B.Tech.

Fifth Semester Examination

Dynamics of Machines (ME-301F)

Q. 1. (i) State D'Alembert's Principle.

Ans. D'Alembert's Principle : It states that the inertia forces and couples, and the external forces and torques on a body together give statical equilibrium

i.e., $F_1 + F_2 + F_3 + \ldots\ldots\ldots\ldots + F_n = 0$

& $T_{g1} + T_{g2} + T_{g3} + \ldots\ldots\ldots\ldots + T_{gn} = 0$

Q. 1. (ii) What is the function of flywheel?

Ans. Function of Flywheel : A flywheel is used to control the variations in speed during each cycle of an engine. A flywheel of suitable dimensions attached to the crankshaft, makes the moment of inertia of the rotating parts quite large and thus, acts as a reservoir of energy.

During the period when the supply of energy is more than required, it stores energy and during the periods the requirements is more than supply, it releases energy.

Q. 1. (iii) Why is balancing of rotating parts necessary for high speed engines?

Ans. The high speed of engines and other machines is a common phenomenon now-a-days. It is therefore, very essential that all the rotating and reciprocating parts should be completely balanced as far as possible. If these parts are not properly balanced, the dynamics forces are set up. These forces not only increase the loads on bearings and stresses in the various members, but also produce unpleasant and even dangerous vibrations.

Q. 1. (iv) A single cylinder reciprocating engine has speed 240 r.p.m., stroke 300 mm, mass of reciprocating parts 50kg, mass of revolving parts at 150 mm radius 37 kg. If two third of the reciprocating parts and all the revolving parts are to be balanced find (i) The balance mass required at a radius of 400 mm, and (ii) The residual unbalanced force when the crank has rotated 60° from top dead center.

Ans. Given : $N = 240\,\text{rpm}$ or $\omega = 2\pi \times \frac{240}{60} = 25.14\ \text{rad/s}$

Stroke $= 300\,\text{mm} = 0.3\,\text{m}$; $m = 50\,\text{kg}$, $m_1 = 37\,\text{kg}$

$r = 150\,\text{mm} = 0.15\,\text{m}$; $c = 2/3$

Balance mass required

Let B = Balance mass required

b = Radius of rotation of the balance mass $= 400\,\text{mm}$

$= 0.4\,\text{m}$

We know that

$$B.b = (m_1 + c.m)r$$

$$B \times 0.4 = \left(37 + \frac{2}{3} \times 50\right) 0.15 = 10.55$$

Or $\qquad B = 26.38 \text{ kg}$ **Ans.**

Residual unbalanced force

Let θ = Crank angle from top dead centre = $60°$

Residual unbalanced force $= m.\omega^2 r \sqrt{(1-c)^2 \cos^2 \theta + c^2 \sin^2 \theta}$

$$= 50(25.14)^2\, 0.15 \sqrt{\left(1-\frac{2}{3}\right)^2 \cos^2 60° + \left(\frac{2}{3}\right)^2 \sin^2 60°}\ N$$

$$= 4740 \times 0.601 = 2849 \text{ N}$$ **Ans.**

Q. 1. (v) What is sensitiveness of governor?

Ans. Sensitiveness of Governor : A governor is said to be sensitive when it readily responds to a small change of speed. The movement of the sleeve for a fractional change of speed is the measure of sensitivity.

$$\text{Sensitiveness} = \frac{\text{Mean speed}}{\text{Range of speed}}$$

$$= \frac{N}{N_2 - N_1}$$

$$= \frac{1}{2}\frac{N_2 + N_1}{N_2 - N_1}$$

Where, N = Mean speed

N_1 = Minimum speed corresponding to full load conditions

N_2 = Maximum load corresponding to no-load conditions.

Q. 1. (vi) State different types of governors.

Ans. Types of Governors : Governors may be classified as :

(i) Centrifugal governors and	(ii) Inertia governors
— Watt governor	— Porter governor
— Proell governor	— Hartnell governor
Hartung governor.	

Q. 1. (vii) Distinguish between brakes and dynamometer.

Ans. A brake is a device by means of which artificial frictional resistance is applied to a moving machine member, in order to retard or stop the motion of a machine.

A dynamometer is a brake but in addition it has a device to measure the frictional resistance knowing the frictional resistance, we may obtain the torque transmitted and hence the power of the engine.

Q. 1. (viii) Write a short note on gyroscope.

Ans. Gyroscope : A gyroscope is a device for measuring or maintaining orientation, based on the principles of conservation of angular momentum. In essence, a mechanical gyroscope is a spinning wheel or disk whose axle is free to take any orientation. This orientation changes much less in response to a given external torque than it would without the large angular momentum associated with the gyroscopes high rate of spin. Since external torque is minimized by mounting the device in gimbals, if orientation remains nearly fixed regardless of any motion of the platform on which it is mounted.

Q. 1. (ix) An aeroplane makes a complete half circle of 50 meters radius, towards left, when flying at 200 km per hour. The rotary engine and the propeller of the plane has a mass of 400 kg and a radius of gyration of 6.3 m. The engine rotates at 2400 rpm clockwise when viewed from the rear. Find the gyroscopic couple on the aircraft and state its effect on it.

Ans. Given, $R = 50\,\text{m}$, $v = 20\,\text{km/hr} = 55.6\,\text{m/s}$, $m = 400\,\text{kg}$.

