B.Tech.

Fifth Semester Examination

Applied Numerical Techniques & Computing (ME-311F)

Q. 1. (a) Write Taylor's series formula for a continuous function in a given interval [a, $a + h$].

Ans. $f(a+h) = f(a) + hf'(a) + hf'(a) + \frac{h^2}{2!} f''(a) + ... + \frac{h^n}{n!} f^n(a + \theta h)$

Q. 1. (b) Write solution of differential equation

$$\frac{dy}{dx} + 2xy = e^{-x^2} \text{ with } y(0) = 1$$

Ans. Given equation $\frac{dy}{dx} + 2xy = e^{-x^2}$

Integrating factor $\quad$ I.F. $= e^{\int 2x.dx} = e^{x^2}$

Solution is : $\quad Y(\text{I.F.}) = \int \theta(\text{I.F.}).dx + c$

$$Ye^{x^2} = \int e^{-x^2}.e^{x^2}.dx + c$$

$$Y.e^{x^2} = x + c$$

$$x = 0,\ y = 1(\text{given})$$

$\Rightarrow \quad c = 1$

So solution is $\quad Y.e^{-x^2} = x + 1$

$$\boxed{Y = e^{-x^2}(x+1)}$$

Q. 1. (c) If one root of equation $x^3 - 10x^2 + 31x - 30 = 0$ is 5. Find the other two roots.

Ans. Since 5 is root of $f(x)$. So $f(x)$ is divisible by $(x - 5)$

$$x - 5 \overline{)x^3 - 10x^2 + 31x - 30}(x^2 - 5x + 6$$

$$x^3 - 5x^2$$

$$- \quad +$$

$$-5x^2 + 31x - 30$$

$$-5x^2 + 25x$$

$$6x - 30$$

$$6x - 30$$

$$0$$

$$\therefore \quad x^3 - 10x^2 + 31x - 30 = 0$$

$$(x-5)(x^2 - 5x + 6) = 0$$

Roots of $(x^2 - 5x + 6)$ is **2 and 3.**

Q. 1. (d) Starting from $x_0 = 1$ one step of N-R method is solving of equation $x^3 + 3x - 7 = 0$ gives value of x_1.

Ans. From N-R method $\quad x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$ ...(1)

Given function is $\quad f(x) = x^3 + 3x - 7$

$$f'(x) = 3x^2 + 3$$

Put $\quad x_0 = 1$

$$f(x_0) = f(1) = 1^3 + 3 \times 1 - 7 = -3$$

$$f'(x_0) = f'(1) = 3 \times (1)^3 + 3$$

$$= 6$$

Substituting x_0, $f(x_0)$ and $f'(x_0)$ value into equation (1)

$$x_1 = 1 - \left(\frac{-3}{6}\right) \times 1 = 1.5 \textbf{ Ans.}$$

Q. 1. (e) Let $x^2 - 117 = 0$. Find the iterative steps of solution using N-R method.

Ans.

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$= x_k - \frac{(x_k^2 - 117)}{2x_k}$$

$$= \frac{1}{2}\left[x_k + \frac{117}{x_k}\right]$$

Q. 1. (f) Which curve represented a differential equation $3y\dfrac{dy}{dx} + 2x = 0$?

Ans.

$$3y\frac{dy}{dx} + 2x = 0$$

$$\frac{dy}{dx} = -\frac{2x}{3y}$$

$$3y\,dy = -2x\,dx$$

$$\int 3y\,dy = \int -2x.\,dx$$

$$\frac{3}{2}y^2 = -2x\frac{x^2}{2} + c$$

$$\boxed{\frac{x^2}{\frac{1}{2}c} + \frac{y^2}{\frac{1}{3}c} = 1}$$

$\Rightarrow$ Represent Ellips.

Q. 1. (g) Write formula of Lagranges mean value theorem.

Ans. (i) If a function $f(x)$ is continuous in closed interval $a \le x \le b$ and

(ii) Differentiable in open interval $x \in (a, b)$

Then there exist at least one value of x in $x \in (a, b)$ such that

$$\boxed{f'(c) = \frac{f(b) - f(a)}{(b-a)}}$$

Q. 1. (h) Let $f = y^x$ what is $\frac{d^2 f}{\partial x\,\partial y}$ at $x = 2, y = 1$.

Ans. $f = y^x, \frac{d^2 f}{dx^2} = ?$ at $x = 2,\ y = 1$

$$f = y^x$$

Take x as constant, we get $\frac{\partial t}{\partial y} = xy^{x-1}$

Now take y as constant :

$$\frac{d^2 f}{\partial x \partial y} = \frac{\partial}{\partial y}(y^{x-1}.x) = y^{x-1} + x.y^{x-1}\ \ln y$$

Whose value at $x = 2$ and $y = 1$ is

$$= 1^{(2-1)}\ (1 + 2\ln 1) = 1 \textbf{ Ans.}$$

Q. 1. (i) The matrix $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 0 & 6 \\ 1 & 1 & p \end{bmatrix}$ has one Eigen value equal to 3. Find the sum of other two Eigen value.

