r_2^2}{I_2} + \frac{K_1 r_1^2}{I_1} + \frac{K_2 r_1^2}{I_1}\right)$$

$$+\frac{K_1 K_3 r_1^2 r_2^2}{I_1 I_2} + \frac{K_2 K_3 r_1^2 r_2^2}{I_1 I_2} + \frac{K_1 K_2 r_1^2 r_2^2}{I_1 I_2} = 0$$

Since, $I_1 = \frac{1}{2} m_1 r_1^2$ and $I_2 = \frac{1}{2} m_2 r_2^2$

$$\frac{K_1 K_2 r_1^2 r_2^2}{\frac{1}{2} m_1 r_1^2 \cdot \frac{1}{2} m_2 r_2^2} = \frac{4 K_1 K_3}{m_1 m_2}$$

$$\frac{K_2K_3r_1^2r_2^2}{I_1I_2} = \frac{K_2K_3r_1^2r_2^2}{\frac{1}{2}m_1r_1^2 + \frac{1}{2}m_2r_2^2} = \frac{4K_2K_3}{m_1m_2}$$

$$\frac{K_1K_3r_1^2r_2^2}{I_1I_2} = \frac{4K_1K_3}{m_1m_2}$$

$$w^4 - w^2\left(\frac{2K_3}{m_2} + \frac{2K_2}{m_2} + \frac{2K_1}{m_1} + \frac{2K_2}{m_1}\right) + \frac{4}{m_1m_2}(K_1K_3 + K_2K_3 + K_1K_2) = 0$$

This is the frequency equation

Putting the values of various terms in the above equation, we get

$$w^4 - w^2\left(\frac{2\times 60\times 10^3}{12} + \frac{2\times 50\times 10^3}{12} + \frac{2\times 40\times 10^3}{10} + \frac{2\times 50\times 10^3}{10}\right)$$

$$+\frac{4}{10\times 12}(40\times 60\times 10^6 + 50\times 60\times 10^6 + 40\times 50\times 10^6) = 0$$

$$w^4 - 36.3333\times 10^3 w^2 + 246.66\times 10^6 = 0$$

$$w^2 = \frac{36.33\times 10^3 \pm\sqrt{(36.33)^2\times 10^6 - 4\times 246.66\times 10^6}}{2}$$

$$w_1 = 165.2 \text{ rad/sec.}$$

$$w_2 = 95.06 \text{ rad/sec.}$$

Q. 6. (a) A I.C. engine has a mass of 40 kg and run at a constant speed 2500 rpm. After it was installed it vibrated with a large amplitude at operating speed. What dynamic vibration absorber should be coupled to the system if the nearest resonant frequency of the combined system has to be at least 25% away from the operating speed.

Ans. Here, $m_1 = 40\text{ kg}$, $N = 2500$ rpm

$$w_2 = 0.75w$$

As $$\left(\frac{w}{w_2}\right)^2 = 1 + \frac{\mu}{2} \pm\sqrt{\mu + \frac{\mu^2}{4}}$$

$$w = \frac{2\pi N}{60} = \frac{2\pi \times 2500}{60} = 261.79 \text{ rad / sec.}$$

$$w_2 = 0.75 w$$

$$= 196.35 \text{ rad / sec.}$$

Using the above relation

$$\left(\frac{261.79}{196.35}\right)^2 = 1 + \frac{\mu}{2} \pm \sqrt{\mu \frac{\mu^2}{4}}$$