$k = 0.3\,\text{m}$, $N = 2400\,\text{rpm}$

Or
$$\omega = 2\pi \times \frac{2400}{60} = 251.4\ \text{rad/s}$$

The mass moment of inertia of the engine and the propeller

$$I = m.k^2 = 400 \times (0.3)^2 = 36\,\text{kg-m}^2$$

& angular velocity of precision

$$\omega_p = \frac{v}{R} = \frac{55.6}{50} = 1.11\ \text{rad/s}$$

We know that gyroscopic couple acting on the aircraft

$$c = I.\omega.\omega_p = 36 \times 251.4 \times 1.11$$

$$= 10046\,\text{N-m}$$

Q. 1. (x) Explain torsion dynamometer.

Ans. Torsion Dynamometer : A torsion dynamometer is used for measuring large powers particularly the power transmitted along the propeller shaft of a turbine or motor vessel. When the power is being transmitted, then the driving end of the shaft twist through a small angle relative to the driven end of the shaft. The amount of twist depends upon many factors such as torque acting on the shaft (T), length of the shaft (l), diameter of the shaft (D) and modulus of rigidity (c) of the material of the shaft

$$\boxed{\frac{T}{J} = \frac{c.\theta}{l}}$$

Section—(A)

Q. 2. (a) A punching machine carries out 6 holes per minute. Each hole of 40 mm dia. in 35 mm thick plate requires 8 N-m of energy/mm² of the sheared area. The punch has a stroke of 95mm. Find the power of the motor required if the mean speed of the flywheel is 20 m/s.

If total fluctuation of speed is not to exceed 3% of the mean speed, determine the mass of the flywheel.

Ans. Given, $d = 40\,\text{mm}$ $k = 0.03$

$t = 35\,\text{mm}$ stroke $= 95\,\text{mm}$

$v = 20\,\text{m/s}$

Power, P = Energy required for punching work/s

$$= \frac{1}{60}(\text{Energy required/minute})$$

$$= \frac{1}{60}(\text{Energy required/hole} \times \text{No. of holes/minute})$$

$$= \frac{1}{60} \times [(\pi dt \times 8) \times 6]$$

$$= \frac{1}{60}[(\pi \times 40 \times 35 \times 8) \times 6]$$

$$= \frac{1}{60}(35186 \times 6)$$

$$= 3519\,\text{N-m/s or } 3519\,\text{W or } 3.519\,\text{kW}$$

Actual time required to punch a hole in 35 mm thick plate

$$= \frac{10}{190} \times 35 = 1.842\,\text{s}$$

Energy required/hole or energy supplied by the motor in 10 seconds

$$= \pi dt \times 8 = 35186\,\text{N-m}$$

Energy supplied by the motor in

$$1.842\text{s} = \frac{35186}{10} \times 1.842$$

$$= 6481\,\text{N-m}$$

Energy supplied by flywheel $= 35186 - 6481 = 28705\,\text{N-m}$

$$k = \frac{e}{2E}$$

$$0.03 = \frac{28705}{2E}$$

Or, $E = 478417$

$$\frac{1}{2}mv^2 = 478417$$

$$\frac{1}{2}m(20)^2 = 478417$$

Or $\qquad m = 2392\text{ kg}$

Q. 2. (b) The turning moment diagram for a multicylinder engine has been drawn to a vertical scale of 1mm = 650 N-m and a horizontal scale of 1mm – 4.5°. The areas above and below the mean torque line are –28, +380, –260, +310, –300, +242, –380, +265 and –229 mm^2.

The fluctuations of the speed is limited to ± 1.8% of the mean speed which is 400 rpm. Density of the rim material is 7000 kg/m^2 and width of the rim is 4.5 times its thickness. The centrifugal stress (hoop stress) in the rim material is limited to 6 N/mm^2. Neglecting the effect of the boss and arms, determine the diameter and cross-section of the flywheel rim.

Ans. $\qquad \rho = 7000\text{ kg/m}^3$

$$f = 6\times 10^6\text{ N/m}^2$$

$$N = 400\text{ rpm} \qquad k = 0.018$$

$$b = 4.5\,t$$

$$f = \rho v^2$$

$$6\times 10^6 = 7000\times v^2$$

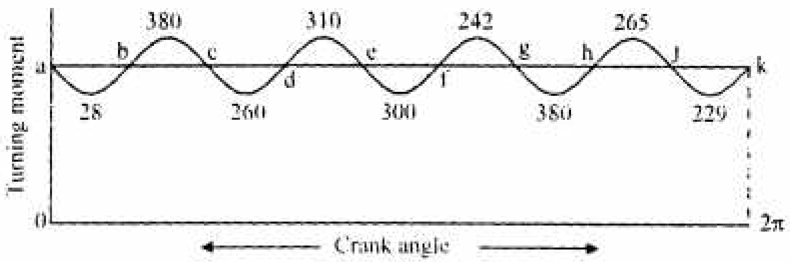
$$v = 29.28\text{ m/s}$$

Or $$\frac{\pi d N}{60} = \frac{\pi\times d\times 400}{60} = 29.28$$

Or $$d = 1.398\text{ m}$$

Let the flywheel KE at $\qquad a = E$

At $\qquad b = E - 28$



At $\qquad c = E - 28 + 380 = E + 352$

At $\qquad d = E + 352 - 260 = E + 92$

At $\qquad e = E + 92 + 310 = E + 402$

At $\qquad f = E + 402 - 300 = E + 102$

At $\qquad g = E + 102 + 242 = E + 344$

At $\qquad h = E + 344 - 380 = E - 36$

At $\quad i = E - 36 + 265 = E + 229$

At $\quad k = E + 229 - 229 = E$

Maximum energy $\quad = E + 402$ (at e)