Ans. Sum of Eigen value of matrix is = sum of diagonal values of present in the matrix

$\therefore$ $1 + 0 + p = 3 + \lambda_2 + \lambda_3$

$$(p+1) = 3 + \lambda_2 + \lambda_3$$

$$\lambda_2 + \lambda_3 = p + I - 3 = (p-2) \quad \textbf{Ans.}$$

Q. 1. (j) Find the sum of Eigen values of the matrix given as $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

Ans. Sum of Eigen value of given matrix = sum of diagonal element of given matrix

$$= 1 + 5 + 1$$

$$= 7 \quad \textbf{Ans.}$$

Section—(A)

Q. 2. (a) For a matrix $[M] = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$ **the transport of matrix is equal to inverse of matrix** $[M]^T = [M]^{-1}$. **Find the value of** x.

Ans.
$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$A^T = A^{-1} \text{ then } A \text{ is orthogonal matrix}$$

$$A \cdot A^{-1} = A^{-1} \cdot A = 1 \text{ and } A^T \cdot A = A \cdot A^T = 1$$

Since m is orthogonal matrix

$$M^T \cdot M = I$$

$$\begin{bmatrix} \frac{3}{5} & x \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix} \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} \left(\frac{3}{5}\right)^2 + x^2 & \frac{3}{5} \cdot \frac{4}{5} + \frac{3}{5} \cdot x \\ \left[\left(\frac{4}{5}\right) \cdot \frac{3}{5} + \frac{3}{5} x\right] & \left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Comparing both side a_{12}

$$a_{12} = \left(\frac{3}{5}\right) \cdot \left(\frac{4}{5}\right) + \left(\frac{3}{5}\right) x = 0$$

$$\left(\frac{3}{5}\right) x = \frac{-3}{5} \cdot \frac{4}{5}$$

$$\boxed{x = \frac{-4}{5}} \quad \textbf{Ans.}$$

Q. 2. (b) In the solution set of following set of linear equation by Gauss elimination using partial pivoting $5x + y + 2z = 34$, $4y - 3z = 12$ and $10x - 2y + z = -4$. Find pivots for elimination of x, y, z.

Ans. The equations are

$$5x + y + 2z = 34$$
$$0x + 4y - 3z = 12$$
$$10x + 2y + z = -4$$

The augmented matrix for Gauss elimination

$$\begin{bmatrix} 5 & 1 & 2 \\ 0 & 4 & -3 \\ 10 & -2 & 1 \end{bmatrix} \begin{bmatrix} 34 \\ 12 \\ -4 \end{bmatrix}$$

Since in the first column maximum element in absolute value is 10, we need to exchange the row 1 with row 3.

$$\begin{bmatrix} 5 & 1 & 2 \\ 0 & 4 & -3 \\ 10 & -2 & 1 \end{bmatrix} \begin{bmatrix} 34 \\ 12 \\ -4 \end{bmatrix} \xrightarrow{R(1,\,3)} \begin{bmatrix} 10 & -2 & 1 \\ 0 & 4 & -3 \\ 5 & 1 & 2 \end{bmatrix} \begin{bmatrix} -4 \\ 12 \\ 34 \end{bmatrix}$$

So pivot for eliminating absolute value is 10.

Now eliminate y, we get column at end and below the diagonal element.

Since $a_{22} = 4$ is already larger in absolute to $a_{32} = 1$

$\therefore$ The pivot element for eliminating y is $a_{22} = 4$ itself.

$\therefore$ The pivot for x and y is 10 and 4.

Q. 3. Fit a parabola $y = ax^2 + bx + c$ in the least square since to the data

x:	**10**	**12**	**15**	**23**	**20**
y:	**14**	**17**	**23**	**25**	**21**

Ans. The normal equations to the curve are :

$$\left.\begin{aligned} \Sigma y &= a\,\Sigma x^2 + b\,\Sigma x + 5c \\ \Sigma xy &= a\,\Sigma x^3 + b\,\Sigma x^2 + c\,\Sigma x \\ \Sigma xy^2 &= a\,\Sigma x^4 + b\,\Sigma x^3 + c\,\Sigma x^2 \end{aligned}\right\} \quad \text{.... (i)}$$