$$1.77 = 1 + \frac{\mu}{2} \pm \sqrt{\mu + \frac{\mu^2}{4}}$$

$$\left(0.77 - \frac{\mu}{2}\right)^2 = \left(\mu + \frac{\mu^2}{4}\right)$$

$$1.77\mu = 0.6049$$

$$\mu = 0.3417$$

And we know that mass ratio

$$\mu = \frac{m_2}{m_1}$$

$$m_1 = 40 \text{ kg}$$

$$m_2 = 0.3417 m_2$$

$$= 0.3417 \times 40 = 13.67 \text{ kg}$$

For vibration absorber, we have

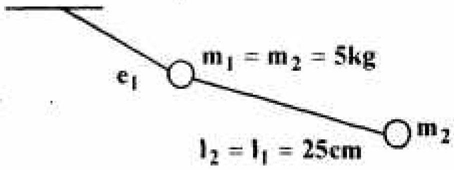
$$w_1 = w_2$$

$$\frac{k_1}{m_1} = \frac{k_2}{m_2}$$

$$(261.79)^2 = \frac{k_2}{13.67}$$

$$k_2 = 1.34 \times 10^6 \text{ N / m.}$$

Q. 6. (b) Determine the nat. frequency of oscillation of the double pendulum as shown in fig. (b).

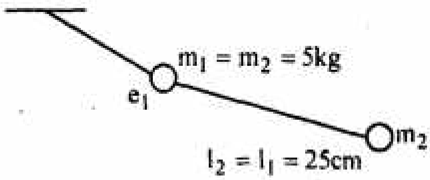


Ans. The equation of motion for two masses are given as

$$\theta_1\ddot{\theta}_1 = -m_1 g l_1\theta_1 - m_2 l_1\theta_2 - m_2\left(l_1\ddot{\theta}_1 + l_2\ddot{\theta}_2\right)l_1$$

$$m_1 l_1^2\ddot{\theta}_1 = -m_1 g l_1\theta_1 - m_2 g l_1\theta_1 - m_2\left(l_1\ddot{\theta}_1 + l_2\ddot{\theta}_2\right)l_1 \qquad ...(1)$$

& $$m_2 l_2^2\ddot{\theta}_2 = -m_2 g l_2\theta_2 - m_2 g l_1 l_2\ddot{\theta}_1$$



$$\ddot{\theta}_2 l_2^2 + g l_2\theta_2 + l_1 l_2\ddot{\theta}_1 = 0$$

$$\ddot{\theta}_2 + \frac{l_1 l_2}{l_2^2}\ddot{\theta}_1 + \frac{g l_2\theta_2}{l_2^2} = 0$$

$$\ddot{\theta}_2 + \frac{l_1}{l_2}\ddot{\theta}_1 + \frac{\theta}{l^2}\theta_2 = 0 \qquad ...(2)$$

Equation (1) can be written in simplified form as

$$\ddot{\theta}_1 + \frac{m_2 l_2}{(m_1 + m_2)l_1}\ddot{\theta}_2 + \frac{g}{l_1}\theta_1 = 0 \qquad ...(3)$$

Let us assume the solution of the form

$$\theta_1 = A_1 \sin wt$$

$$\theta_2 = A_2 \sin wt$$

So, $$\ddot{\theta}_1 = -w^2 A_1 \sin wt,$$

$$\ddot{\theta}_2 = -w^2 A_2 \sin wt$$

Putting these terms in equation (2) & (3) we get

$$-w^2 A_2 + \frac{l_1}{l_2}\left(-w^2 A_1\right) + \frac{g}{l_2} A_2 = 0$$

$$\left(-w^2 + \frac{g}{l_2}\right) A_2 - w^2 \frac{l_1}{l_2} A_1 = 0$$

$$\frac{A_1}{A_2} = \frac{-w^2 + \frac{g}{l_2}}{w^2 \frac{l_1}{l_2}} \qquad \ldots(4)$$

And $\quad -w^2 A_1 + \frac{m_2 l_2}{(m_1 + m_1) l_1}\left(-w^2 A_2\right) + \frac{g}{l_1} A_1 = 0$

$$\left(-w^2 + \frac{g}{l_1}\right) A_1 - \frac{m_2 l_2 w^2}{(m_1 + m_2) l_1} A_2 = 0$$

$$\frac{A_1}{A_2} = \frac{m_2 l_2 w^2}{(m_1 + m_2) l_1 \left(-w^2 + \frac{g}{l_1}\right)}$$