Minimum energy $\quad = E - 36$ (at h)

Maximum fluctuation of energy,

$$e_{max} = (E + 402) - (E - 36) \times \text{Horizontal scale} \times \text{Vertical scale}$$

$$= 438 \times \left(4.5 \times \frac{\pi}{180}\right) \times 650$$

$$= 22360 \text{ N-m}$$

$$K = \frac{e}{mk^2\omega^2}$$

$$0.018 = \frac{22360}{m\left(\frac{1.398}{2}\right)^2 \left(\frac{2\pi \times 400}{60}\right)^2}$$

$$m = 1449 \text{ kg}$$

$$\text{Density} \times \text{Volume} = 1449$$

$$p \times (\pi d) \times t \times 4.5t = 1449$$

$$7000 \times \pi \times 1.398 \times t \times 4.5t = 1449$$

$$t = 0.1023 \text{ m or } 102.3 \text{ mm}$$

$$b = 4.5 \times 102.3 = 460.5 \text{ mm}$$

Q. 3. The torque delivered by a two-stroke engine is represented by

$$T = (1000 + 300 \sin 2\theta - 500 \cos 2\theta) \text{ N-m}$$

Where, θ is the angle turned by the crank from the inner dead centre. The engine speed is 250 rpm. The mass of the flywheel is 400 kg and radius of gyration 400 mm.

Determine :

(i) The power developed

(ii) The total percentage fluctuations of speed

(iii) The angular acceleration of flywheel when the crank has rotated through an angle of 60° from the inner-dead centre.

(iv) The maximum angular acceleration and retardation of the flywheel.

Ans. The torque being a function of 2θ, the cycle is repeated every 180° of the crank rotation

(i) $$T_{mean} = \frac{1}{\pi}\int_0^{\pi} T.d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} (1000 + 300\sin 2\theta - 500\cos 2\theta)d\theta$$

$$= \frac{1}{\pi}\left[1000\theta - \frac{300}{2}\cos 2\theta - \frac{500}{2}\sin 2\theta\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[1000\theta - \frac{300}{2}\cos 2\pi - 0\right] - (0 - 150 - 0)$$

$$= 1000 \text{ N-m}$$

$$\therefore \quad P = T.\omega = 1000 \times \frac{2\pi \times 250}{60} = 26180 \text{ W}$$

Or $= 26.18$ kW

(ii) At any instant, $\Delta T = T - T_{mean}$

$$= (1000 + 300\sin 2\theta - 500\cos 2\theta) - 1000$$

$$= 300\sin 2\theta - 500\cos 2\theta$$

ΔT is zero, when $300\sin 2\theta - 500\cos 2\theta = 0$

Or $300\sin 2\theta = 500\cos 2\theta$

Or $\tan 2\theta = 5/3$

Or $2\theta = 59° \text{ or } 239°$

$$\theta = 29.5° \text{ or } 119.5°$$

$$e_{max} = \int_{29.5°}^{119.5°} \Delta T d\theta$$

$$= \int_{29.5°}^{119.5°} (300\sin 2\theta - 500\cos 2\theta)d\theta$$

$$= [-150\cos 2\theta - 250\sin 2\theta]_{29.5°}^{119.5°}$$

$$= 583.1 \text{ N-m}$$

$$K = \frac{e}{mk^2\omega^2} = \frac{583.1}{400 \times (0.4)^2 \times \left(\frac{2\pi \times 250}{60}\right)^2}$$

$$= 0.01329 \text{ or } 1.329\%$$

(iii) Acceleration is produced by excess or deficit torque at any moment

$$\Delta T = 300\sin 2\theta - 500\cos 2\theta$$

When $\theta = 60°$

$$\Delta T = 509.8 \text{ N-m}$$

$$I\alpha = mk^2\alpha = 509.8$$

Or $\quad 400 \times (0.4)^2 \times \alpha = 509.8$

$$\alpha = 7.966 \text{ rad/s}$$

(iv) For ΔT_{max} and ΔT_{min}

$$\frac{d}{d\theta}(\Delta T) = \frac{d}{d\theta}(300 \sin 2\theta - 500 \cos 2\theta)d\theta = 0$$

Or $\quad 2 \times 300 \cos 2\theta + 2 \times 500 \sin 2\theta = 0$

$$600 \cos 2\theta = -1000 \sin 2\theta$$

Or $\quad \tan 2\theta = -0.6$

Or $\quad 2\theta = 149.04°$ and $329.04°$

When $\quad 2\theta = 149.04°$, $T = 1583.1$ N-m, $\Delta T = 583.1$ N-m

When $\quad 2\theta = 329.04°$, $T = 416.9$ N-m, $\Delta T = -583.1$ N-m

As values of ΔT at maximum and minimum torque T are same, maximum acceleration is equal to maximum retardation.

Or $\quad \Delta T = mk^2\alpha = 583.1$

Or $\quad 400 \times (0.4)^2 \times \alpha = 583.1$

Maximum acceleration or retardation, $\alpha = 9.11 \text{ rad/s}^2$

Section—(B)

Q. 4. The following data apply to an outside cylinder uncoupled locomotive :

Mass of rotating parts per cylinder = 360 kg.