The value of Σx, Σx^2, ..., etc. are calculated by means of following table :

x	y	x^2	x^3	x^4	xy	x^2y
10	14	100	1000	10000	140	1400
12	17	144	1728	207625	204	2448
15	23	225	3375	50625	345	5175
23	25	529	12167	279841	575	13225
20	21	400	8000	160000	420	8400
Σx	Σy	$\Sigma x^2 =$	$\Sigma x^3 = 26270$	$\Sigma x^3 = 521202$	$\Sigma xy = 1684$	$\Sigma x^2 y = 30648$

Substituting the values from table in equation (i)

$$100 = 1398a + 80b + 5c$$

$$1684 = 26270a + 1398b + 80c$$

$$30648 = 521202a + 26270b + 1398$$

$$a = -0.07,\ b = 3.03,\ c = -8.89$$

The required equation is

$$y = -0.07x^2 + 3.03x - 8.89 \text{ Ans}$$

Section—(B)

Q. 4. For data given below, find the equation to best fitting exponential curve of form $Y = ae^{bx}$

x	1	2	3	4	5	6
y	1.6	4.5	13.8	40.2	125	300

Ans. $Y = ae^{bx}$

Take log $\log y = \log a + bx \log e$

which is of form $\Rightarrow$ $y = A + Bx$

Where $Y = \log y$, $A = \log a$, $B = b \log e$

x	y	$Y = \log y$	x^2	xy
1	1.6	.2041	1	.2041
2	4.5	.6532	4	1.3064
3	13.8	1.1399	9	3.4197
4	40.2	1.6042	16	6.4197

5	125	2.6042	25	6.4168
				10.48545
6	300	2.4771	36	14.8626
$\Sigma x = 21$		$\Sigma Y = 8.1754$	$\Sigma x^2 = 91$	$\Sigma xY = 36.69$

Normal equations are $\Sigma Y = mA + B\Sigma x$

$$\Sigma xY = A\Sigma x + B\Sigma x^2$$

Here $m = 6$

$\therefore$ From equation (1) $8.1759 = 6A + 21B$

$$36.6941 = 21A + 91B$$

$$\begin{cases} A = -0.2534 \\ B = 0.4617 \end{cases}$$

$a = \text{antilog } A = \text{antilog } (\bar{1}.7466) = 0.5580$

$$b = \frac{B}{\log e} = \frac{.4617}{.4343} = 1.0631$$

Here required equation is $y = \dfrac{0.4617}{0.4343} = 1.0631$

Required equation is $\boxed{Y = 0.5580\, e^{1.0631}}$ **Ans.**

Q. 5. Transform linear form by substituting $v = y^{1-n}$ of the equation

$$\frac{dy}{dt} + p(t) \,.\, y = q(t) \,.\, y^n, \quad n > 0$$

Ans. Given $\dfrac{dy}{dt} + p(t) \,.\, y = q(t)\, y^n ; n > 0$

$$v = y^{1-n}$$

$$\frac{dv}{dt} = (1-n)\, y^{-n} \frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{1}{(1-n)\, y^{-n}} \frac{dv}{dt}$$

Substituting in given differential equation

$$\frac{1}{(1-n)\, y^{-n}} \frac{dv}{dt} + p(t) \,.\, y = q(1) \,.\, y^n$$

Multiply by $(1-n)\, y^{-n}$, we get

$$\frac{1}{(1-n)\,y^{-n}}\,\frac{dv}{dt}+p(t)\,y=q(t)\,.\,y^{n}$$

Multiply by $(1-n)\,y^{-n}$, we get

$$\frac{dv}{dt}+p(t)(1-n)\,y^{1-n}=q(t)(1-n)$$

Now since $y^{1-n}=v$, we get

$$\boxed{\frac{dv}{dt}+(1-n)\,p.v=(1-n).\,q}$$

Section—(C)

Q. 6. The equation $e^{x}-1=0$ is required to solve using N-R method with initial guess $x_0=-1$, then after one step N-R method estimate of the solution.

Ans. Equation $e^{x}-1=0$ here is $f(x)=e^{x}-1$

$$f'(x)=e^{x}$$

The Newton Raphson iterative equation is

$$x_{i+1}=x_i-\frac{f(x_i)}{f'(x_i)}$$

$$f(x_i)=e^{x_i}-1$$

$$f'(x_i)=e^{x_i}$$

$$v_{i+1}=x_i-\frac{e^{x_i}-1}{e^{x_i}}$$

$$x_{i+1}=x_i-\frac{(e^{x_i}-1)}{e^{x_i}}$$

Now, put $i=0$

Put $x_0=-1$ as given

$$X_{i+1}=\frac{x_i\,.\,e^{x_i}-(e^{x_i}-1)}{e^{x_i}}=\frac{e^{x_i}\,(x_i-1)+1}{e^{x_i}}$$