The frequency equation can be written as,

$$\frac{-w^2 + \frac{g}{l_2}}{w^2 \frac{l_1}{l_2}} = \frac{m_2 l_2 w^2}{(m_1 + m_2) l_1 \left(-w^2 + \frac{g}{l_1}\right)}$$

$$m_2 w^4 = \left(-w^2 + \frac{g}{l_2}\right)(m_1 + m_2)\left(-w^2 + \frac{\theta}{l_1}\right)$$

$$w^4 = \frac{m_1 + m_2}{m_2}\left(w^4 - w^2 \frac{g}{l_1} - w^2 \frac{g}{l_2} + \frac{g^2}{l_1 l_2}\right) = 0$$

$$w^4 - \frac{m_1+m_2}{m_2}\left(w^4 - w^2\frac{g}{l_1} - w^2\frac{g}{l_2} + \frac{g^2}{l_1l_2}\right) = 0$$

$$w^4\left(1 - \frac{m_1+m_2}{m}\right) + \frac{m_1+m_2}{m_2}w^2g\left(\frac{1}{l_1}+\frac{1}{l_2}\right) - \left(\frac{m_1+m_2}{m_2}\right)\frac{g^2}{l_1l_2} = 0$$

$$w^4\frac{m_1}{m_2} - \left(\frac{m_1+m_2}{2}\right)w^2\frac{(l_1+l_2)}{l_1l_2}g + \frac{(m_1+m_2)}{m_2}\cdot\frac{g^2}{l_1l_2} = 0$$

$$w^4 - \frac{(m_1+m_2)w^2}{m_1}\frac{(l_1+l_2)g}{l_1l_2} + \frac{(m_1+m_2)g^2}{m_1l_1l_2} = 0$$

This is the frequency equation

$$m_1 = m_2 = 5\text{kg},\ l_1 = l_2 = 25\text{cm}$$

$$w^4 - \frac{2m}{m}w^2\frac{2l}{l^2}g + \frac{2mg^2}{ml^2} = 0$$

$$w^4 - 4w^2\frac{g}{l} + \frac{2g^2}{l^2} = 0$$

Given $l = 0.25$ m

$$w^4 - \frac{4w^2\times 9.8}{0.25} + \frac{2\times 9.8^2}{(0.25)^2} = 0$$

$$w^4 - 156.8w^2 + 3073.28 = 0$$

$$w^2 = \frac{1568 \pm \sqrt{(156.8)^2 - 4\times 3073.28}}{2}$$

$$w_{1,2}^2 = \frac{156.8 \pm 110.87}{2}$$

$$w_1 = 11.56\text{ rad / sec.}$$

$$w_2 = 4.8\text{ rad / sec.}$$

Q. 7. (a) Determine the frequency equation is transverse vibration for a uniform beam of length l having one end fixed and other simply supported.

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Ans. Transverse vibration of beams : Net forces acting on the element,

$$Q - \left(Q + \frac{\partial Q}{\partial x} dx\right) = dm \text{ acceleration.}$$

$$-\frac{\partial \theta}{\partial x} dx = (\rho A dx) \frac{\partial^2 y}{\partial t^2}$$

$$\frac{\partial \theta}{\partial x} + \rho A \frac{\partial^2 y}{\partial t^2} = 0 \quad ...(1)$$

Considering the moment about A, we get

$$M - \left(M + \frac{\partial M}{\partial x} dx\right) + \left(\theta + \frac{\partial \theta}{\partial x} dx\right) = 0$$

$$-\frac{\partial M}{\partial x} + \theta + \frac{\partial \theta}{\partial x} dx = 0$$

So, $\theta = \frac{\partial M}{\partial x}$ higher order derivatives are neglected $\left(\frac{\partial \theta}{\partial x} dx = 0\right)$

Or $$\frac{\partial \theta}{\partial x} = \frac{\partial^2 M}{\partial x^2} \quad ...(2)$$