Mass of reciprocating parts per cylinder = 300 kg.

Angle between cranks = 90°

Crank radius = 0.3 m

Cylinder centres = 1.75 m

Radius of balance masses = 0.75 m.

Wheel center = 1.45 m

If whole of the rotating and two thirds of reciprocating parts are to be balanced in planes of the driving wheels.

Find : (i) Magnitude and angular positions of balance masses.

(ii) Speed in kilometers per hour at which the wheel will lift off the rails when the load on each driving wheel is 30 kN and the diameter of tread of driving wheels is 1.8 m, and

(iii) Swaying couple at speed arrived at in (ii) above.

Ans. Given, $\quad m_1 = 360$ kg, $\quad m_2 = 300$ kg, $\quad \angle AOD = 90°$

$r_A = r_B = 0.3$ m, $\quad a = 1.75$ m, $\quad r_B = r_C = 0.75$ m;

$C = 2/3$

We know that the equivalent masses of the rotating parts to be balanced per cylinder

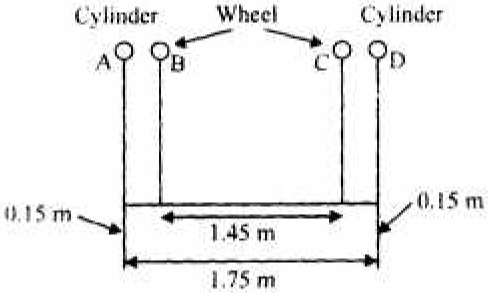
$$m = m_A = m_D = m_1 + c.m_2 = 360 + \frac{2}{3} \times 300$$

$$= 560\,\text{kg}$$

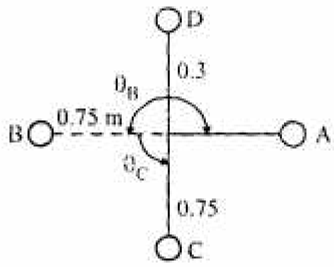
Magnitude and angular position of balance masses

Let m_B and m_C = Magnitude of the balance masses and

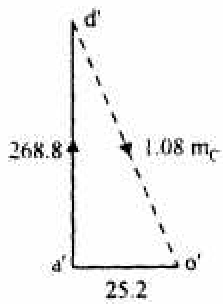
θ_B and θ_C = Angular position of the balance masses m_B and m_C from the crank A.



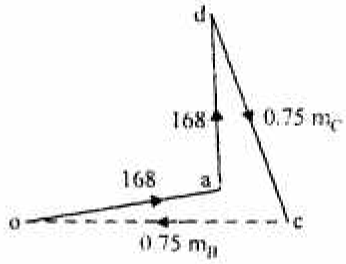
(a) Position of planes



(b) Position of masses



(c) Couple polygon



(d) Force polygon

Assuming the plane of wheel B as the reference plane

Plane (1)	Mass (m/kg) (2)	Radius $(r)m$ (3)	Centrifugal force $\div \omega^2 (m.r)$ kg-m (4)	Distance from plane $B(l)$ (5)	Couple $\div \omega^2 (m.r.l.)$ kg-m^2 (6)
A	560	0.3	168	− 0.15	− 25.2
$B(R.P.)$	m_B	0.75	$0.75\,m_B$	0	0
C	m_C	0.75	$0.75\,m_C$	1.45	$108\,m_C$
D	560	0.3	168	1.6	268.8

(b) Speed at which the wheel will lift off the rails

Give $P = 30\,kN = 30 \times 10^3\,N; \quad D = 1.8\,m$

Let ω = Angular speed at which the wheels will lift off the rails in rad/s and

v = Corresponding linear speed in km/h

Each balancing mass

$$m_B = m_C = 249\,kg$$

Balancing mass for reciprocating parts

$$B = \frac{c.m_2}{m} \times 249 = \frac{2}{3} \times \frac{300}{560} \times 247$$

$$= 89\,kg$$

&

$$v = \omega \times D/2 = 21.2 \times 1.8/2 = 19.08\,m/s$$

$$= 19.08 \times \frac{3600}{1000} = 68.7\,km/h$$

(c) Swaying couple at speed $\omega = 21.1$ rad/s

$$= \frac{a(1-c)}{\sqrt{2}} \times m_2 \omega^2 . r$$

$$= \frac{175\left[1 - \frac{2}{3}\right]}{\sqrt{2}} \times 300(21.2)^2\,0.3$$

$$= 16687\,N\text{-}m$$

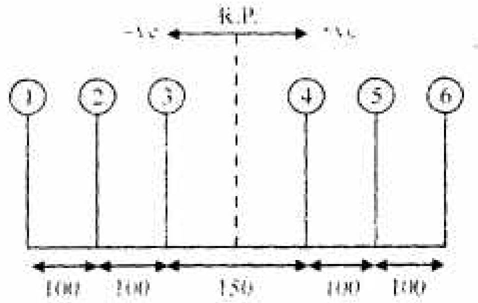
Q. 5. The firing order in a 6-cylinder vertical four stroke in-line engine in 1-4-2-6-3-5. The piston stroke is 100 mm and the length of each connecting rod is 200 mm. The pitch distances between the cylinder centre lines are 100 mm, 100 mm, 150 mm, 100 mm and 100 mm respectively. The reciprocating mass per cylinder is 1 kg and the engine runs at 3000 rpm.