$$x_i=\frac{e^{x_0}\,(x_0-1)+1}{e^{x_0}}$$

Now put $i=0$

$$x_i=\left[\frac{e^{-1}\,(x_0-1)+1}{e^{x_0}}\right]$$

$$x_i=\left[\frac{e^{-1}\,(-2)+1}{e^{x_0}}\right]$$

$$x_i = [e^{-1}(-2)+1]\, e^{-1}$$

$$\boxed{x_i = 0.71828}$$

Q. 7. Solve by Cardon's method

$$x^3 + 3x - 14 = 0$$

Ans. $x = (u+v)\ldots$ assumed

Cubing $x = (u+v)$

$$x^3 = (u+v)^3 = u^3 + v^3 + 3uv(u+v)$$

$$x^3 - 3u.v.x - (u^3 + v^3) = 0$$

Comparing get $u.v = -1$

$$u^3.v^3 = -1$$

& $u^3 + v^3 = 14$

$\therefore u^3$ and v^3 are root of $t^2 - 14t - 1 = 0$

$$t = \frac{14 \pm \sqrt{196+4}}{2} = \frac{(14 \pm 10\sqrt{2})}{2}$$

$$= (7 \pm 5\sqrt{2})$$

Let $u^3 = 7 + 5\sqrt{2}$

$$v^3 = 7 \pm 5\sqrt{2}$$

$\therefore u$ and v will of form $a \pm b\sqrt{2}$

$$(a + b\sqrt{2})^3 = (7 + 5\sqrt{2})$$

$$a^3 + 2\sqrt{2}b^3 + 3ab\sqrt{2} = (a + b\sqrt{2}) = 7 + b\sqrt{2}$$

$$(a^3 + 6ab^2) + (2b^3 + 3a^2b)\sqrt{2} = 7 + 5\sqrt{2}$$

Let $u^3 = 7 + 5\sqrt{2}, \quad v^3 = (7 - 5\sqrt{2})$

u and v will be of form $(a \pm b\sqrt{2})$

Dividing the given equation by $(x-2)$

$$\begin{array}{c|cccc} 2 & 1 & 0 & 3 & -14 \\ & & 2 & 4 & 14 \\ \hline & 1 & 2 & 7 & \boxed{0} \end{array}$$

$\therefore$ Depressed equation is

$$x^2 + 2x + 7 = 0$$

$$x = -1 \pm 1\sqrt{6}$$

$$x = 2, -1, \pm 1\sqrt{6}$$

Section—(D)

Q. 8. Find solution of differential Equation

$$\frac{d^2y}{dx^2} + p\frac{dy}{dx} + (q+1)y = 0$$

Ans. Deduce that $\log(1+x) < x - \frac{x^2}{2} + \frac{x^3}{3}$ for $x > 0$

By Maclarian theorem with remainder R_3, we have

$$f(x) = f(0) + x\left[f^{I}(0) + \frac{x^2}{2!} f^{II}(0) + \frac{x^3}{3!} f^{III}(\theta)x\right]$$

$$f(x) = \log(1+x), \; f(0) = 0$$

$$f^{I}(x) = \frac{1}{1+x}, \; f^{I}(0) = 0$$

$$f^{II}(x) = -\frac{1}{1+x^2}, \; f^{II}(0) = -1$$

$$f^{III}(x) = \frac{2}{(1+x)^3} \quad f^{III}(\theta x) = \frac{2}{(1+\theta x)^2}$$

Substituting in equation (i), we get

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3(1+\theta x)^3}$$

Since $x > 0$ and $\theta > 0, \theta x > 0$

or $\theta > 0, \theta x > 0$

$$(1+\theta x)^3 > 1 \text{ i.e., } \frac{1}{(1+\theta x)^3} < 1$$

$$\therefore \quad x - x - \frac{x^2}{2} + \frac{x^3}{3(1+\theta x)^3} = x - \frac{x^2}{2} + \frac{x^3}{3}$$

$$\boxed{\log(1+x) < \left(x - \frac{x^2}{2} + \frac{x^3}{3}\right)} \quad \textbf{Ans.}$$

Q. 9. If $f(x) = \log(1 + x)$, $x > 0$ using Talyor theorem. Show that for $0 < \theta < 1$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3(1+\theta . x)^3}$$

Ans. Given equation is

$$\frac{d^2 y}{dx^2} + p\frac{dy}{dx} + (q+1)y = 0$$

$$\Rightarrow \quad [D^2 + p.D + (q+1)]\; Y = 0$$

Put $p = 4$ and $q = 3$

$$(D^2 + 4D + 4)Y = 0$$

$$D^2 + 4D + 4 = 0$$

$$(D+2)^2 = 0$$

$$D = -2, -2$$

$$\Rightarrow \quad Y = (C_1 x + C_2)e^{-2x}$$

$$\boxed{Y = x . e^{-2x}} \quad \textbf{Ans.}$$