From the above two equations (1) & (2), we get

$$\frac{\partial^2 M}{\partial x^2} = -\rho A \frac{\partial^2 y}{\partial t^2} \quad ...(3)$$

We know that from beam theory that

$$M = -EI \frac{\partial^2 y}{\partial x^2}$$

So, $$\frac{\partial^2 M}{\partial x^2} = -EI \frac{\partial^2 y}{\partial x^2} \quad ...(4)$$

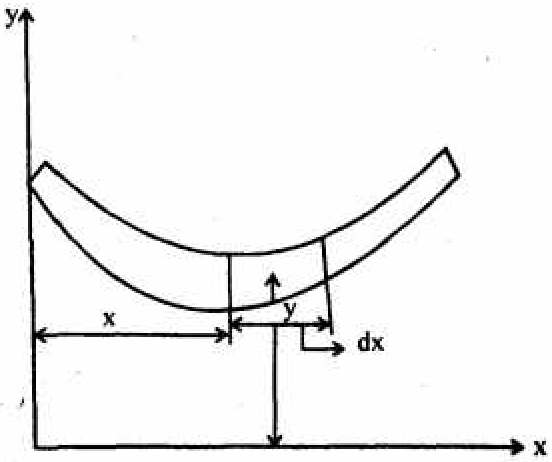
Comparing equation (3) & (4) we get

$$EI \frac{\partial^2 y}{\partial x^2} + \rho A \frac{\partial^2 y}{\partial t^2} = 0$$

n/

$$\frac{\partial^2 y}{\partial x^2} + \left(\frac{\rho A}{EI}\right)\frac{\partial^2 y}{\partial t^2} = 0 \qquad ...(5)$$

This is the general equation for transverse vibrate which is different from wave equation.



Let us assume the solution of the form

$$y = y(x)\sin(wt + \phi) \qquad ...(6)$$

Where y(x) is the shape of the beam for principal mode of vibrations equation (5) can be written with the help of the above equation.

$$\frac{\partial^4 y}{\partial x^4} - c^4 y = 0 \qquad ...(7)$$

Where, $$c^4 = \frac{\rho A}{EI} w^2$$

This is fourth-order differential equation. To find the solution, let us assume

$$y = e^{\lambda x}$$

So, $$\frac{\partial^4 y}{\partial x^4} = \lambda^4 e^{\lambda x}$$

Equation (7) can be written as

$$\lambda^4 e^{\lambda x} - c^4 e^{\lambda x} = 0$$

$$\lambda^4 - c^4 = 0 \quad ...(8)$$

$$(\lambda + c)(\lambda - c)(\lambda^2 + c^2) = 0$$

$$\lambda_{1,2} = \pm c$$

$$\lambda_{3,4} = +ic \text{ where, } i = \sqrt{-1}$$

$$e^{cn} = \cosh cx + \sinh cx$$

$$e^{-cx} = \cosh cx - \sinh cx$$

$$e^{icx} = \cos cx + i \sin cx$$

$$e^{-icx} = \cos cx - i \sin cx$$

So, the solution of differential equation can be written as

$$y = c_1(\cosh cx + \sinh cx) + c_2(\cos cx - \sinh cx)$$

$$+ c_3(\cos cx + i \sin(x)) + c_4(\cos cx - i \sin(x))$$

$$= (c_1 + c_2)\cosh cx + (c_1 - c_2)\sinh cx + (c_3 + c_4)\cos(x + (c_3 - c_4)i \sin cx)$$

$$y(x, t) = A \cosh cx + B \sinh cx + C \cos cx + D \sin cx \quad ...(9)$$

&

$$A = c_1 + c_2$$

$$B = c_1 - c_2$$

$$C = c_3 + c_4$$

$$D = i(c_3 - c_4)$$

Where A, B, C and D are constants.

Q. 7. (b) A uniform circular shaft (mp supported beam) of length l is deflected by a force P applied at a point distance c from one end. Find the resulting transverse vibrations when the load is suddenly removed.