Determine the out of balance primary and secondary forces and couples on this engine taking a plane midway between the cylinder 3 and 4 as the reference plane.

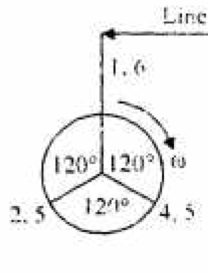
Ans. Given, $L = 100\,mm$ or $r = \frac{L}{2} = 50\,mm = 0.05\,m.$

$$l = 200\,mm$$

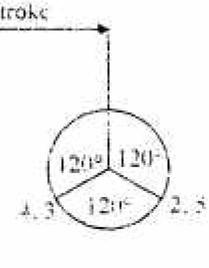
$$m = 1\,kg; \qquad N = 300\,rpm$$



(a) Position of planes



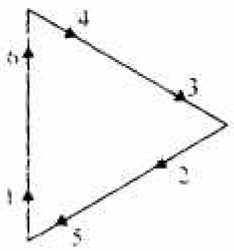
(b) Primary crank position



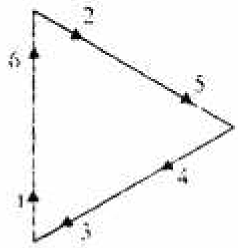
(c) Secondary crank position

With reference to plane midway between the cylinders 3 and 4, the data may be tabulated as

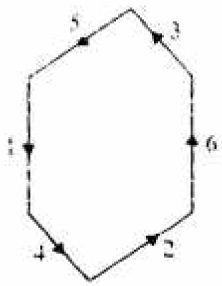
Plane (1)	Mass (m) kg	Radius (r) m	Centrifugal force $\div \omega^2$ ($m.r$) kg-m	Distance from plane 3(l) m	Couple $\div \omega^2$ ($m.r.l.$) kg-m^2
1	1	0.05	0.05	− 0.275	− 0.01375
2	1	0.05	0.05	− 0.175	− 0.00875
3	1	0.05	0.05	− 0.075	− 0.00375
4	1	0.05	0.05	+ 0.075	+ 0.00375
5	1	0.05	0.05	+ 0.175	+ 0.00875
6	1	0.05	0.05	+ 0.275	+ 0.01375



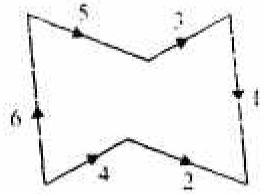
Primary force polygon



Secondary force polygon



Primary couple polygon



Secondary couple polygon

From figure (d) and (e) we see that the primary and secondary force polygons are closed. Therefore there are no out of balance primary and secondary forces. Thus, the engine is balanced for primary and secondary forces.

Also primary and secondary couple polygons are closed figures. Therefore there are no out of balance primary and secondary couples. Thus, the engine is balanced for primary and secondary couples.

Section—(C)

Q. 6. A porter governor has equal arms each 250 mm long and pivoted on the axis of rotation. Each ball has a mass of 5 kg and the mass of the central lead on the sleeve is 25 kg. The radius of rotation of the ball is 150 mm when the governor begins to lift and 200 mm when the governor is at maximum speed.

Find the range of speed, sleeve lift, governor effort and power of the governor in the following cases :

(i) When the friction at the sleeve is neglected, and

(ii) When the friction at the sleeve is equivalent to 10N.

Ans. Given : $BP = BD = 250\,\text{mm}$, $m = 5\,\text{kg}$; $M = 25\,\text{kg}$

$r_1 = 150\,\text{mm}$; $r_2 = 200\,\text{mm}$; $F = 10\,\text{N}$

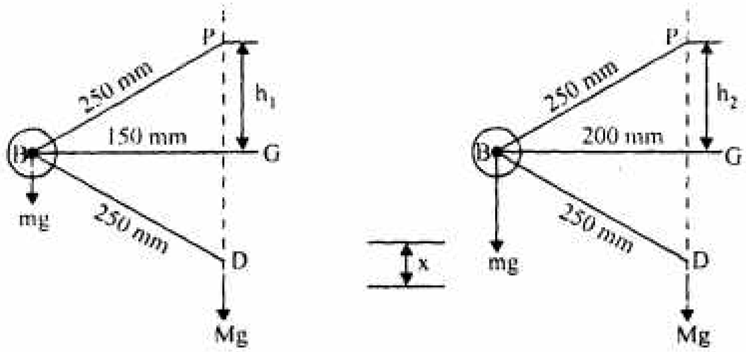
(i) When the friction at the sleeve is neglected

Let, N_1 = Minimum speed, and

N_2 = Maximum speed

$$h_1 = PG\sqrt{(BP)^2 - (BG)^2} = \sqrt{(250)^2 - (150)^2} = 200\,\text{mm}$$

$$= 0.2\,\text{mm}$$



(a) Minimum position **(b) Maximum position**

$$h_2 = PG = \sqrt{(BP)^2 - BG^2} = \sqrt{250^2 - 200^2}$$

$$= 150\,\text{mm} = 0.15\,\text{m}$$

We know that. $$N_1^2 = \frac{m+M}{m} \times \frac{895}{h_1} = \frac{5+25}{5} \times \frac{895}{0.2}$$

$$= 26850$$

$\therefore$ $$N_1 = 164 \text{ rpm}$$

& $$(N_2)^2 = \frac{m+M}{m} \times \frac{895}{h_2} = \frac{5+25}{5} \times \frac{895}{0.15} = 35800$$

$\therefore$ $$N_2 = 189 \text{ rpm}$$

Range of Speed : We know that range of speed

$$= N_2 - N_1 = 189 - 164 = 25 \text{ rpm}$$

Sleeve Lift :

$$x = 2(h_1 - h_2) = 2(200 - 150) = 100 \text{ mm} = 0.1 \text{ m}$$ **Ans.**

Governor Effort :

Let C = Percentage increase in speed

We know that increase in speed or range of speed

$$C.N_1 = N_2 - N_1 = 25 \text{ rpm}$$

$$C = \frac{25}{N_1} = \frac{25}{164} = 0.152$$

We know that governor effort

$$P = C(m+M)g = 0.152(5+25)9.81$$

$$= 44.7 \text{ N}$$ **Ans.**

Power of the governor $= P.x = 44.7 \times 0.1 = 4.47$ N-m

(ii) When the friction at the sleeve is taken into account

$$(N_1)^2 = \frac{m.g + (M.g - F)}{m.g} \times \frac{895}{h_1}$$

$$= \frac{5 \times 9.81 + (25 \times 9.81 - 10)}{5 \times 9.81} \times \frac{895}{0.2}$$

$$= 25938$$

$\therefore$ $$N_1 = 161 \text{ rpm}$$

& $$(N_2)^2 = \frac{m.g + (M.g + F)}{m.g} \times \frac{895}{h_2}$$

$$= \frac{5 \times 9.81 + (25 \times 9.81 + 10)}{5 \times 9.81} \times \frac{895}{0.15} = 37016$$

$$N_2 = 192.4 \text{ rpm}$$

Range of Speed

$$N_2 - N_1 = 192.4 - 161 = 31.4 \text{ rpm } \textbf{Ans.}$$

Sleeve Lift : The sleeve lift (x) will be given as

$$x = 100 \text{ mm} = 0.1 \text{ m } \textbf{Ans.}$$

Governor Effort : Let C = Percentage increase in speed

We know that increase in speed or range of speed.

$$C.N_1 = N_2 - N_1 = 31.4 \text{ rpm}$$

$$C = \frac{31.4}{N_1} = \frac{31.4}{161} = 0.195$$

Governor Effort :

$$P = C(m.g + M.g + F) = 0.195(5 \times 9.81 + 25 \times 9.81 + 10) \text{ N}$$

$$= 57.4 \text{ N } \textbf{Ans.}$$

Power of the Governor :

Power of the governor $= P.x = 57.4 \times 0.1 = 5.74$ N-m **Ans.**

Q. 7. In a Hartnell governor, the lengths of ball and sleeve arms of a bell crank lever are 120 mm and 100 mm respectively. The distance of the fulcrum of the bell crank lever from the governor axis is 140 mm. Each governor ball has a mass of 4 kg. The governor run at a mean speed of 300 rpm with the ball arms vertical and sleeve arms horizontal. For an increase of speed of 4 percent, the sleeve moves 10 mm upwards. Neglecting friction, find :

(i) The minimum equilibrium speed if the total sleeve movement is limited to 20 mm

(ii) The spring stiffness

(iii) The sensitiveness of the governor, and

(iv) The spring stiffness if the governor is to be isochronous at 300 rpm.

Ans. Given, $x = 120 \text{ mm} = 0.12 \text{ m};$

$y = 100 \text{ mm} = 0.1 \text{ m};$

$r = 140 \text{ mm} = 0.14 \text{ m}$

$m = 4 \text{ kg};$

$N = 300 \text{ rpm};$

or $\omega = 2\pi \times \frac{300}{60} = 31.42 \text{ rad/s}$

$h_1 = 10 \text{ mm} = 0.01 \text{ m}; \ h = 20 \text{ mm} = 0.02 \text{ m}$

(i) Minimum equilibrium speed

Let N_1 = Minimum equilibrium speed

r_1 = Radius of rotation in the minimum position i.e., when the sleeve moves downward, and

r_2 = Radius of rotation in the maximum position i.e., when the sleeve moves upward.

Since the increase in speed is 4% therefore maximum speed

$$N_2 = N + 0.04N = 1.04N = 1.04 \times 300 = 312 \text{ rpm}$$

$$\omega_2 = 2\pi \times \frac{312}{60} = 32.7 \text{ rad/s}$$

We know that lift of the sleeve for the maximum position

$$h_2 = h - h_1 = 0.02 - 0.01 = 0.01 \text{ m}$$

For the minimum position

$$\frac{h_1}{y} = \frac{r - r_1}{x} \text{ or } r_1 = r - h_1 \times \frac{x}{y} = 0.14 - 0.01 \times \frac{0.12}{0.1}$$

$$= 0.128 \text{ m}$$

For maximum position

$$\frac{h_2}{y} = \frac{r_2 - r}{x} \text{ or } r_2 = r + h_2 \times \frac{x}{y} = 0.14 + 0.01 \times \frac{0.12}{0.1} = 0.152 \text{ m}$$

Centrifugal force in the mean position.