Ans. If c is the displacement at a distance x from left and it becomes $c + \frac{\partial x}{\partial x} dx$ at a distance $x + dx$. So, strain of the element is given by

$$E = \frac{\frac{\partial c}{\partial x}dx}{dx} = \frac{\partial u}{\partial x} \qquad ...(1)$$

Let $A \rightarrow$ Cross-sectional area of bar.

$\rho \rightarrow$ Density of material.

$E \rightarrow$ Modules of elasticity of material.

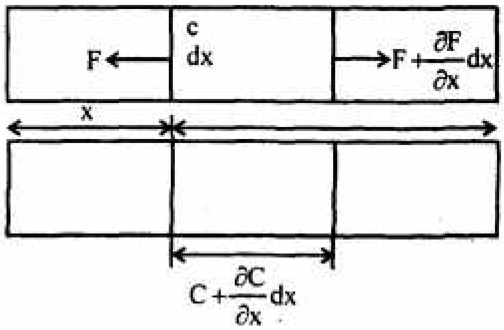
$F \rightarrow$ Force acting axially on the bar.

Net force acting on the element.

$$\left(F + \frac{\partial F}{\partial x}dx\right) - F = \text{(mass)} \times \text{(acceleration of the element)}$$

$$= dm \times \frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial F}{\partial x}dx = (\rho dx A)\left(\frac{\partial^2 A}{\partial t^2}\right) \qquad ...(2)$$



We know that $\frac{F}{A} = 0$, where σ is the stress, so

$$F = \sigma A$$

$$\frac{\partial F}{\partial x} = \frac{\partial \sigma}{\partial x}A$$

$$\left(\frac{\partial F}{\partial x}\right)dx = \left(\frac{\partial \theta}{\partial x}\right) dxA \qquad ...(3)$$

Equation (2) can be written with the help of above equation as

$$\left\{\frac{\partial\sigma}{\partial x}\right\}dxA = (\rho dxA)\left\{\frac{\partial^2 u}{\partial t^2}\right\} \qquad ...(4)$$

According to Hooke's law

$$\frac{\text{Stress}}{\text{Strain}} = E$$

$$\frac{\sigma}{\epsilon} = E$$

$$\sigma = \epsilon E$$

$$\frac{\partial\sigma}{\partial x} = \frac{\partial \epsilon}{\partial x}E$$

$$\left(\frac{\partial\sigma}{\partial x}\right)dx\,A = \left(\frac{\partial \epsilon}{\partial x}\right)dx\,AE \qquad ...(5)$$

We have

$$\left(\frac{\partial \epsilon}{\partial x}\right)dx\,AE = \rho dxA\left(\frac{\partial^2 u}{\partial t^2}\right)$$

$$\frac{E}{q}\frac{\partial \epsilon}{\partial x} = \frac{\partial^2 u}{\partial t^2}$$

$$\left(\frac{E}{\rho}\right)\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial^2 u}{\partial t^2}$$

$$\frac{E}{\rho}\left(\frac{\partial^2 u}{\partial x^2}\right) = \frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{a^2}\frac{\partial^2 u}{\partial t^2} \qquad ...(6)$$

Where $\quad a^2 = \dfrac{E}{\rho}$

A solution of the form as in equation (5)

$$u(x,t) = X(x)T(t)$$

So, $$X(n) = A\sin\frac{\rho x}{a} + B\cos\frac{\rho x}{a},\ T(t) = C\sin\rho t + D\cos\rho t$$

Into the general solution as

$$u(x,t) = \sum_{n=1}^{\infty}\left(A\sin\frac{\rho}{a}x + B\cos\frac{\rho x}{a}\right)(C\sin\rho t + D\cos\rho t).$$