$$F_c = m\omega^2 r = 4(31.42)^2 0.14 = 553 \text{ N}$$

Centrifugal force in the minimum position

$$F_{c1} = m(\omega_1)^2 r_1 = 4 \times \left(\frac{2\pi N_1}{60}\right)^2 0.128$$

$$= 0.0056 (N_1)^2 \qquad \text{... (1)}$$

Centrifugal force in the maximum position

$$F_{c2} = m(\omega_2)^2 r_2 = 4 \times (32.7)^2 0.152 = 650 \text{ N}$$

Centrifugal force at any instant

$$F_c = F_{c1} + (F_{c2} - F_{c1})\left(\frac{r - r_1}{r_2 - r_1}\right)$$

$$553 = F_{c1} + (650 - F_{c1})\left(\frac{0.14 - 0.128}{0.152 - 0.128}\right)$$

$$= 0.5F_{c1} + 325$$

$$F_{c_1} = \frac{553-325}{0.5} = 456\text{ N} \quad \text{.... (ii)}$$

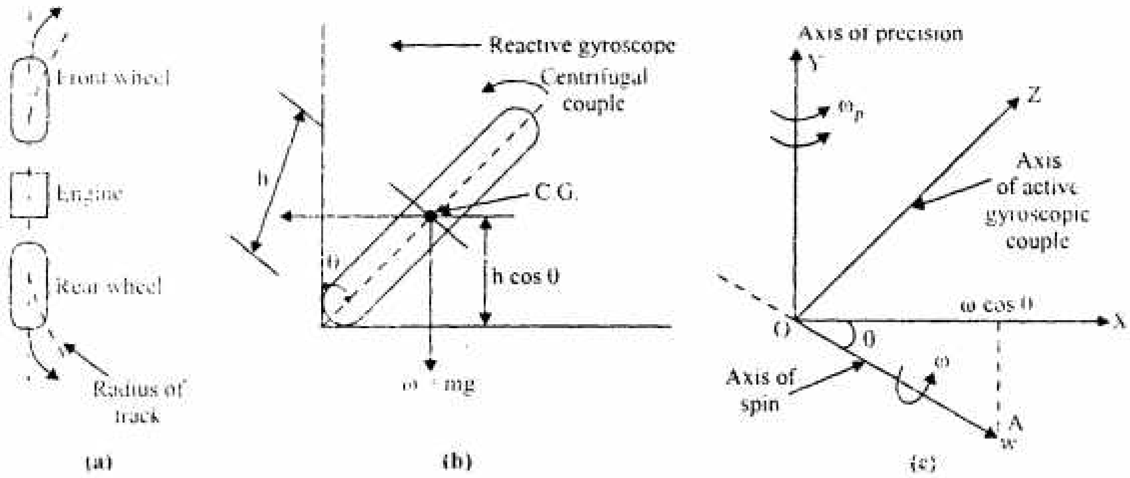
From equations (i) and (ii)

$$(N_1)^2 = \frac{456}{0.0056} = 81428 \text{ or } N_1 = 285.4 \text{ rpm Ans.}$$

Section—(D)

Q. 8. (a) Discuss the effect of the gyroscopic couple on a two wheeled vehicle taking a turn.

Ans. Consider a two wheel vehicle taking a right turn



Let m = Mass of the vehicle and its rider in kg

W = Weight of the vehicle and its rider in newtons = mg

h = Height of the centre of gravity of the vehicle and rider

r_w = Radius of the wheels

R = Radius of track or curvature

I_w = Mass moment of inertia of each wheel

I_E = Mass moment of inertia of the rotating parts of the engine

ω_E = Angular velocity of the engine

ω_W = Angular velocity of the wheels

G = Gear ratio = $\frac{\omega_E}{\omega_w}$

v = Linear velocity of the vehicle = $\omega_w \times r_w$

θ = Angle of heel. It is inclination of the vehicle to the vertical for equilibrium.

(i) Effect of Gyroscopic Couple

$$v = \omega_w \times r_w \text{ or } \omega_w = \frac{v}{r_w}$$

& $$\omega_E = G.\omega_w = G \times \frac{v}{r_w}$$

$\therefore$ Total $$(I \times \omega) = 2I_w \times \omega_w \pm I_E \times \omega_E$$

$$= 2I_w \times \frac{v}{r_w} \pm I_E \times G \times \frac{v}{r_w} = \frac{v}{r_w}(2I_w \pm GI_E)$$

& velocity of precision $\omega_p = \dfrac{v}{R}$

When the wheels move over the curved path, the vehicle is always inclined at an angle θ with the vertical plane. This angle is known as the angle of heel. In other words, axis of spin is inclined to the horizontal at an angle θ.

$\therefore$ Gyroscopic couple $C_1 = I.\omega\cos\theta \times \omega_p = \dfrac{v}{r_w}(2I_w \pm GI_E)\cos\theta \times \dfrac{v}{R}$

$$= \frac{v^2}{R}(2I_w \pm GI_E)\cos\theta$$

(ii) Effect of Centrifugal Couple

Centrifugal force, $F_c = \dfrac{mv^2}{R}$

This force acts horizontally through the centre of gravity (C.G.) along the outward direction.

Centrifugal couple $C_2 = F_c \times h\cos\theta = \left(\dfrac{mv^2}{R}\right)h\cos\theta$

Since the centrifugal couple has a tendency to overturn the vehicle.

$\therefore$ Total overturning couple.

$$C_o = \text{Gyroscopic couple} + \text{Centrifugal couple}$$

$$= \frac{v^2}{R.r_w}(2I_w + GI_E)\cos\theta + \frac{mv^2}{R} \times h\cos\theta$$

$$= \frac{v^2}{R}\left[\frac{2I_w + GI_E}{r_w} + m^h\right]\cos\theta$$

ω_e know that balancing couple = $m.g.h.\sin\theta$

The balancing couple acts in clockwise direction when seen from the front of the vehicle. Therefore for stability, the overturning couple must be equal to the balancing couple i.e..

$$\frac{v^2}{R}\left[\frac{2I_w + GI_E}{r_w} + mh\right]\cos\theta = mgh\sin\theta$$

The value of angle of heel (θ) may be determined so that the vehicle does not skid.