Q. 8. Write short notes on the following :

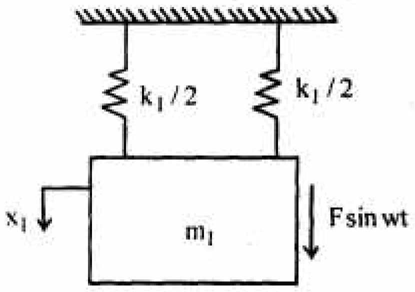
(i) Vibration Absorber

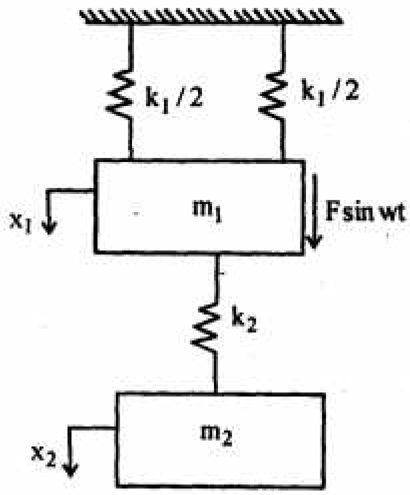
(ii) Holzer (type) method Multidegree freedom system.

(iii) Ralighs method

(iv) Secondary vertical speed in rotating shafts.

Ans. (i) Vibration Absorber : When a structure externally excited has undesirable vibrations, it becomes necessary to eliminate them by coupling some vibrating system to it. The vibrating system is known as vibration absorber.

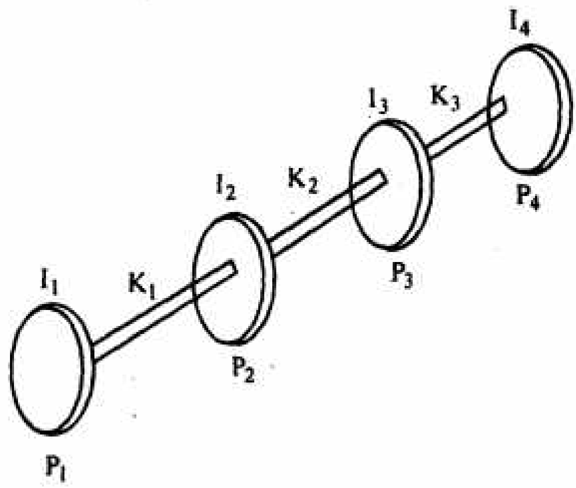




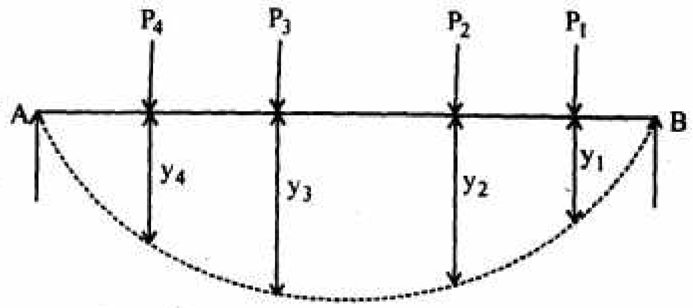
In such cases the excitation frequency is nearly equal to the natural frequency of the structure or machine. The mass which is excited can have zero amplitude of vibration and the spring mass system which is coupled to it vibrates freely. Vibration absorbers are used to control structural resonance.

(ii) Holzer (type) Method Multidegree Freedom System :

Holzer Method : This is trial and error method used to find the natural frequency and mode shape of multimass lumped parameter system. This can be applied to both free and forced vibrations. This method can be used for the analysis of damped, undamped, semidefinite systems with fixed ends having linear and angular motions. First of all, a trial frequency of the system is assumed. A solution is found when the trail frequency satisfies the constraints of the system.



(iii) Rayleighs Method : This is the energy method to find the frequency. This method is used to find the natural frequency of the system when transverse point loads are acting on the beam or shaft.



Good estimate of fundamental frequency can be made by assuming the suitable deflection curve for the fundamental mode. The maximum kinetic energy is equated to maximum potential energy of the system to determine the natural frequency.