Q. 8. (b) Find the angle of inclination w.r.t. the vertical of a two wheeler negotiating a turn. Given : combined mass of the vehicle with its rider 250 kg; moment of inertial of the engine flywheel 0.3 kg-m², moment of inertia of each road wheel 1 kg-m²; speed of engine flywheel 5 times that of road wheels and in the same direction; height of centre of gravity of rider with vehicle 0.6 m; two wheeler speed 90 km/hr; wheel radius 300 mm; radius of turn 50 m.

Ans. Given. $m = 250\,\text{kg}$; $I_E = 0.3\,\text{kg-m}^2$; $I_w = 1\,\text{kg-m}^2$;

$\omega_E = 5W_w$ or $G = \frac{\omega_E}{\omega_W} = 5$; $h = 0.6\,\text{m}$

$v = 90\,\text{km/hr} = 25\,\text{m/s}$; $r_w = 300\,\text{mm} = 0.3\,\text{m}$; $R = 50\,\text{m}$

Let θ = Angle of inclination w.r.t. the vertical of a two wheeler

We know that gyroscopic couple

$$C_1 = \frac{V^2}{R \times r_w}(2I_w + GI_E)\cos\theta = \frac{(25)^2}{50 \times 0.3}(2 \times 1 + 5 \times 0.3)\cos\theta$$

$$= 146\cos\theta \text{ N-m}$$

& centrifugal couple

$$C_2 = \frac{mv^2}{R} \times h\cos\theta = \frac{250 \times (25)^2}{50} \times 0.6\cos\theta$$

$$= 1875\cos\theta \text{ N-m}$$

$\therefore$ Total overturning couple

$$= C_1 + C_2 = 146\cos\theta + 1875\cos\theta$$

$$= 2021\cos\theta \text{ N-m}$$

We know that balancing couple

$$= mgh\sin\theta = 250 \times 9.81 \times 0.6\sin\theta$$

$$= 1471.5\sin\theta \text{ N-m}$$

Since the overturning couple must be equal to the balancing couple for equilibrium condition, therefore

$$2021\cos\theta = 1471.5\cos\theta$$

$$\therefore \quad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{2021}{1471.5} = 1.3734 \text{ or}$$

$$\theta = 53.94° \textbf{ Ans.}$$

Q. 9. The turbine rotor of a ship has a mass of 2000 kg and rotates at a speed of 3000 rpm. Clockwise when looking from a stem. The radius of gyration of the rotor is 0.5 m.

Determine the gyroscopic couple and its effects upon the ship when the ship is steering to the right in a curve of 100 mm radius at a speed of 16.1 knots (1 knot = 1855 m/hr).

Calculate also the torque and its effects when the ship is pitching in simple harmonic motion, the bow falling with its maximum velocity. The period of pitching is 50 seconds and the total angular displacement between the two extreme positions of pitching is 12°. Find the maximum acceleration during pitching motion.

Ans. Given, $m = 2000\,\text{kg}$; $N = 3000\,\text{rpm}$ or $\omega = 2\pi \times \frac{3000}{60}$

$$= 314.2\,\text{rad/s}$$

$$k = 0.5\,\text{m},\ R = 100\,\text{m},\quad v = 16.1\,\text{knots} = 16.1 \times \frac{1855}{3600}$$

$$= 8.3\ \text{m/s}$$

Gyroscopic Couple : We know that mass moment of inertia of the rotor,

$$I = m.k^2 = 2000(0.5)^2 = 500\,\text{kg-m}^2$$

Angular velocity of precision,

$$\omega_P = \frac{v}{R} = \frac{8.3}{100} = 0.083\,\text{rad/s}$$

$\therefore$ Gyroscopic couple,

$$C = I.\omega.\omega_P = 500 \times 314.2 \times 0.083$$

$$= 13040\,\text{N-m} = 13.04\,\text{kN-m}$$

When rotor rotates clockwise when looking from a stern and the ship stress to the right, the effect of the reactive gyroscopic couple is to raise the stern and lower the bow.

Torque during pitching

Given, $t_p = 50\,\text{s};\ 2\phi = 12°$ or $\phi = 6° = 6 \times \frac{\pi}{180}$

$$= 0.105\,\text{rad}$$

We know that angular velocity of simple harmonic motion

$$\omega_1 = \frac{2\pi}{t_p} = \frac{2\pi}{50} = 0.1257\,\text{rad/s}$$

& maximum angular velocity of precision,

$$\omega_{p\,max} = \phi\omega_1 = 0.105 \times 0.1257$$

$$= 0.0132 \text{ rad/s}$$

∴ Torque or maximum gyroscopic couple during pitching

$$C_{max} = I.\omega.\omega_{p\,max} = 500 \times 314.2 \times 0.0132$$

$$= 2074 \text{ N-m} \quad \textbf{Ans.}$$

Maximum acceleration during pitching

$$\alpha_{max} = \phi(\omega_1)^2 = 0.105(0.1257)^2$$

$$= 0.00166 \text{ rad/s}^2 \quad \textbf{Ans.}$$